

### **Cambridge Assessment International Education**

Cambridge International General Certificate of Secondary Education

#### **ADDITIONAL MATHEMATICS**

0606/11

Paper 1 May/June 2019

MARK SCHEME
Maximum Mark: 80



This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

### **Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

#### GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

#### **GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always whole marks (not half marks, or other fractions).

#### **GENERIC MARKING PRINCIPLE 3:**

#### Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit
  is given for valid answers which go beyond the scope of the syllabus and mark scheme,
  referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these
  features are specifically assessed by the question as indicated by the mark scheme. The
  meaning, however, should be unambiguous.

### **GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

#### **GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

#### GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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#### MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

#### Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

#### **Abbreviations**

awrt answers which round to cao correct answer only

dep dependent

FT follow through after error isw ignore subsequent working nfww not from wrong working

oe or equivalent

rot rounded or truncated

SC Special Case soi seen or implied

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Question	Answer	Marks	Guidance
1(a)	E	B1	
	E	В1	
1(b)	R P	B2	<b>B1</b> for $P \subset R$ and $Q \subset R$ <b>B1</b> for $P \cap Q = \emptyset$
2(i)	4	B1	
2(ii)	$120^{\circ} \text{ or } \frac{2\pi}{3}$	B1	
2(iii)	2 2 2 30 30 30 30 40 50 40 50 41	В3	<b>B1</b> for a complete curve starting at $(-90^{\circ}, 3)$ and finishing at $(90^{\circ}, -5)$ <b>B1</b> for $-5 \le y \le 3$ for a complete curve Minimum point(s) at $y = -5$ Maximum point(s) at $y = 3$ <b>DepB1</b> for a fully correct sine curve satisfying both the above and passing through $(-60^{\circ}, -1), (0^{\circ}, -1)$ and $(60^{\circ}, -1)$
3(i)	-12	B1	
3(ii)	$(2 \times -3 - 1)(k - 3) - 12 = 23$ oe or $2(-3)^2 + (2k - 1)(-3) - k - 12 = 23$	M1	
	k = -2	A1	

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Question	Answer	Marks	Guidance
3(iii)	$(2x-1)(x-2)-12 = -25$ $2x^2 - 5x + 15 = 0$	M1	expansion and simplification to a 3 term quadratic equation equated to zero, using <i>their k</i> .
	Discriminant: $25 - (4 \times 2 \times 15)$ = -95	M1	using discriminant for their three term quadratic equation
	which is < 0 so no real solutions	A1	cao for correct discriminant and correct conclusion
4(i)	<i>a</i> = 256	B1	
	$8 \times 2^7 \times bx [= 256x]  \text{oe}$	M1	
	or $\frac{8 \times 7 \times 2^6 \times (bx)^2}{2} \left[ = cx^2 \right]$ oe		
	$b = \frac{1}{4}$ oe, $c = 112$	A2	A1 for each
4(ii)	$\left(256 + 256x + 112x^{2}\right)\left(4x^{2} - 12 + \frac{9}{x^{2}}\right)$	B1	$for \left(4x^2 - 12 + \frac{9}{x^2}\right)$
	Terms independent of x are $(256 \times (-12)) + (112 \times 9)$ $= -3072 + 1008$	M1	adding and selecting $(their256 \times their(-12)) + (their112 \times their9)$
	=-2064	A1	
5(i)	$v = 20 \times \frac{1}{\sqrt{3^2 + 4^2}} \binom{3}{4}$ oe	M1	finding and using the magnitude of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
	$v = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$	A1	
5(ii)	$\mathbf{r}_p = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ 16 \end{pmatrix} t$	M1	correct use of position vector and <i>their</i> velocity vector
		A1	

Question	Answer	Marks	Guidance
5(iii)		M1	equating position vectors of both particles at time <i>t</i> and solve either equation for <i>t</i>
	t=4	A1	
	Position vector of collision $\begin{pmatrix} 49 \\ 66 \end{pmatrix}$	A1	
6	Method 1		
	$3x^2 - 2x + 1 = 2x + 5$ leading to	M1	equating the equations of the line and the curve and rearranging to obtain a three term quadratic equated to zero
	$3x^2 - 4x - 4 = 0$	A1	
	$x = -\frac{2}{3} \text{ and } x = 2$	A1	
	$\int_{-\frac{2}{3}}^{2} \left(2x + 5 - \left(3x^2 - 2x + 1\right)\right) dx$	M1	subtraction (either way round)
	$\int_{-\frac{2}{3}}^{2} \left(4 + 4x - 3x^2\right)  \mathrm{d}x$	M1	integration to $Ax + Bx^2 + Cx^3$
	$\left[4x + 2x^2 - x^3\right]_{-\frac{2}{3}}^2$	A1	for $4x + 2x^2 - x^3$ oe
	$(8+8-8) - \left(-\frac{8}{3} + \frac{8}{9} + \frac{8}{27}\right)$ $= 8\frac{40}{27}$	M1	Dep on preceding M1 correct use of limits
	$= \frac{256}{27} \text{ or } 9.48 \text{ or } 9\frac{13}{27}$	A1	

