## Cambridge International AS \& A Level

## THINKING SKILLS

9694/03
Paper 3 Problem Analysis and Solution
For examination from 2020

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SPECIMEN PAPER

You must answer on the enclosed answer booklet.
You will need: Answer booklet (enclosed)
Calculator

\section*{INSTRUCTIONS}
- Answer all questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.

Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

\section*{INFORMATION}
- The total mark for this paper is 50 .
- The number of marks for each question or part question is shown in brackets [ ].

1 Bill owns three stores which sell models. Every Monday, Bill sends supplies to the stores from his warehouse. All of the supplies are packaged into boxes of the same size and 20 of these boxes can be loaded onto the van for each delivery.

The distances, in kilometres, between Bill's warehouse and the three stores are shown below.
\begin{tabular}{|c|c|c|c|}
\hline \begin{tabular}{c} 
Bill's \\
Warehouse
\end{tabular} & \multicolumn{3}{|c}{} \\
\cline { 1 - 1 } 5 & 8 & \begin{tabular}{c} 
Model \\
Solutions
\end{tabular} & \\
\hline 7 & 6 & \begin{tabular}{c} 
Model \\
Emporium
\end{tabular} & \\
\hline 8 & 10 & \begin{tabular}{c} 
Model \\
Market
\end{tabular} \\
\hline
\end{tabular}

Bill sometimes needs to make more than one trip when delivering the boxes. He wants to know the total distance that he needs to travel for any of the trips that deliver to just two of the stores before returning to the warehouse. The three trips are:
\[
\begin{aligned}
& \text { Warehouse } \rightarrow \text { Model Solutions } \rightarrow \text { Model Emporium } \rightarrow \text { Warehouse } \\
& \text { Warehouse } \rightarrow \text { Model Solutions } \rightarrow \text { Model Market } \rightarrow \text { Warehouse } \\
& \text { Warehouse } \rightarrow \text { Model Emporium } \rightarrow \text { Model Market } \rightarrow \text { Warehouse }
\end{aligned}
\]

Bill has already worked out that he needs to travel a total distance of 20 km if he makes the first trip.
(a) What are the total distances that Bill needs to travel to make each of the other trips?

This morning, Bill needs to deliver 12 boxes to Model Solutions, 8 boxes to Model Emporium and 9 boxes to Model Market. He cannot load all of the boxes onto the van, so Bill has decided to deliver to one of the stores first and then return to the warehouse where he will load all the boxes for the other two stores. When he has completed the deliveries he will return the van to the warehouse.
(b) Which store should Bill deliver to first in order to make the total distance travelled as short as possible? What is this shortest distance?

Bill hires the van at a cost of \(\$ 6\) per km travelled. It is also possible for him to hire a larger van, which would hold 25 boxes, but this would cost \(\$ 7\) per km travelled.

Bill already knows that he will need to deliver 13 boxes to Model Solutions and 12 boxes to Model Market next week, but does not yet know how many boxes will need to be delivered to Model Emporium.
(c) (i) If Model Emporium does not require any boxes delivered next week, how much would Bill save by hiring the larger van rather than the smaller one?
(ii) If Model Emporium does require some boxes delivered next week, what is the smallest number of boxes that could be needed at Model Emporium to make it cheaper for Bill to hire the larger van?

2 Empuda is a sport that resembles tennis, in that two players compete against each other by hitting a ball over a net.

An empuda match consists of 20 strands. Each strand begins when one player delivers the ball, and continues until someone scores a grod, a torf or a lenk. The deliverer alternates from strand to strand, and the number of points scored depends upon whether the winner of the strand is the deliverer or the recipient, as detailed below.
\begin{tabular}{|l|c|c|c|}
\hline \multirow{2}{*}{\begin{tabular}{c} 
Winner of \\
Strand
\end{tabular}} & \multicolumn{3}{|c|}{ Points Scored } \\
\cline { 2 - 4 } & Grod & Torf & Lenk \\
\hline Deliverer & 1 & 3 & 5 \\
\hline Recipient & 2 & 5 & 9 \\
\hline
\end{tabular}
(a) (i) What is the greatest number of points that one player can score in an empuda match?
(ii) What is the greatest possible number of points that a player can win an empuda match by, having won fewer strands than the loser?

