

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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### **FURTHER MATHEMATICS**

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages.

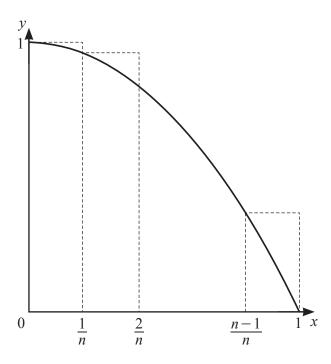
(a)	Use standard results from the list of formulae (MF19) to find the Maclaurin's series for $y$ in term of $x$ up to and including the term in $x^4$ .	ns 2]
		•••
(b)	Deduce the value of $\frac{d^4y}{dx^4}$ when $x = 0$ .	
	dx <sup>+</sup>	
		• • •
(c)	Use your answer to part (a) to find an approximation to $\int_0^{\frac{1}{2}} y  dx$ , giving your answer as a ration fraction in its lowest terms.	 al 2]
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$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4x^3y}{x^4 + 5} = 6x$$

for which $y = 1$ when $x = 1$ . Give your answer in the form $y = f(x)$ .	[7]

3



The diagram shows the curve with equation  $y = 1 - x^2$  for  $0 \le x \le 1$ , together with a set of *n* rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of the rectangles, show that

	$\int_0^1 (1-x^2)^{-x^2}$	$^{2})\mathrm{d}x < \frac{4n^{2}-6}{6}$	$\frac{+3n-1}{6n^2}$ .	[4]
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(b)	Use a similar method to find, in terms of $n$ , a lower bound for $\int_0^1 (1-x^2) dx$ . [4]

		•••••
(b)	Use de Moivre's theorem to show that $\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1$ .	[4
		•••••
		•••••

(c)	Use the results	of parts (a) and	d (b) to express	each real root of	of the equation
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in the form $\cos k\pi$ , where $k$ is a rational number.	[4]

5 The curve C has parametric equations

$$x = 3t + 2t^{-1} + at^3$$
,  $y = 4t - \frac{3}{2}t^{-1} + bt^3$ , for  $1 \le t \le 2$ ,

where a and b are constants.

(a)	It is given	that $a = \frac{2}{3}$	and $b = -\frac{1}{2}$ .
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Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{25}{4}(t^2 + t^{-2})^2$ and find the exact length of $C$ .	[6]

It is given instead that $a = b = 0$ .
Find the value of $\frac{d^2y}{dx^2}$ when $t = 1$ .

6	The	matrix	P	is	given	by

$$\mathbf{P} = \begin{pmatrix} 1 & 6 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & -3 \end{pmatrix}.$$

Use the characteristic equation of <b>P</b> to find $\mathbf{P}^{-1}$ .	[:

<b>(b)</b>	Find the matrix <b>A</b> such that
	$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}.$ [4]
(c)	State the eigenvalues and corresponding eigenvectors of $\mathbf{A}^3$ . [2]

7 It is given that  $y = x^2 w$  and

$$x^{2} \frac{d^{2} w}{dx^{2}} + 4x(x+1) \frac{dw}{dx} + (5x^{2} + 8x + 2) w = 5x^{2} + 4x + 2.$$

(a)	Show that $\frac{d^{2}y}{dx^{2}} + 4\frac{dy}{dx} + 5y = 5x^{2} + 4x + 2.$ [4]

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(a)	Starting from the definitions of tanh and sech in terms of exponentials, prove that	
	$1 - \tanh^2 x = \operatorname{sech}^2 x.$	[3]
<b>a</b> .)		
(b)	Using the substitution $u = \tanh x$ , or otherwise, find $\int \operatorname{sech}^2 x \tanh^2 x  dx$ .	[2]
It is	given that, for $n \ge 0$ , $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \tanh^2 x  dx$ .	•••••
	Show that, for $n \ge 2$ ,	
	$(n+1)I_n = \left(\frac{4}{5}\right)^3 \left(\frac{3}{5}\right)^{n-2} + (n-2)I_{n-2}.$	[5]
	[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$ .]	
		••••••

Find the value of $I_4$ . [3]

(d)

## **Additional Page**

If you use the following fined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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