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Question	Answer	Marks	Guidance
6	$\frac{\text{Method 2}}{3x^2 - 2x + 1} = 2x + 5$ leading to	М1	equating the line and the curve and rearranging to obtain a three term quadratic equated to zero
	$3x^2 - 4x - 4 = 0$	A1	
	$x = -\frac{2}{3} \text{ and } x = 2$	A1	
	Area of trapezium = $\frac{1}{2} \left( \frac{11}{3} + 9 \right) \times \frac{8}{3}$	B1	area of the trapezium, allow unsimplified
	Area under curve = $\int_{-\frac{2}{3}}^{2} 3x^2 - 2x + 1  dx$	M1	integration to $Ax + Bx^2 + Cx^3$
	$= \left[ x^3 - x^2 + x \right]_{-\frac{2}{3}}^2$	A1	for $x^3 - x^2 + x$
	$= \left( (8-4+2) - \left( -\frac{8}{27} - \frac{4}{9} - \frac{2}{3} \right) \right)$ $6\frac{38}{27}$	M1	DepM1 for correct use of limits.
	Shaded Area = $\frac{152}{9} - \frac{200}{27}$ = $\frac{256}{27}$ or 9.48 or $9\frac{13}{27}$	A1	
7(a)	27 27 Method 1		
. (4)	$\log_3 x + \frac{\log_3 x}{\log_3 9} = 12$	B1	change to base 3 logarithm
	$\frac{3\log_3 x}{2} = 12$ $x = 3^8 \text{ or } \sqrt[3]{3^{24}}$	M1	simplification and dealing with base 3 logarithms to obtain a power of 3
	x = 6561	A1	

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Question	Answer	Marks	Guidance
7(a)	$\frac{\text{Method 2}}{\log_9 x} + \log_9 x = 12$	B1	change to base 9
	$3\log_9 x = 12$ $x = 9^4 \text{ or } \sqrt[3]{9^{12}}$	M1	simplification and dealing with base 9 logarithms to obtain a power of 9
	x = 6561	A1	
7(b)	Method 1 $\log_4 (3y^2 - 10) = \log_4 (y - 1)^2 + \frac{1}{2}$	В1	use of power rule
	$\log_4 \frac{3y^2 - 10}{\left(y - 1\right)^2} = \frac{1}{2}$	B1	<b>DepB1</b> for use of division rule
	$\frac{3y^2 - 10}{\left(y - 1\right)^2} = 2$	B1	for $\frac{1}{2} = \log_4 2$
	$y^2 + 4y - 12 = 0$	M1	<b>Dep</b> on first two B marks simplification to a three term quadratic.
	y = 2 only	A1	
7(b)	Method 2 $\log_4 (3y^2 - 10) = \log_4 (y - 1)^2 + \frac{1}{2}$	B1	use of power rule
	$\log_4(3y^2 - 10) = \log_4(y - 1)^2 + \log_4 2$	B1	for log <sub>4</sub> 2
	$3y^2 - 10 = 2(y - 1)^2$	B1	Dep on first B1 use of the multiplication rule
	$y^2 + 4y - 12 = 0$	M1	<b>Dep</b> on first and third B marks. simplification to a 3 term quadratic
	y = 2 only	A1	

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Question	Answer	Marks	Guidance
8(i)	f>-1	B1	or $f(x) > -1$ , $y > -1$ , $(-1, \infty)$ , $\{y: y > -1\}$
8(ii)	$e^{y} = \frac{x+1}{5} \text{ oe}$	M1	a complete valid method to obtain the inverse function
	$y = \ln\left(\frac{x+1}{5}\right)$ or $f^{-1}(x) = \ln\left(\frac{x+1}{5}\right)$ oe	A1	
	Domain $x > -1$ or $(-1, \infty)$	B1	FT their (i) or correct
8(iii)	g(1) = 5  so  fg(1) = f(5)	M1	evaluation using correct order of operations
	$5e^5 - 1 = 741$	A1	awrt 741 or 5e <sup>5</sup> –1
8(iv)	$g^{2}(x) = (x^{2} + 4)^{2} + 4$	M1	correct use of g <sup>2</sup>
	$x^{4} + 8x^{2} + 16 + 4 = 40$ $(x^{2} + 4)^{2} = 36$ or $x^{4} + 8x^{2} - 20 = 0$ $(x^{2} + 10)(x^{2} - 2) = 0$	M1	<b>DepM1</b> for forming and solving a quadratic in $x^2$
	$x = \pm \sqrt{2}$ only	A1	
9(i)	Method 1		
	$600\pi = 2\pi r^2 + 2\pi rh$	B1	
	$h = \frac{600\pi - 2\pi r^2}{2\pi r}$	M1	making <i>h</i> subject from a two term expression for SA.
	$V = \pi r^2 h$ $V = \pi r^2 \left( \frac{600\pi - 2\pi r^2}{2\pi r} \right)$ $V = \pi r^2 \left( \frac{300}{r} - r \right)$ $V = 300\pi r - \pi r^3$	A1	correct substitution and manipulation to obtain given answer

