Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS	**9709/32**
Paper 3	**May/June 2018**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	**M1**	$8\left(2^x\right)^2-18\left(2^x\right)+9=0$
	Obtain $2^x=\frac{3}{2}$ and $2^x=\frac{3}{4}$ or equivalent	**A1**	
	OR: Obtain $2^x=\frac{3}{2}$ by solving an equation	**B1**	
	Obtain $2^x=\frac{3}{4}$ by solving an equation	**B1**	
	Use correct method for solving an equation of the form $2^x=a$, where $a>0$	**M1**	
	Obtain **final** answers x = 0.585 and x = – 0.415 only	**A1**	The question requires 3 s.f. Do not ISW if they go on to reject one value
		4	

Question	Answer	Marks	Guidance
2	Use correct tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**	$\frac{1}{\tan\theta}+\frac{1-\tan\theta\tan 45}{\tan\theta+\tan 45}=2$ Allow M1 with $\tan 45°$ $=\frac{1}{\tan\theta}+\frac{1-\tan\theta}{\tan\theta+1}$
	Obtain a correct equation in any form	**A1**	With values substituted
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**	
	Obtain answer $x = 30°$	**A1**	One correct solution
	Obtain answer $x = 150°$	**A1**	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1		
	Reduce to $\tan^2\theta=\frac{1}{3}, \sin^2\theta=\frac{1}{4}, \cos^2\theta=\frac{3}{4}$ or $\cot^2\theta=3$ etc. A1		
		5	

Question	Answer	Marks	Guidance
3(i)	Fully justify the given statement	**B1**	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
3(ii)	Separate variables and attempt integration of at least one side Obtain terms ln y and $\frac{1}{2}x$	**B1 B1**	Must be working from $\int \frac{1}{y}\,dy = \int k\,dx$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y \rvert = \ldots$ scores A0
		5	

Question	Answer	Marks	Guidance
4(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\dfrac{\sin x}{1+\cos x} \times \dfrac{1-\cos x}{1-\cos x} = \dfrac{\sin x - \sin x\cos x}{1-\cos^2 x}$ M1A1 $= \dfrac{2\sin x - 2\sin x\cos x}{2-2\cos^2 x}$		Given answer so check working carefully
	$= \dfrac{2\sin x - \sin 2x}{1-\cos 2x}$ M1A1		
		4	
4(ii)	State integral of the form $a\ \ln(1+\cos x)$	**M1***	If they use the substitution $u = 1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	**A1**	
	Substitute correct limits in correct order	**M1(dep)***	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**	
	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	**B1**	$3x^2+6xy+3x^2\frac{dy}{dx}-3y^2\frac{dy}{dx}=0$
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**	Given answer so check working carefully
	Obtain the given answer	**A1**	
		4	
5(ii)	Equate derivative to – 1 and solve for *y*	**M1***	
	Use their $y = -2x$ or equivalent to obtain an equation in *x* or *y*	**M1(dep*)**	
	Obtain answer (1, – 2)	**A1**	
	Obtain answer ($\sqrt[3]{3}$, 0)	**B1**	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question	Answer	Marks	Guidance
6(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	M1	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2 \sin\theta$
	State a correct equation in any form	A1	Given answer so check working carefully
	Obtain the given answer correctly	A1	
		3	
6(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	M1	e.g. $\mathrm{f}(\theta) = \frac{\pi}{2} + \sin\theta$ $\begin{cases} \mathrm{f}(2.2) = 2.37... > 2.2 \\ \mathrm{f}(2.4) = 2.24... < 2.4 \end{cases}$ or $\mathrm{f}(\theta) = \theta - \frac{\pi}{2} - \sin\theta$ $\begin{cases} \mathrm{f}(2.2) = -0.17... < 0 \\ \mathrm{f}(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	A1	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">6(iii)</td><td>Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once</td><td>M1</td><td rowspan="3">e.g.
<table>
<tr><td>2.2</td><td>2.3</td><td>2.4</td></tr>
<tr><td>2.3793</td><td>2.3165</td><td>2.2463</td></tr>
<tr><td>2.2614</td><td>2.3054</td><td>2.3512</td></tr>
<tr><td>2.3417</td><td>2.3129</td><td>2.2814</td></tr>
<tr><td>2.2881</td><td>2.3079</td><td>2.3288</td></tr>
<tr><td>2.3244</td><td></td><td>2.2970</td></tr>
<tr><td>2.3000</td><td></td><td>2.3185</td></tr>
<tr><td>2.3165</td><td></td><td>2.3041</td></tr>
<tr><td>2.3054</td><td></td><td>2.3138</td></tr>
<tr><td>2.3129</td><td></td><td>2.3072</td></tr>
</table></td></tr>
<tr><td>Obtain final answer 2.31</td><td>A1</td></tr>
<tr><td>Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)</td><td>A1</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer		Marks	Guidance
7(i)	Substitute in *uv*, expand the product and use $i^2 = -1$		M1	
	Obtain answer $uv = -11 - 5\sqrt{3}i$		A1	
	EITHER:	Substitute in *u/v* and multiply numerator and denominator by the conjugate of *v*, or equivalent	M1	
		Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	A1	
		Obtain final answer $-1 + \sqrt{3}i$	A1	
	OR:	Substitute in *u/v* , equate to $x + iy$ and solve for x or for y	M1	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$		A1	
	Obtain final answer $-1 + \sqrt{3}\,i$		A1	
			5	

Question	Answer	Marks	Guidance
7(ii)	Show the points *A* and *B* representing *u* and *v* in relatively correct positions	**B1**	
	Carry out a complete method for finding angle *AOB*, e.g. calculate arg(*u*/*v*) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	**M1**	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan\left(a-b\right) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	**A1**	Given answer so check working carefully
		3	

Question	Answer	Marks	Guidance
8(i)	Use correct product or quotient rule	**M1**	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 2$ with no errors seen	**A1**	
		4	
8(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$	**M1***	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	**A1**	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	**A1**	
	Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	**M1(dep*)**	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
9(i)	Use a correct method to find a constant	**M1**	
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		4	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(2+x^2\right)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**	The ft is on *A*, *B* and *C*. $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}...\right) + \frac{x+2}{2}\left(1-\frac{x^2}{2}...\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3, by $Bx + C$, where $BC \neq 0$	**M1**	
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**	
		5	

Question	Answer		Marks	Guidance
10(i)	Equate at least two pairs of components and solve for s or for t		M1	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for s or t, e.g. $s = -6$, $t = -11$		A1	
	Verify that all three equations are not satisfied and the lines fail to intersect		A1	
	State that the lines are not parallel		B1	
			4	
10(ii)	*EITHER*:	Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a + 3b - c = 0$	B1	
		Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	A1	
	OR:	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	A1	
			3	

Question	Answer		Marks	Guidance
10(iii)	*EITHER*:	State position vector or coordinates of the mid-point of a line segment joining points on l and m, e.g. $\frac{3}{2}\mathbf{i}+\mathbf{j}+\frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing l and m B1
		Use the result of (**ii**) and the mid-point to find d	**M1**	Use average of distances to find equation of p. M1
		Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR:	Using the result of part **(ii)**, form an equation in d by equating perpendicular distances to the plane of a point on l and a point on m	**M1**	
		State a correct equation, e.g. $\left\|\frac{14-d}{\sqrt{35}}\right\| = \left\|\frac{-d}{\sqrt{35}}\right\|$	**A1**	
		Solve for d and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
			3	