**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**Cambridge International Advanced Level**

# MARK SCHEME for the October/November 2015 series

# 9694 THINKING SKILLS

**9694/31** Paper 3 (Problem Analysis and Solution), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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**1 (a) What is the chance that this approach would not decide who gets the ice-cream? [2]**

2/8 (or equivalent)

*Award 1 mark for eight possible outcomes OR an answer of 3/4.*

**(b) (i) If John and Luke had each selected a number at random, what would Mark's chance of getting the ice-cream have been? [1]**

3/9 equally likely outcomes = 1/3 (or equivalent)

**(ii) What would Luke's chance of winning have been if he had selected 1? [1]**

John would get it unless he also selected 1 in which case Mark would have it. 0

**(c) (i) Knowing what Luke would do, what was the best strategy for John to maximise his chance of getting the ice-cream, and what was his chance of success? [2]**

Since with no 1 he can only win with a 3: always select 3 [1 mark].
Chance of success is 1/2 [1 mark].

**(ii) What was Luke's chance of getting the ice-cream if John used this strategy? [1]**

Since Mark or John would get the ice-cream, 0.

**(iii) If Luke and John both avoid 1 and each, independently, toss a coin to select 2 or 3, what is the chance that John will get the ice-cream? [1]**

John only wins if he gets a 3 and Luke a 2, so 1/4.

**(d) Describe a simple strategy for them to use. [1]**

Strategy must yield equal chance of John and Luke winning, with no communication.
e.g. One of them always gives 2 and the other alternates between 1 and 3.
OR One of them always gives 2 and the other randomly chooses 1 and 3.
OR They start with 1 and 2 and then each takes what the other did last time.

**(e) Is there a strategy that Luke can use so he can expect to get the ice-cream more than half the time, whatever John does? Explain your reason. [1]**

No, because, if there were, John could use the same strategy, and they can't both get more than half. Some appeal to symmetrical relation between players.

**2 (a) (i) How many slices of bread would be left at the end of the third day? [1]**

(5 + 4 + 5) × 20 – 88 × 3 OR 12 – 8 + 12 = <u>16</u>

**(ii) When was the first day when there would have been no bread left at the end of the day? [1]**

12 – 8 + 12 – 8 – 8 = 0, so first <u>Friday/5th day</u>

**(iii) How many slices would be left at the end of the first three weeks? [1]**

<u>12</u>

**(b) (i) Did this approach lead to more or less bread being bought over the first three weeks, or was it the same? [1]**

Nine 5s for Fred, eight 5s for bakery (92 in total v 93 in total) so <u>less</u>.
<u>Some evidence of a comparison needed</u> – e.g. 1 loaf less, or comparative figures (1860 v. 1890).

**(ii) By how many did the total number of slices consumed over the first three weeks differ from Fred's prediction? [1]**

(1848 – 1827 =) <u>21</u>
<u>1 slice per day</u>

**(c) Would Fred's original plan have resulted in a shortage, and, if so, on which day? [1]**

Fred's 5s are never later than the corresponding one, so <u>No</u>
*Accept without explanation*

**(d) (i) What was the minimum number of slices left over that resulted in the bakery delivering 4 loaves? [2]**

<u>11</u>

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Eaten – 80 | 6 | 8 | 4 | 11 | 1 | 4 | 6 | 6 | 8 | 5 | 4 | 7 | 7 | 9 | 11 | 6 | 12 | 8 | 8 | 9 | 7 |
| Slices left | 14 | 6 | 22 | **11** | 10 | 26 | 20 | 14 | 6 | 21 | 17 | 10 | 23 | 14 | 3 | 17 | 5 | 17 | 9 | 20 | |
| Evening | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su |

*1 mark for calculating any number of slices left over that led to an order of 4 loaves (shaded cells in table).*

**(ii) Does this guarantee there will be enough every day if a similar usage continues? Explain your answer. [2]**

No, there could be (at least) 92 wanted. [1 mark]
If previous day had only 11 slices (FT from (i)) only 80 more would be ordered, which could lead to as few as 91 slices available. [1 mark]

*No mark for judgment only.*

**3 (a) How much does each token cost in the first round of the game?** [1]

50 tokens gives a price of $50, the remaining 75 will reduce the price by a further $37.50, so the selling price will be $12.50.

**(b) If all four players in a game each requested 40 tokens in the first round, what will the maximum allocation per person be? [1]**

They would be divided equally between the four players, so the maximum allocation would be 31 tokens (and there would be 1 left).

**(c) If the four players in a game requested 20, 26, 50 and 80 tokens in the first round, what will be the numbers of tokens that the four players receive? [2]**

20, 26, 39 and 39 (in any order)

*1 mark for 20, 26, x, x (where x is between 26 and 38 inclusive)*

**(d) In the first round of another game, Joseph requested 80 tokens but only received 40. Give an example of the numbers of tokens that could have been requested by the other three players if 2 tokens remained unsold. [2]**

If 2 tokens were not given out then there must be at least three players who did not get as many tokens as they ordered. Those three players must have got 40 each, so the fourth player must have ordered just 3 tokens.
The three numbers must be 3 and two numbers which are at least 40.

*Award 1 mark for 3 numbers which give Joseph 40*
*OR for working which shows the optimum being split 3 ways (including any solution which leaves 2 tokens unsold, e.g. 6, 39+, 39+, 80: minimum value can be 3n for positive integer 'n' < 10).*

**(e) Eliza knows that each of her opponents will request just one token in any round where the price is $10 or more, and request 50 tokens if the price is less than $10. Show that Eliza can receive a total of 99 tokens by the end of the second round. [2]**

Pay $25 for 2 tokens in round 1; the price in round 2 will be $10; buy 97 tokens in round 2.

