

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

763995814

FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

1 Let *a* be a positive constant.

(a) Sketch the curve with equation $y = \frac{ax}{x+7}$. [2]

(b) Sketch the curve with equation $y = \left| \frac{ax}{x+7} \right|$ and find the set of values of x for which $\left| \frac{ax}{x+7} \right| > \frac{a}{2}$.

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(a)	Find	d a cubic equation whose roots are α^2 , β^2 , γ^2 .]
(b)	It is	given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$.	
	(i)	Find the value of p .	1

	••••••				
			 	 	•••••
	of $\alpha^3 + \beta^3 + \gamma$	3.			[2
Find the value o					
Find the value o			 	 	•••••
ind the value c				 	
ind the value c					
Find the value c					
Find the value c					
Find the value o					

	e curve C has equation $y = \frac{x^2}{2x+1}$.	
(a)	Find the equations of the asymptotes of <i>C</i> .	[3
		•••••
(b)	Find the coordinates of the stationary points on C .	[3
(b)	Find the coordinates of the stationary points on <i>C</i> .	[3
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(b)	Find the coordinates of the stationary points on <i>C</i> .	
(b)		

(c) Sketch *C*. [3]

4 (a) By first expressing $\frac{1}{r^2-1}$ in partial fractions, show that

$$\sum_{r=2}^{n} \frac{1}{r^2 - 1} = \frac{3}{4} - \frac{an + b}{2n(n+1)},$$

where a and b are integers to be found.	[5]

١	Deduce the value of $\sum_{r=1}^{\infty}$	r^2	1.				
		•••••	••••••		 	 	
				•••••	 	 •••••	•••••
		•••••			 	 	
				•••••	 •••••	 	
		•••••			 	 	
			2n	n			
	Find the limit, as $n \rightarrow$	∞ , of	$\sum_{n=+1}^{\infty} \frac{1}{r^2}$	$\frac{n}{-1}$.			
	Find the limit, as $n \rightarrow$	∞, of	$\sum_{n=+1}^{\infty} \frac{1}{r^2}$	<u>" – 1</u> .	 	 	
	Find the limit, as $n \rightarrow$	∞, of	$\sum_{n=+1}^{\infty} \frac{1}{r^2}$	<u>"</u> - 1 · · · · · · · · · · · · · · · · · ·	 	 	
	Find the limit, as $n \rightarrow$	∞, of	$\sum_{n=+1}^{\infty} \frac{1}{r^2}$	<u>-1</u> .			
	Find the limit, as $n \rightarrow$	∞, of 	$\sum_{n=+1}^{\infty} \frac{r^2}{r^2}$	" -1.		 	
	Find the limit, as $n \rightarrow$	∞, of r	$\sum_{n=1}^{\infty} \frac{1}{r^2}$	" -1 · · · · · · · · · · · · · · · · · ·			
	Find the limit, as $n \rightarrow$	∞, of 	$\sum_{n=1}^{\infty} \frac{r^2}{r^2}$	<u>"</u> -1.			
	Find the limit, as $n \rightarrow$	∞, of r	$\sum_{n=1}^{\infty} \frac{1}{r^2}$	" –1.			
	Find the limit, as $n \rightarrow$						

The lines l_1 and l_2 have equations $\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{j} + 6\mathbf{k})$

I	Find the shortest distance between l_1 and l_2 .
•	
•	
•	

The plane Π contains l_1 and is parallel to the vector $\mathbf{i} + \mathbf{k}$. **(b)** Find the equation of Π , giving your answer in the form ax + by + cz = d. [4] (c) Find the acute angle between l_2 and Π . [3]

(a)	The transformation in the x - y plane represented triangle of area $d \text{ cm}^2$.	by A^{-1} transforms a triangle of area 30 cm ²
	Find the value of d .	
(b)	Prove by mathematical induction that, for all po	ositive integers n ,
()	$\mathbf{A}^n = \begin{pmatrix} 2^n \\ 2^n - \end{pmatrix}$	
	\2"-	1 1/

$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}.$	
Find the value of n .	

- 7 The curve C_1 has polar equation $r = \theta \cos \theta$, for $0 \le \theta \le \frac{1}{2}\pi$.
 - (a) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

 $2\theta \tan \theta - 1 = 0$

and verify that this equation has a root between 0.6 and 0.7.	
curve C_2 has polar equation $r = \theta \sin \theta$, for $0 \le \theta \le \frac{1}{2}\pi$. The curves C_1 and denoted by O , and at another point Q .	and C_2 intersect a
Find the polar coordinates of Q , giving your answers in exact form.	

(c)	Sketch C_1 and C_2 on the same diagram.	[3]
(d)	Find, in terms of π , the area of the region bounded by the arc OQ of C_1 and the arc OQ of C_2	. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any must be clearly shown.	

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