

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

2388089987

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

()	Prove by induction that $u_n = 2^n - 1$ for all positive integers n .	[5]
h)		•••••
	Deduce that u is divisible by u for $n \ge 1$	[2
(~)	Deduce that u_{2n} is divisible by u_n for $n \ge 1$.	[2]
(~)		
(~)	Deduce that u_{2n} is divisible by u_n for $n \ge 1$.	

J	Use standard results from the List of Formulae (MF19) to show that $S_n = \frac{4}{3}n(4n^2 - 1)$.
•	
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Express $\frac{n}{S_n}$ in partial fra	ctions and find $\sum_{n=1}^{\infty}$	$\frac{n}{S_n}$ in terms of N .	[4
Deduce the value of $\sum_{n=1}^{\infty}$	$\frac{n}{S}$.		[1
n=1			

4 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a)	Show that A is non-singular.	[3]

The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}$.

(b)	Find the value of k .	[3]

Find the equations of the invariant lines, through the origin, of the transferesented by CAB .	[3]

[2]

The curve C has polar equation $r = a \tan \theta$, where a is a positive constant and $0 \le \theta \le \frac{1}{4}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole.

5

						1	
b)	Find the exact value of	f the area o	f the region	bounded by (C and the half-	-line $\theta = \frac{1}{4}\pi$.	[4

	C has Cartesia	an equation)	$\sqrt{-\sqrt{a^2-x^2}}$	•		
•••••	•••••				••••••	
					•••••	
			•••••			
Using your	answer to pa	urt (b), deduce	e the exact va	alue of $\int_0^{\frac{1}{2}a\sqrt{2}}$	$\frac{x^2}{\sqrt{a^2 - x^2}} \mathrm{d}x.$	
Using your	answer to pa	urt (b), deduce	e the exact va	alue of $\int_0^{\frac{1}{2}a\sqrt{2}}$	$\frac{x^2}{\sqrt{a^2 - x^2}} \mathrm{d}x.$	
Using your	answer to pa	nrt (b), deduc	e the exact va	alue of $\int_0^{\frac{1}{2}a\sqrt{2}}$	$\frac{x^2}{\sqrt{a^2 - x^2}} \mathrm{d}x.$	
Using your	answer to pa	urt (b), deduc	e the exact va	alue of $\int_0^{\frac{1}{2}a\sqrt{2}}$	$\frac{x^2}{\sqrt{a^2 - x^2}} \mathrm{d}x.$	

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(a)	Find the equations of the asymptotes of <i>C</i> .	[3
(b)	Show that C has no turning points.	[:
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(b)		

(c) Sketch *C*, stating the coordinates of the intersections with the axes.

[3]

Sketch the curve with equation $y = \left \frac{10 + x - 3}{2x - 3} \right $	$\frac{2x^2}{3}$ and find in exact form the set of values of x for
which $\left \frac{10 + x - 2x^2}{2x - 3} \right < 4$.	$\left \frac{2x^2}{3} \right $ and find in exact form the set of values of x for [6]

a)		[4]
b)	Find the distance between l_2 and Π .	[3]
		•••••

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The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

Silow that I	has position	27	27 K un	a state a veet	or equation is	<i>n 1 Q</i> .	[8]
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Additional Page

If you use the following lined page to complete the answer(s) to any must be clearly shown.	

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