

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER				CANDIDATE NUMBER		

5 8 5 0 0 5 4 7 6 3

FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has 12 pages. Blank pages are indicated.

1 Two randomly selected groups of students, with similar ranges of abilities, take the same examination in different rooms. One group of 140 students takes the examination with background music playing. The other group of 210 students takes the examination in silence. Each student is awarded a grade for their performance in the examination and the numbers from each group gaining each grade are shown in the following table.

		Grade awarded							
	A B C								
Background music	49	51	40						
Silence	93	68	49						

Test at the 10% significance level whether grades awarded are independent of whether back music is playing during the examination.	cground [6]

2

The times, in milliseconds, taken by a computer to perform a certain task were recorded on 10 randomly chosen occasions. The times were as follows.

	ilcoxon	signed-	rank tes	t at the 5	% signit	ficance l	evel to te	est this claim.	
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3	The continuous	random	variable X ha	s probability	density	v function	f given	by

$$f(x) = \begin{cases} \frac{3}{16} (2 - \sqrt{x}) & 0 \le x < 1, \\ \frac{3}{16\sqrt{x}} & 1 \le x \le 9, \\ 0 & \text{otherwise.} \end{cases}$$

a)	Find $E(X)$.	[3]

The random variable *Y* is such that $Y = \sqrt{X}$.

Find the probability density function of <i>Y</i> .	[5]
	•••••

4	A company has two different machines, X and Y, each of which fills empty cups with coffee. The
	manager is investigating the volumes of coffee, x and y, measured in appropriate units, in the cups filled
	by machines X and Y respectively. She chooses a random sample of 50 cups filled by machine X and a
	random sample of 40 cups filled by machine <i>Y</i> . The volumes are summarised as follows.

$$\sum x = 15.2$$
 $\sum x^2 = 5.1$ $\sum y = 13.4$ $\sum y^2 = 4.8$

The manager claims that there is no difference between the mean volume of coffee in cups filled by machine X and the mean volume of coffee in cups filled by machine Y.

Test the manager's claim at the 10% significance level.	[9]

5

A large number of children are competing in a throwing competition. The distances, in metres, thrown by a random sample of 8 children are as follows.

				1	9.8	22	2.1	24.	4	21.3	5	20.8	3	26.3	,	23.7		25.0					
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(h)	Find a 95% confidence interval for the population mean distance thrown.	[3
(<i>D)</i>	i ind a 7370 confidence interval for the population inean distance unlown.	Į-

)	Find the probability generating function, $G_X(t)$, of X .	[2]
	Is obtained by Rassa, is $G_{\gamma}(t)$. The coefficient of t in $G_{\gamma}(t)$ is $\frac{7}{12}$.	
	$\Gamma:=1$ $C:=A$	[2]
)	Find $G_{\gamma}(t)$.	[3]
)	Find $G_{\gamma}(t)$.	[3]
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)	Find $G_{\gamma}(t)$.	[3]
)	Find G _Y (t).	[3]
)	Find G _y (t).	[3]
)	Find G _y (t).	[3]

The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

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Additional Page

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