*1 mark for correctly identifying a round 2 price based on Eliza's choice in round 1.*

**(f) At the start of round 5 in another game the price of tokens was $5.50 less than it had been in round 4 of the game. Give an example of the number of tokens available at the start of round 4 and the total number requested in that round that could lead to this. [3]**

The number of extra tokens available must be less than 10 (as the game has not ended) and each extra token added $ 1 to the price if the price was $ 50 or more and $ 0.50 to the price if it was below $ 50. The only options are:

| Tokens at start of round 4 | Price ($) | Number sold | Tokens at start of round 5 | Price ($) |
|---|---|---|---|---|
| <u>45</u> | 55 | <u>4</u> | 51 | 49.50 |
| <u>46</u> | 54 | <u>3</u> | 53 | 48.50 |
| <u>47</u> | 53 | <u>2</u> | 55 | 47.50 |
| <u>48</u> | 52 | <u>1</u> | 57 | 46.50 |

*Full marks awarded for the two underlined figures in any row.*
*If 3 marks cannot be awarded, award 1 mark for each of the following:*

- identification that the two numbers must lie on either side of 50.
- two prices that differ by $ 5.50 identified and the corresponding numbers of tokens calculated for each.

**(g) Near the end of one game Jill is the only player with any money left. She has $ 50 and at the start of the current turn there are 130 tokens available. What is the largest number of extra tokens that she can buy before the game ends? [4]**

Tokens will cost $ 10 this turn and she must buy at least one to keep the game going.
If she buys just one then there will be 139 available next round and the cost will be $ 5.50 each. She will have $ 40 left.
If she again buys just one then there will be 148 available next round and the cost will be $ 1 each. She will have $ 34.50 left.
She will need to buy 9 next, so that there will only be 149 available next round and the cost will be $ 0.50 each. She will have $ 25.50 left.
She can now continue to buy sets of 10 each time, keeping the price at $ 0.50 until she runs out of money, so she will manage to buy another 51 tokens.
She will be able to buy another <u>62 tokens</u>.

*If 4 marks cannot be awarded, award 1 mark for each of the following:*

- Identifying that she should buy just one each round until there are more than 140 tokens available.
- Identifying that on the round where there are 148 available she needs to buy 9 to keep the game going.
- Identifying that she can buy sets of 10 tokens until she runs out of money.

**4 (a) How many points did the Jays gain from their match yesterday? [1]**

Answer: 4 (3 points for a win by one goal + 1 point for first goal)

**(b) Which team had lost exactly half of its previous matches this season, before yesterday? [1]**

Answer: Rooks (had lost 6 matches out of 12)

**(c) How many matches have the Ravens won this season by two goals or more? Justify your answer. [2]**

Answer: 1 (2 marks, dependent on supporting evidence)

1 mark for recognition that the Ravens scored 1 point in yesterday's loss
OR 1 mark for recognition that 22 points are accounted for by the draws and first goal bonuses.

**(d) Of all the matches played so far this season, how many have been 0 – 0 draws? [2]**

Answer: 4 (52 matches played, 48 "first goal" points) (2 marks)

*If 2 marks cannot be awarded, award 1 mark for a clear attempt to subtract 'first goal' points ("48" sufficient) from the total matches played (13 games, 8 teams, multiplier).*

*Alternatively, award 1 mark for a clear attempt to subtract the total number of points scored from 5 × the total number of matches played. (The total number of points is 256, but would be 260 (5 × 52) if a goal had been scored in all 52 matches.)*

**(e) Give the final score of this match, and explain how this score resulted in the Choughs gaining 3 points and the Jackdaws 2 points. [2]**

(Choughs) 4 (Jackdaws) 3 (1 mark) therefore Choughs score 3 points & Jackdaws 1 point.
Jackdaws scored the first goal (1 mark) so Jackdaws get an extra point.

**(f) Any one of the top four teams could still win the league. Explain why the Jackdaws cannot win the league. [2]**

EITHER the Jays or Choughs (or both) will gain at least 2 points next week (because they are playing each other) (so will have at least 37 points).
OR
The Jackdaws could achieve 37 points (32 + maximum of 5 points) (1 mark).

Given equal scores, the Jackdaws would have fewer 'first goal' points (than either the Jays or the Choughs), so could not win (1 mark).

**(g) Give a set of scores for next week's matches that would result in the Ravens winning the league with fewer matches won than any of the other teams, regardless of which teams score first.** **[3]**

To win the league, the Ravens must score at least 38 points, because the Choughs and/or the Jays will advance to at least 37 points with a greater number of 'first goal' points.

To have fewer wins than any other team, they must not win, and the Crows and Rooks must both win.

For the first goal not to be a consideration, a draw for the Ravens can only guarantee 2 points, so neither the Choughs nor the Jays must gain more than 2 points.

*1 mark for each of these conditions satisfied:*

<u>Choughs v Jays : draw AND Magpies v Ravens : draw</u>
<u>Choughs v Jays : 0 – 0 draw</u>
<u>Jackdaws v Rooks : Rooks win AND Nutcrackers v Crows : Crows win</u>

**(h) There has been a proposal to allow two more teams to join the league from next season. If this proposal is accepted, how many more matches will be played altogether during a season than at present?** **[2]**

Answer: <u>34</u> (90 – 56) [2 marks]

*If 2 marks cannot be awarded, award 1 mark for seeing*

10 × 9 × 2 (= 180)
OR 10 × 9 (= 90)
OR 8 × 7 × 2 (= 112)
OR 8 × 7 (= 56)
OR (10 × 9 × 2) – (8 × 7 × 2) (= 68)
OR ((10 × 10 × 2) – (8 × 8 × 2))/2 (= 36)

SC1: 18 – 14 = 4 matches (more per team)