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Question	Answer	Marks	Guidance
9(i)	Method 2		
	$600\pi = 2\pi r^2 + 2\pi rh$	B1	
	$600\pi r = 2\pi r^3 + 2\pi r^2 h$	M1	multiplying both sides by r
	$\frac{600\pi r - 2\pi r^3}{2} = \pi r^2 h$ $V = \pi r^2 h$ $V = 300\pi r - \pi r^3$	A1	correct manipulation to obtain $\pi r^2 h$
9(ii)	$\frac{\mathrm{d}V}{\mathrm{d}r} = 300\pi - 3\pi r^2$	M1	differentiation of given formula to $A+Br^2$
	When $\frac{dV}{dr} = 300\pi - 3\pi r^2 = 0$	M1	equating to zero and attempt to solve
	r = 10	A1	
	$V = 2000\pi$ or 6280 or 6283	A1	
	$\frac{d^2V}{dr^2} = -6\pi r ,  \frac{d^2V}{dr^2} < 0$ so maximum	B1	cao for $\frac{d^2V}{dr^2} = -6\pi r$ , $\frac{d^2V}{dr^2} = -60\pi$ or other correct method leading to maximum
10(i)	Method 1		
	$\lg y = A + Bx^2$	B1	statement soi
	16 = A + 6B $4 = A + 2B$	M1	one correct equation
	leading to $A = -2$ and $B = 3$	A2	A1 for each
10(i)	Method 2		
	$\lg y = A + Bx^2$	B1	statement soi
	Gradient = $B$ B = 3	B1	
	16 = A + 6B or $4 = A + 2B$	M1	a correct equation
	A = -2	A1	

Question	Answer	Marks	Guidance
10(i)	Method 3 $\lg y - 4 = 3(x^2 - 2)$ or $\lg y - 16 = 3(x^2 - 6)$	M1	correct equation or for correct method for finding constant.
	OR $4 = 3(2) + c$ or $16 = 3(6) + c$		
	$\lg y = A + Bx^2$	B1	statement soi by their A and B
	Hence $y = 10^{3x^2 - 2}$ B = 3	B1	
	A = -2	A1	
10(ii)	$y = 10^{-2+3\left(\frac{1}{\sqrt{3}}\right)^2}$	M1	correct use of <i>their A</i> and <i>B</i>
	y = 0.1 oe	A1	
10(iii)	$2 = 10^{3x^2 - 2}$	M1	correct use of their A and B
	$     \lg 2 = 3x^2 - 2 \\     x = \sqrt{\frac{\lg 2 + 2}{3}} $	M1	complete correct method to solve for x
	x = 0.876	A1	

Question	Answer	Marks	Guidance
11(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 + 1)(2x - 3)^{-\frac{1}{2}} + 2x(2x - 3)^{\frac{1}{2}}$	M1	differentiation of a product
		B1	for $\frac{d}{dx}(2x-3)^{\frac{1}{2}} = \frac{1}{2} \times 2(2x-3)^{-\frac{1}{2}}$ oe
		A1	all else correct i.e. $\frac{dy}{dx} = (x^2 + 1)f(x) + 2x(2x - 3)^{\frac{1}{2}}$
	$= (2x-3)^{-\frac{1}{2}} (x^2+1+2x(2x-3))$	M1	correctly taking out a factor of $(2x-3)^{-\frac{1}{2}}$ or correctly using $(2x-3)^{\frac{1}{2}}$ as denominator
	$=\frac{5x^2 - 6x + 1}{(2x - 3)^{\frac{1}{2}}}$	A1	denominator
11(ii)	When $x = 2$ , $y = 5$	B1	
	$\frac{dy}{dx} = 9$ , so gradient of normal = $-\frac{1}{9}$	M1	substitution to obtain gradient and correct method for gradient of normal
	Equation of normal $y-5=-\frac{1}{9}(x-2)$	M1	DepM1 for equation of normal
	x+9y-47=0 or $-x-9y+47=0$	A1	Must be in this form