



# **Cambridge International AS & A Level**

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## **FURTHER MATHEMATICS**

**9231/11**

Paper 1 Further Pure Mathematics 1

**May/June 2020**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 Let  $a$  be a positive constant.

- (a) Sketch the curve with equation  $y = \frac{ax}{x+7}$ . [2]

- (b) Sketch the curve with equation  $y = \left| \frac{ax}{x+7} \right|$  and find the set of values of  $x$  for which  $\left| \frac{ax}{x+7} \right| > \frac{a}{2}$ .  
[4]

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- 2 The cubic equation  $6x^3 + px^2 - 3x - 5 = 0$ , where  $p$  is a constant, has roots  $\alpha, \beta, \gamma$ .

(a) Find a cubic equation whose roots are  $\alpha^2, \beta^2, \gamma^2$ .

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(b) It is given that  $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$ .

(i) Find the value of  $p$ .

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- (ii) Find the value of  $\alpha^3 + \beta^3 + \gamma^3$ . [2]

- 3 The curve  $C$  has equation  $y = \frac{x^2}{2x+1}$ .

(a) Find the equations of the asymptotes of  $C$ .

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(b) Find the coordinates of the stationary points on  $C$ .

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(c) Sketch  $C$ .

[3]

- 4 (a) By first expressing  $\frac{1}{r^2 - 1}$  in partial fractions, show that

$$\sum_{r=2}^n \frac{1}{r^2 - 1} = \frac{3}{4} - \frac{an+b}{2n(n+1)},$$

where  $a$  and  $b$  are integers to be found.

[5]

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- (b) Deduce the value of  $\sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$ . [1]

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- (c) Find the limit, as  $n \rightarrow \infty$ , of  $\sum_{r=n+1}^{2n} \frac{n}{r^2 - 1}$ . [4]

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- 5 The lines  $l_1$  and  $l_2$  have equations  $\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$  and  $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{j} + 6\mathbf{k})$  respectively.

(a) Find the shortest distance between  $l_1$  and  $l_2$ .

[5]

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The plane  $\Pi$  contains  $l_1$  and is parallel to the vector  $\mathbf{i} + \mathbf{k}$ .

- (b) Find the equation of  $\Pi$ , giving your answer in the form  $ax + by + cz = d$ .

[4]

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- (c) Find the acute angle between  $l_2$  and  $\Pi$ .

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6 Let  $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ .

- (a) The transformation in the  $x$ - $y$  plane represented by  $\mathbf{A}^{-1}$  transforms a triangle of area  $30\text{ cm}^2$  into a triangle of area  $d\text{ cm}^2$ .

Find the value of  $d$ .

[3]

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- (b) Prove by mathematical induction that, for all positive integers  $n$ ,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}. \quad [5]$$

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- (c) The line  $y = 2x$  is invariant under the transformation in the  $x$ - $y$  plane represented by  $\mathbf{A}^n \mathbf{B}$ , where  
 $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}$ .

Find the value of  $n$ .

[5]

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- 7 The curve  $C_1$  has polar equation  $r = \theta \cos \theta$ , for  $0 \leq \theta \leq \frac{1}{2}\pi$ .

- (a) The point on  $C_1$  furthest from the line  $\theta = \frac{1}{2}\pi$  is denoted by  $P$ . Show that, at  $P$ ,

$$2\theta \tan \theta - 1 = 0$$

and verify that this equation has a root between 0.6 and 0.7.

[5]

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The curve  $C_2$  has polar equation  $r = \theta \sin \theta$ , for  $0 \leq \theta \leq \frac{1}{2}\pi$ . The curves  $C_1$  and  $C_2$  intersect at the pole, denoted by  $O$ , and at another point  $Q$ .

- (b) Find the polar coordinates of  $Q$ , giving your answers in exact form.

[2]

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(c) Sketch  $C_1$  and  $C_2$  on the same diagram.

[3]

(d) Find, in terms of  $\pi$ , the area of the region bounded by the arc  $OQ$  of  $C_1$  and the arc  $OQ$  of  $C_2$ . [7]

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**Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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