
THINKING SKILLS

9694/31

Paper 3 Problem Analysis and Solution

October/November 2016

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of 7 printed pages, 1 blank page, and 1 insert.



- 1 Kate is planning the layout of her new restaurant. To make the plans easier, she thinks of the dining area as a grid of squares as shown below:

	1	2	3	4	5	6	7	8	9	10
A										
B										
C										
D										
E										
F										
G										

The shaded squares (at grid references G1 and G10) represent the entrance to the dining area and the access to the kitchen, so both of these squares must be empty.

Each table in the restaurant consists of one or more units. Each unit takes up one square and each chair will occupy a square adjacent to one of the units. Every unit must have exactly two chairs next to it, on opposite sides of the unit.

A table is described as a table for the maximum number of customers who could be seated at it; so, for example, two units positioned next to each other would be a table for 4.

There must always be a route of empty squares to reach any of the chairs in the restaurant from both the entrance and the access to the kitchen.

Chairs for different tables can only be in adjacent squares if they are back to back. Chairs for one table can never be adjacent to any of the units of another table.

(a) Explain why it would not be possible to place units at A2 and B2 to form a table for 4. [1]

(b) How many different positions are there for a table for 6? [3]

Kate is considering having only tables for 4 in the restaurant.

(c) What is the largest number of tables for 4 that could be fitted into the room? Give an example of the positions for the tables which achieves this, by listing the grid references. [3]

Kate wants a layout which does not include any tables for more than 6, but does include at least one table for 2, 4 and 6.

(d) Show, by listing the grid references of the tables, that it is possible to fit 26 customers in the room. [3]

- 2 A codon is a genetic code, represented by three letters, which determines the nature of a building block used in the production of proteins. There are four possibilities (U, C, A or G) for each letter in a codon. The 64 possible codons and the building blocks they produce are as follows:

UUU → <i>phe</i>	UCU → <i>ser</i>	UAU → <i>tyr</i>	UGU → <i>cys</i>
UUC → <i>phe</i>	UCC → <i>ser</i>	UAC → <i>tyr</i>	UGC → <i>cys</i>
UUA → <i>leu</i>	UCA → <i>ser</i>	UAA → <i>stp</i>	UGA → <i>stp</i>
UUG → <i>leu</i>	UCG → <i>ser</i>	UAG → <i>stp</i>	UGG → <i>trp</i>
CUU → <i>leu</i>	CCU → <i>pro</i>	CAU → <i>his</i>	CGU → <i>arg</i>
CUC → <i>leu</i>	CCC → <i>pro</i>	CAC → <i>his</i>	CGC → <i>arg</i>
CUA → <i>leu</i>	CCA → <i>pro</i>	CAA → <i>gln</i>	CGA → <i>arg</i>
CUG → <i>leu</i>	CCG → <i>pro</i>	CAG → <i>gln</i>	CGG → <i>arg</i>
AUU → <i>ile</i>	ACU → <i>thr</i>	AAU → <i>asn</i>	AGU → <i>ser</i>
AUC → <i>ile</i>	ACC → <i>thr</i>	AAC → <i>asn</i>	AGC → <i>ser</i>
AUA → <i>ile</i>	ACA → <i>thr</i>	AAA → <i>lys</i>	AGA → <i>arg</i>
AUG → <i>met</i>	ACG → <i>thr</i>	AAG → <i>lys</i>	AGG → <i>arg</i>
GUU → <i>val</i>	GCU → <i>ala</i>	GAU → <i>asp</i>	GGU → <i>gly</i>
GUC → <i>val</i>	GCC → <i>ala</i>	GAC → <i>asp</i>	GGC → <i>gly</i>
GUA → <i>val</i>	GCA → <i>ala</i>	GAA → <i>glu</i>	GGA → <i>gly</i>
GUG → <i>val</i>	GCG → <i>ala</i>	GAG → <i>glu</i>	GGG → <i>gly</i>

For example, if the codon is ACU, the building block *thr* is made.

A mutation occurs when a **single** letter of the code is changed, e.g. if ACA is changed to ACU.

- (a) How many possible mutations are there for each codon? [1]

A **silent** mutation is one where the resulting building block stays the same. For example, CAA changed to CAG still produces *gln*.

