Cambridge International AS Level

**Cambridge Assessment International Education**
Cambridge International Advanced Subsidiary Level

**MATHEMATICS** **9709/21**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

| |
|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br><br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Attempt to solve quadratic equation in $e^x$ | **M1** | Either directly or using substitution $u = e^x$ |
| | Obtain $e^x = \frac{1}{3}$, $e^x = 27$ | **A1** | $e^x = \frac{1}{3}$, $e^x = 27$ may be implied if $u = e^x$ is stated |
| | Use correct process at least once for solving $e^x = c$ where $c > 0$ | **M1** | |
| | Obtain $-\ln 3$ from a correct solution | **A1** | Condone use of $x = e^x$ |
| | Obtain $3\ln 3$ from a correct solution | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Either | | |
| | State or imply equation $\ln y = \ln A + \ln B \ln x$ | **B1** | |
| | Equate gradient of line to $\ln B$ | **M1** | |
| | Obtain $\ln B = 1.6486...$ and hence $B = 5.2$ | **A1** | |
| | Substitute appropriate values to find $\ln A$ | **M1** | |
| | Obtain $\ln A = 1.2809...$ and hence $A = 3.6$ | **A1** | |
| | Or | | |
| | State or imply equation $\ln y = \ln A + \ln B \ln x$ | **B1** | |
| | Use given coordinates to obtain a correct equation | **B1** | Equations are $4.908 = \ln A + 2.2 \ln B$ and $11.008 = \ln A + 5.9 \ln B$ |
| | Use given coordinates to obtain a second correct equation and attempt to solve both equations simultaneously to obtain at least one of the unknowns $\ln A$ or $\ln B$ | **M1** | |
| | Obtain $\ln B = 1.6486...$ and hence $B = 5.2$ | **A1** | |
| | Obtain $\ln A = 1.2809...$ and hence $A = 3.6$ | **A1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Or | | |
| | Use given coordinates to obtain a correct equation | **B1** | Equations are $e^{4.908} = AB^{2.2}$ and $e^{11.008} = AB^{5.9}$ |
| | Use given coordinates to obtain a second correct equation | **B1** | |
| | Solve to obtain B | **M1** | M mark dependent on both previous B marks |
| | $B = 5.2$ | **A1** | |
| | $A = 3.6$ | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Rewrite integrand as $4e^{2x} + 4e^{-x}$ | **B1** | |
| | Integrate to obtain form $k_1e^{2x} + k_2e^{-x}$ where $k_1 \neq 4, k_2 \neq 4$ | **M1** | |
| | Obtain correct $2e^{2x} - 4e^{-x}$ | **A1** | |
| | Apply limits correctly, retaining exactness | **M1** | Dependent on previous M1 |
| | Obtain $2e^4 - 4e^{-2} + 2$ or exact similarly simplified equivalent | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | Use quotient rule or equivalent | **M1** | Obtaining two terms in numerator and $(2x+1)^2$ in denominator for a quotient |
| | Obtain correct $\frac{\frac{5}{x}(2x+1)-10\ln x}{(2x+1)^2}$ or equivalent, or $\frac{5}{x}(2x+1)^{-1}-10\ln x(2x+1)^{-2}$ or equivalent | **A1** | Obtaining one term with $(2x+1)^{-1}$ oe and a second term with $(2x+1)^{-2}$ oe for a product<br>Condone poor use of brackets if recovered later |
| | Substitute $x=1$ to obtain $\frac{15}{9}$ or $\frac{5}{3}$ or equivalent, www | **A1** | |
| | | **3** | |
| 4(ii) | Equate numerator to zero and attempt relevant arrangement | **M1** | For M1, need to see at least one line of working after either $10+\frac{5}{x}-10\ln x=0$ or their numerator (which must have at least 2 terms, one involving $\ln x$) = 0 |
| | Confirm $x=\frac{x+0.5}{\ln x}$ | **A1** | AG; necessary detail needed |
| | | **2** | |
| 4(iii) | Use iteration process correctly at least once | **M1** | |
| | Obtain final answer 3.181 | **A1** | |
| | Show sufficient iterations to 6 sf to justify answer or show sign change in interval $(3.1805, 3.1815)$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -4\sin 2\theta + 3\cos\theta$ | **B1** | B1 may be implied |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} / \frac{\mathrm{d}x}{\mathrm{d}\theta}$ in terms of $\theta$ or with 1 already substituted | **M1** | |
| | Obtain or imply $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3\sin\theta}{-4\sin 2\theta + 3\cos\theta}$ | **A1** | |
| | Substitute 1 to obtain 1.25 | **A1** | Or greater accuracy 1.252013… |
| | | **4** | |
| 5(ii) | Equate denominator of first derivative to zero | **M1** | |
| | Use $\sin 2\theta = 2\sin\theta\cos\theta$ | **A1** | |
| | Obtain $\sin\theta = \frac{3}{8}$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Substitute $x=-2$ and equate to zero | **M1** | |
| | Obtain $-8+4a-28+a+1=0$ or equivalent and hence $a=7$ | **A1** | |
| | Attempt either division by $x+2$ and reach partial quotient $x^2+kx$, where $k$ is numeric or use of identity or inspection or synthetic division | **M1** | Synthetic division:<br>–2 \| 1 \| 7 \| 14 \| 8<br>\| \| –2 \| –10 \| –8<br>\| 1 \| 5 \| 4 \| 0 |
| | Obtain quotient $x^2+5x+4$ soi | **A1** | |
| | Conclude with $(x+1)(x+2)(x+4)$ | **A1** | |
| | | **5** | |

