Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | |

CANDIDATE NUMBER | | | |

5678150535

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.

JC18 06_9231_13/2R

CAMBRIDGE International Examinations

[Turn over

BLANK PAGE

1 The variables x and y are such that $y = -1$ when $x = 0$ and

$$\left(x + \frac{dy}{dx}\right)^3 = y^2 + x.$$

(i) Find the value of $\frac{dy}{dx}$ when $x = 0$. [1]

....

....

....

(ii) Find also the value of $\frac{d^2y}{dx^2}$ when $x = 0$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

2 **(i)** Verify that

$$\frac{n(e-1)+e}{n(n+1)e^{n+1}} = \frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}}.$$ [1]

Let $S_N = \sum_{n=1}^{N} \frac{n(e-1)+e}{n(n+1)e^{n+1}}$.

(ii) Express S_N in terms of N and e. [2]

Let $S = \lim_{N \to \infty} S_N$.

(iii) Find the least value of N such that $(N+1)(S - S_N) < 10^{-3}$. [3]

3 **(i)** Use de Moivre's theorem to show that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$ [3]

(ii) Hence find all the roots of the equation

$$x^4 - 6x^2 + 1 = 0$$

in the form $\tan q\pi$, where q is a positive rational number. [5]

4 The curve C has equation

$$y = \frac{x^2 + 7x + 6}{x - 2}.$$

(i) Find the coordinates of the points of intersection of C with the axes. [2]

(ii) Find the equation of each of the asymptotes of C. [3]

(iii) Sketch C. [3]

5 It is given that $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{A}$ with corresponding eigenvalue λ.

(i) Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ and state the corresponding eigenvalue. [3]

....

....

....

....

....

....

....

....

....

It is given that

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}.$$

(ii) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{A}^3 + \mathbf{I} = \mathbf{PDP}^{-1},$$

where $\mathbf{I}$ is the 2×2 identity matrix. [5]

....

....

....

....

....

....

....

....

....

....

6 The equation

$$9x^3 - 9x^2 + x - 2 = 0$$

has roots α, β, γ.

(i) Use the substitution $y = 3x - 1$ to show that $3\alpha - 1$, $3\beta - 1$, $3\gamma - 1$ are the roots of the equation

$$y^3 - 2y - 7 = 0.$$ [2]

....

....

....

....

....

....

....

....

....

....

The sum $(3\alpha - 1)^n + (3\beta - 1)^n + (3\gamma - 1)^n$ is denoted by S_n.

(ii) Find the value of S_3. [2]

....

....

....

....

....

....

....

....

....

(iii) Find the value of S_{-2}. [4]

7 The lines l_1 and l_2 have vector equations

$$\mathbf{r} = a\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

respectively. It is given that l_1 and l_2 intersect.

(i) Find the value of the constant a. [3]

....

....

....

....

....

....

....

....

....

....

....

....

The point P has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$.

(ii) Find the perpendicular distance from P to the plane containing l_1 and l_2. [4]

....

....

....

....

....

....

....

....

(iii) Find the perpendicular distance from P to l_2. [4]

8 The curves C_1 and C_2 have polar equations, for $0 \leqslant \theta \leqslant \pi$, as follows:

$$C_1\colon r = a,$$
$$C_2\colon r = 2a|\cos\theta|,$$

where a is a positive constant. The curves intersect at the points P_1 and P_2.

(i) Find the polar coordinates of P_1 and P_2. [2]

....

....

....

....

....

....

....

....

....

(ii) In a single diagram, sketch C_1, C_2 and their line of symmetry. [3]

(iii) The region R enclosed by C_1 and C_2 is bounded by the arcs OP_1, P_1P_2 and P_2O, where O is the pole. Find the area of R, giving your answer in exact form. [5]

9 For the sequence u_1, u_2, u_3, …, it is given that $u_1 = 8$ and

$$u_{r+1} = \frac{5u_r - 3}{4}$$

for all r.

(i) Prove by mathematical induction that

$$u_n = 4\left(\tfrac{5}{4}\right)^n + 3,$$

for all positive integers n. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Deduce the set of values of x for which the infinite series

$$(u_1 - 3)x + (u_2 - 3)x^2 + \ldots + (u_r - 3)x^r + \ldots$$

is convergent. [2]

(iii) Use the result given in part **(i)** to find surds a and b such that

$$\sum_{n=1}^{N} \ln(u_n - 3) = N^2 \ln a + N \ln b.$$ [3]

10 It is given that $t \neq 0$ and

$$t\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 9tx = 3t^2 + 1.$$

(i) Show that if $y = tx$ then

$$\frac{d^2y}{dt^2} + 9y = 3t^2 + 1.$$ [3]

....

....

....

....

....

....

....

....

....

....

....

(ii) Find x in terms of t, given that $x = \frac{1}{9}\pi$ and $\frac{dx}{dt} = \frac{2}{3}$ when $t = \frac{1}{3}\pi$. [9]

....

....

....

....

....

....

....

....

....

11 Answer only **one** of the following two alternatives.

EITHER

(i) Show that

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \mathrm{e}^x \cos x \,\mathrm{d}x = \tfrac{1}{2}\left(\mathrm{e}^{\frac{1}{2}\pi} + \mathrm{e}^{-\frac{1}{2}\pi}\right). \qquad [4]$$

(ii) It is given that, for $n \geqslant 0$,

$$I_n = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{2x} \cos^n x \, dx.$$

Show that, for $n \geqslant 2$,

$$4I_n = n(n-1) \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{2x} \sin^2 x \cos^{n-2} x \, dx - nI_n,$$

and deduce the reduction formula

$$(n^2 + 4)I_n = n(n-1)I_{n-2}.$$ [6]

(iii) Using the result in part **(i)** and the reduction formula in part **(ii)**, find the y-coordinate of the centroid of the region bounded by the x-axis and the arc of the curve $y = e^x \cos x$ from $x = -\frac{1}{2}\pi$ to $x = \frac{1}{2}\pi$. Give your answer correct to 3 significant figures. [4]

OR

Let V be the subspace of $\mathbb{R}^4$ spanned by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ -5 \\ 5 \\ 6 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ -3 \\ 15 \\ 18 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_4 = \begin{pmatrix} 0 \\ -2 \\ 10 \\ 8 \end{pmatrix}.$$

(i) Show that the dimension of V is 3. [3]

(ii) Express $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. [2]

....

....

....

....

....

....

....

....

(iii) Write down a basis for V. [1]

....

....

....

....

Let $\mathbf{M} = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 2 & -5 & -3 & -2 \\ 0 & 5 & 15 & 10 \\ 2 & 6 & 18 & 8 \end{pmatrix}$.

(iv) Find the general solution of $\mathbf{Mx} = \mathbf{v}_1 + \mathbf{v}_2$. [6]

....

....

....

....

....

....

....

....

The set of elements of $\mathbb{R}^4$ which are not solutions of $\mathbf{Mx} = \mathbf{v}_1 + \mathbf{v}_2$ is denoted by W.

(v) State, with a reason, whether W is a vector space. [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.