Eight players are competing today at Nyhope Empuda Club for the Rulane Cup. The competition is organised as a league, with all the participants playing each other once. League positions are decided by the number of matches won. Where two or more participants have the same number of wins, the total number of points scored overall becomes the deciding factor.

Today's Order of Play, and a summary of each player's performances in matches completed so far, are as follows.
\begin{tabular}{|c|c|c|c|}
\hline Time & Court 1 & Court 2 & Court 3 \\
\hline \(09: 00\) & Serrano v Walker & Bagge v Lyne & Fiander v Sharif \\
\hline \(09: 50\) & Fiander v Serrano & Lyne v Walker & Brown v Knuttall \\
\hline \(10: 40\) & Bagge v Brown & Fiander v Walker & Knuttall v Sharif \\
\hline \(11: 30\) & Brown v Lyne & Serrano v Sharif & Bagge v Knuttall \\
\hline \(12: 20\) & Bagge v Fiander & Knuttall v Serrano & Brown v Walker \\
\hline \(13: 10\) & -------------- & Lyne v Sharif & ------------- \\
\hline \(14: 00\) & Sharif v Walker & Knuttall v Lyne & Brown v Fiander \\
\hline \(14: 50\) & Knuttall v Walker & Fiander v Lyne & Bagge v Serrano \\
\hline \(15: 40\) & Brown v Serrano & Bagge v Sharif & Fiander v Knuttall \\
\hline \(16: 30\) & Lyne v Serrano & Bagge v Walker & Brown v Sharif \\
\hline
\end{tabular}
\begin{tabular}{|l|c|c|c|c|c|c|c|c|}
\cline { 2 - 10 } \multicolumn{1}{c|}{} & \multicolumn{9}{c|}{ Points Scored Against: } \\
\cline { 2 - 10 } \multicolumn{1}{c|}{} & Bagge & Brown & Fiander & Knuttall & Lyne & Serrano & Sharif & Walker \\
\hline J. Bagge & & 32 & 18 & 37 & 22 & & & \\
\hline I. Brown & 41 & & & 57 & 40 & & & 47 \\
\hline A. Fiander & 34 & & & & & 22 & 45 & 36 \\
\hline P. Knuttall & 33 & 48 & & & & 43 & 32 & \\
\hline B. Lyne & 49 & 20 & & & & & & 32 \\
\hline K. Serrano & & & 39 & 24 & & & 36 & 45 \\
\hline C. Sharif & & & 27 & 32 & & 18 & & \\
\hline R. Walker & & 12 & 24 & & 41 & 38 & & \\
\hline
\end{tabular}
(b) Which two players will not play on all three courts today?

Normally, when both players have the same score after 20 strands, 2 further strands are played (and again if necessary) until a winner emerges. In today's event, however, due to time constraints, a tied match counts as a win for both players.
(c) One of the matches played earlier today was tied. Which two players both registered a win as a result?

The highest-scoring match so far today has been Ian Brown's 57-48 defeat of Philip Knuttall. The points scored by the two players were as follows:
\begin{tabular}{|l|c|c|}
\cline { 2 - 3 } \multicolumn{1}{c|}{} & Brown & Knuttall \\
\hline 9 points & 5 & 4 \\
\hline 5 points & 0 & 1 \\
\hline 3 points & 2 & 1 \\
\hline 2 points & 2 & 1 \\
\hline 1 point & 2 & 2 \\
\hline
\end{tabular}
(d) There was only one strand in which 5 points were scored. Did Philip Knuttall win this strand with a torf or a lenk? Explain your answer.
(e) In the match currently being played, Craig Sharif has made a spectacular start and now leads Brett Lyne 33-0 after 6 strands.

How many grods, how many torfs and how many lenks has Craig scored?
(f) Ian Brown is the only player to have won all of his matches so far, and consequently he is top of the league at present.

Who is currently in second place, and who is currently in third place?