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="11">6(ii)</td><td><u>Either</u></td><td></td><td></td></tr>
<tr><td>State $(2x+1)(2x+2)(2x+4)=3(x+1)(x+2)(x+4)$</td><td>M1</td><td>Following their complete factorised form</td></tr>
<tr><td>Obtain $x=-1$ and $x=-2$</td><td>A1</td><td>Calculator not permitted so necessary detail needed</td></tr>
<tr><td>Cancel common factors to obtain linear equation or factorise to find corresponding factor</td><td>M1</td><td></td></tr>
<tr><td>Obtain $x=\frac{8}{5}$ or equivalent</td><td>A1</td><td></td></tr>
<tr><td><u>Or</u></td><td></td><td></td></tr>
<tr><td>State $(2x+1)(2x+2)(2x+4)=3(x+1)(x+2)(x+4)$ or $(2x)^3+7(2x)^2+14(2x)+8=3(x^3+7x^2+14x+8)$</td><td>M1</td><td>Following their completed factorised form,<br>Must see $8x^3$ and $28x^2$ if using second statement without bracketed terms in $2x$</td></tr>
<tr><td>Expand and simplify to obtain $5x^3+7x^2-14x-16=0$</td><td>A1</td><td>Must be equated to 0 for A1</td></tr>
<tr><td>Attempt complete factorisation of cubic with leading term $5x^3$ (may make use of synthetic division)</td><td>M1</td><td>Synthetic division:
<table>
<tr><td rowspan="2">–2</td><td>5</td><td>7</td><td>–14</td><td>–16</td></tr>
<tr><td></td><td>–10</td><td>6</td><td>16</td></tr>
<tr><td></td><td>5</td><td>–3</td><td>–8</td><td>0</td></tr>
</table></td></tr>
<tr><td>Obtain $(x+1)(x+2)(5x-8)=0$ and conclude $x=-1,\ x=-2,\ x=\frac{8}{5}$</td><td>A1</td><td>Calculator not permitted so necessary detail needed</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | State $R=\sqrt{29}$ or 5.385… | **B1** | |
| | Use appropriate trigonometry to find $\alpha$ | **M1** | Allow M1 for $\tan\alpha=\pm\frac{2}{5}$ or $\pm\frac{5}{2}$ oe |
| | Obtain 0.3805 with no errors seen | **A1** | Or greater accuracy 0.3805063… |
| | | **3** | |
| 7(ii) | State that equation is $5\cos\theta-2\sin\theta=4$ | **B1** | |
| | Evaluate $\cos^{-1}(k/R)-\alpha$ to find one value of $\theta$ | **M1** | Allow M1 from their $\sqrt{29}\cos(\theta\pm\alpha)$ |
| | Obtain 0.353 | **A1** | Or greater accuracy 0.35307… |
| | Carry out correct method to find second value | **M1** | |
| | Obtain 5.17 and no extra solutions in the range | **A1** | Or greater accuracy 5.16909…<br><br>If working consistently in degrees, then no A marks are available, B1, M1, M1 max |
| | | **5** | |
| 7(iii) | State integrand as $\frac{1}{29}\sec^2(\frac{1}{2}x+0.3805)$ | **B1 FT** | Following their answer from part (i), must be in the form $R\cos(\theta\pm\alpha)$ |
| | Integrate to obtain form $k\tan(\frac{1}{2}x+\text{their }\alpha)$ | **M1** | |
| | Obtain $\frac{2}{29}\tan(\frac{1}{2}x+0.3805)+c$ | **A1** | |
| | | **3** | |