Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME				
CENTRE NUMBER			CANDIDATE NUMBER	

* 7 9 5 9 7 8 1 4 4 2 *

FURTHER MATHEMATICS **9231/12**

Paper 1 **October/November 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **26** printed pages and **2** blank pages.

JC18 11_9231_12/RP

CAMBRIDGE International Examinations

1 The roots of the cubic equation

$$x^3 - 5x^2 + 13x - 4 = 0$$

are α, β, γ.

(i) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [3]

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

2 It is given that

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(i) Find the eigenvalue of $\mathbf{A}$ corresponding to the eigenvector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. [1]

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(ii) Write down the negative eigenvalue of $\mathbf{A}$ and find a corresponding eigenvector. [3]

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(iii) Find an eigenvalue and a corresponding eigenvector of the matrix $\mathbf{A} + \mathbf{A}^6$. [2]

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3 The curve C has polar equation $r = a\cos 3\theta$, for $-\frac{1}{6}\pi \leqslant \theta \leqslant \frac{1}{6}\pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C, showing full working. [3]

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(iii) Using the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, find a cartesian equation of C. [3]

4 **(i)** Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 4\sin t.$$ [7]

(ii) State an approximate solution for large positive values of t. [1]

5 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 6 & 5 & -1 & 3 \\ 9 & 8 & -2 & 5 \\ -3 & -2 & 0 & -1 \end{pmatrix}.$$

(i) Find the rank of $\mathbf{M}$. [3]

Let K be the null space of T.

(ii) Find a basis for K. [3]

(iii) Find the general solution of

$$\mathbf{Mx} = \begin{pmatrix} 2 \\ 5 \\ 8 \\ -2 \end{pmatrix}.$$ [3]

6 It is given that $y = e^x u$, where u is a function of x. The rth derivatives $\frac{d^r y}{dx^r}$ and $\frac{d^r u}{dx^r}$ are denoted by $y^{(r)}$ and $u^{(r)}$ respectively. Prove by mathematical induction that, for all positive integers n,

$$y^{(n)} = e^x\left(\binom{n}{0}u + \binom{n}{1}u^{(1)} + \binom{n}{2}u^{(2)} + \dots + \binom{n}{r}u^{(r)} + \dots + \binom{n}{n}u^{(n)}\right).$$ [8]

[You may use without proof the result $\binom{k}{r} + \binom{k}{r-1} = \binom{k+1}{r}$.]

7 Let

$$S_N = \sum_{r=1}^{N}(3r+1)(3r+4) \quad \text{and} \quad T_N = \sum_{r=1}^{N} \frac{1}{(3r+1)(3r+4)}.$$

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = N(3N^2 + 12N + 13).$$ [3]

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(ii) Use the method of differences to show that

$$T_N = \frac{1}{12} - \frac{1}{3(3N+4)}.$$ [3]

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(iii) Deduce that $\frac{S_N}{T_N}$ is an integer. [2]

(iv) Find $\lim_{N\to\infty} \frac{S_N}{N^3 T_N}$. [2]

8 **(i)** By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^6$, where $z = \cos\theta + \mathrm{i}\sin\theta$, express $\cos^6\theta$ in the form

$$\tfrac{1}{32}(p + q\cos 2\theta + r\cos 4\theta + s\cos 6\theta),$$

where p, q, r and s are integers to be determined. [6]

(ii) Hence find the exact value of

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos^6\left(\tfrac{1}{2}x\right)\,\mathrm{d}x.$$ [4]

9 The curve C has equation

$$y = \frac{5x^2 + 5x + 1}{x^2 + x + 1}.$$

(i) Find the equation of the asymptote of C. [2]

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(ii) Show that, for all real values of x, $-\frac{1}{3} \leqslant y < 5$. [4]

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(iii) Find the coordinates of any stationary points of C. [2]

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(iv) Sketch C, stating the coordinates of any intersections with the y-axis. [2]

10 The position vectors of the points A, B, C, D are

$$\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, \qquad -\mathbf{i} + 3\mathbf{k}, \qquad m\mathbf{j} + 4\mathbf{k},$$

respectively, where m is a constant.

(i) Show that the lines AB and CD are parallel when $m = \frac{3}{2}$. [1]

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(ii) Given that $m \neq \frac{3}{2}$, find the shortest distance between the lines AB and CD. [5]

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(iii) When $m = 2$, find the acute angle between the planes ABC and ABD, giving your answer in degrees. [6]

11 Answer only **one** of the following two alternatives.

EITHER

The curve C is defined parametrically by

$$x = 18t - t^2 \quad \text{and} \quad y = 8t^{\frac{3}{2}},$$

where $0 < t \leqslant 4$.

(i) Show that at all points of C,

$$\frac{d^2y}{dx^2} = \frac{3(9+t)}{2t^{\frac{1}{2}}(9-t)^3}.$$ [4]

(ii) Show that the mean value of $\frac{d^2y}{dx^2}$ with respect to x over the interval $0 < x \leqslant 56$ is $\frac{3}{70}$. [4]

(iii) Find the area of the surface generated when C is rotated through 2π radians about the x-axis, showing full working. [6]

OR

Let $I_n = \int_1^{\sqrt{2}} (x^2 - 1)^n \, dx$.

(i) Show that, for $n \geqslant 1$,

$$(2n + 1)I_n = \sqrt{2} - 2nI_{n-1}.$$ [5]

(ii) Using the substitution $x = \sec\theta$, show that

$$I_n = \int_0^{\frac{1}{4}\pi} \tan^{2n+1}\theta \sec\theta \, d\theta.$$ [4]

(iii) Deduce the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sin^7\theta}{\cos^8\theta}\,\mathrm{d}\theta.$$

[5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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