

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

699578043

FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10 \,\mathrm{m\,s^{-2}}$.

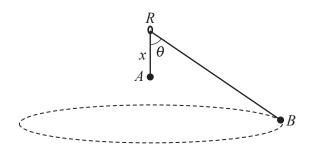
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

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Find, in terms of <i>u</i> , the speed of <i>P</i> at time $\frac{2}{3}T$ after projection.		
	Find, in terms of u , the speed of P at time $\frac{2}{3}T$ after projection.	



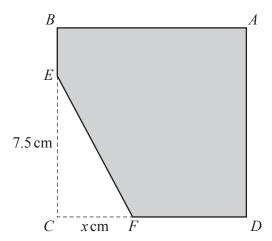
A light inextensible string of length a is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass 3m. The other end of the string is attached to a particle B of mass B. The particle B hangs in equilibrium at a distance B vertically below the ring. The angle between B and B is B (see diagram). The particle B moves in a horizontal circle with constant angular speed B?

Show that $\cos \theta = \frac{1}{3}$ and find x in terms of a.	[5]

fixe	end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a point A . The other end of the spring is attached to a particle P of mass m . The spring hangs with P ically below A . The particle P is released from rest in the position where the extension of the spring a .
(a)	Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

•••••

(a)



A uniform square lamina ABCD has sides of length 10 cm. The point E is on BC with EC = 7.5 cm, and the point F is on DC with CF = x cm. The triangle EFC is removed from ABCD (see diagram). The centre of mass of the resulting shape ABEFD is a distance \overline{x} cm from CB and a distance \overline{y} cm from CD.

Show that $\overline{x} = \frac{400 - x^2}{80 - 3x}$ and find a corresponding expression for \overline{y} .	[4]
	•••••

The shape ABEFD is in equilibrium in a vertical plane with the edge DF resting on a smooth horizontal surface.

constants to be determine	ined.				[3
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A particle P is moving along a straight line with acceleration 3ku - kv where v is its velocity at time t,

1)	Find the time taken for P to achieve a velocity of $2u$.	[3]

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strik with	article P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle area a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α in the barrier. The coefficient of restitution between P and the barrier is e . As a result of the impact direction of motion of P is turned through 90° .
(a)	Show that $\tan^2 \alpha = \frac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

A hollow cylinder of radius a is fixed with its axis horizontal. A particle P, of mass m, moves in part of a vertical circle of radius a and centre O on the smooth inner surface of the cylinder. The speed of P when it is at the lowest point A of its motion is $\sqrt{\frac{7}{2}ga}$.

The particle P loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical through O.

Show that $\theta = 60^{\circ}$.	

Additional Page

If you use the following lined page to complete the answer(s) to any must be clearly shown.	

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