3 Elections in France always take place on Sundays, and people are often called upon to vote on two consecutive Sundays. Voting is voluntary. On each Sunday people may only vote for one candidate in that round.

In the first round, a candidate who gets more than half of the votes cast is elected, so long as these were the votes of at least a quarter of the electorate. (The electorate consists of all the people entitled to vote.)

Otherwise, there is a second vote. This vote is restricted to the two candidates from the first round with the two highest numbers of votes, and any other candidate who had a first-round vote of at least an eighth of the electorate. The candidate with the most votes in the second round is elected. A candidate may withdraw after the first round, and the second round only happens if there are still at least two candidates.

The threshold of an eighth of the electorate used to be a tenth of the electorate.
The people in Cambronne-sur-Pierre had to vote on the second Sunday. They were annoyed to find that there were exactly the same three candidates as on the first Sunday.
(a) Give an example of percentages of the electorate voting for each of the three candidates on the first Sunday that could have led to this situation.

Assume for the rest of this question that no two candidates in a round get exactly the same number of votes, and that the electorate contains at least 30000 people.
(b) (i) What is the theoretical maximum possible number of candidates in the second round?
(ii) What was the theoretical maximum possible number of candidates in the second round before the threshold was changed?

The first round votes in an election several years ago, when the threshold was still a tenth, were:
\begin{tabular}{|c|c|c|c|c|}
\hline Alain & Bernard & Clothard & David & Emile \\
\hline 4273 & 53 & 5370 & 10502 & 651 \\
\hline
\end{tabular}

David was not elected on the first Sunday, and a second round was held.
(c) (i) Which rule stopped David from being elected on the first Sunday?
(ii) What does this indicate about the number of people entitled to vote who did not vote?
(iii) Alain qualified for the second round. What does this indicate about the size of the electorate?

There were, in fact, 42070 people entitled to vote. If Bernard had not been a candidate, 22 of his voters would have voted for David, and the rest for Alain.
(d) What difference, if any, would that have made? Explain your answer briefly.

4 Claudel is a sculptor who is considering how she can best make a profit from her skills. She makes her calculations based on the following:
- She will work 200 hours in each calendar month.
- Each commission will earn her \(\$ 900\), after paying for materials, and takes 30 hours of sculpting work.
- Each commission also requires 10 hours of unskilled work. She can either do this herself or hire an assistant. The minimum wage for an assistant is \(\$ 10\) per hour.
- New commissions are always available.
(a) Show that Claudel is able to make \(\$ 4500\) per month if she does the unskilled work herself.

When doing her calculations, Claudel decides to include the appropriate fraction of earnings for any commission that is only partially completed. For example, if she is halfway through a commission at the end of a month, she considers that as \(\$ 450\) earned. When paying an assistant, she treats the unskilled work in the same way.
(b) If Claudel hires a part-time assistant at the beginning of the first month, what is the maximum profit she can make by the end of that month?
(c) What is the maximum that Claudel could pay an assistant per hour and ensure that she still makes the same amount of profit in the first year as she would on her own?

Claudel decides that she will pay any unskilled assistants that she hires at the minimum wage of \(\$ 10\) per hour. She can either do the sculpting work herself, or pay skilled artisans at a rate of \(\$ 15\) per hour. Each assistant or artisan will work no more than 200 hours per calendar month. Each artisan requires continual support and quality control: supervising their work requires her to spend 12 minutes with each artisan per hour of work. This time is not spent sculpting - and therefore the artisan only spends 48 minutes sculpting in every hour (which amounts to 4 hours sculpting, in every 5 hours of paid work). Skilled artisans will not do unskilled work.
(d) How much profit could Claudel make per month if she employed one skilled artisan full-time and a part-time unskilled assistant?

Claudel is considering hiring several artisans and doing no work herself other than supervising.
(e) How many artisans working full time would make it necessary to hire a second assistant?

She discovers that if she pursues her plan, she will have to register as a 'small business'. The only extra cost involved in this process is that of registering employees, which will cost \(\$ 1000\) per year for each of her employees, paid in advance.
(f) Calculate the maximum profit Claudel could make per year.

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