- (b) List all the possible silent mutations of the codon CUG. [2]

- (c) Give four examples of codons with **exactly two** possible silent mutations. [3]

A sequence of codons produces a sequence of building blocks, which makes up a protein. For example:

	GAG	AGA	AUA	AAA	GAA	CUA	AGA...
produces	<i>glu</i>	<i>arg</i>	<i>ile</i>	<i>lys</i>	<i>glu</i>	<i>leu</i>	<i>arg...</i>

It is useful to be able to identify which company has produced a particular protein. To achieve this, a 'watermark' can be extracted from a sequence of codons. Each company is assigned its own watermark. Since there are many different sequences of codons that would produce the same protein, the company is usually able to choose a sequence of codons that bears its watermark.

The watermark can be read by looking at the third letter in each codon:

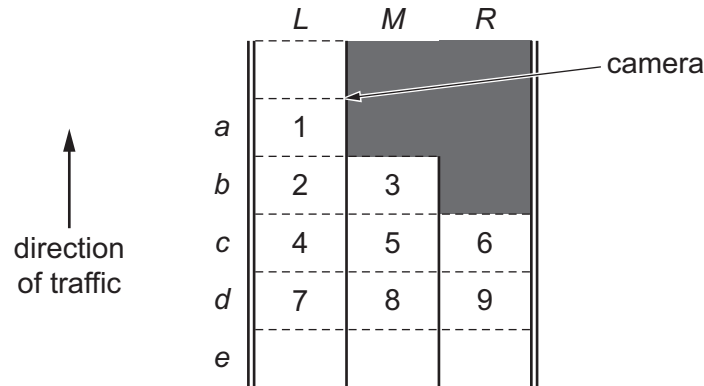
- if it is either U or A then a 0 is read;
- if it is either C or G then a 1 is read.

- (d) Give a sequence of codons which would produce the sequence of building blocks in the example above (*glu arg ile lys glu leu arg*) and bear the watermark 1010101. [3]

- (e) Which two building blocks, if needed, might not have a codon which would fit the desired watermark? [1]

- 3 Traffic officers are modelling the flow of traffic on busy multi-lane highways in situations where lanes are closed.

They envisage a simple scenario involving 9 cars on a 3-lane highway where two of the lanes are being closed. The traffic filters into the lane on the left, where it passes a camera. This is shown in the diagram below.



The flow of traffic is modelled by updating the diagram to represent the position one second later. Each second, each car can move one cell up or left, but there can never be two cars in the same cell.

When car 1 moves up past the camera, there are several possibilities for how the other cars could move during the same second. For example, car 2 could move to L_a , car 4 to L_b , car 5 to L_c , car 8 to M_c and car 9 to M_d .

- (a) What is the shortest amount of time that it could take for car 6 to pass the camera? [1]
- (b) Which of the nine cars could be the last to pass the camera? List all of them. [2]

When two lanes of traffic merge, the traffic slows. The officers now assume that a car moving left takes a second longer than moving up. This is modelled as a second of delay after the car has moved into its new lane.

- (c) Show that car 4 could pass the camera in 5 seconds, if it allows car 3 to leave in front of it. You should use diagrams to show the positions of the relevant cars at the end of each second. [3]
- (d) What is the shortest time that car 6 could take to pass the camera? State a possible order of cars passing the camera that would enable this to happen, including any gaps where no cars passed during that 1-second time interval. [2]
- (e) Consider a case where the nine cars in the diagram pass the camera in the minimum total time. State a possible order of the cars passing the camera, including any gaps. [2]
- (f) If there were 30 cars on the highway, using all three lanes, what is the shortest amount of time in which all of them could pass the camera? [2]

The traffic officers are considering what really happens when two cars could both move into the same space (one moving up, one moving left). They now assume that, in this case, the car moving to the left always takes priority over the car moving up. They model this situation, considering **only** the nine cars shown on the original diagram.

- (g) Given this assumption, state the order in which the nine cars would pass the camera, including any gaps. [3]

- 4 In the *Nyneskwaird Sudoku Tournament* competitors attempt six sudoku puzzles. The difficulty level of the puzzles increases as the tournament progresses and each puzzle has a time limit.

Points are awarded to the first three competitors to complete a puzzle as follows:

First	12 points
Second	8 points
Third	5 points

Upon successfully completing a puzzle, all competitors have their times recorded as the number of minutes (with the seconds ignored). Each competitor scores a number of points equal to the difference between the time limit and the time recorded. For instance, if a puzzle has a time limit of 50 minutes, a competitor who successfully completes it in 42 minutes 59 seconds scores 8 points. The points for the recorded time and any points for finishing in the first three are added to give the competitor's score for that puzzle.

Each competitor's final score is the sum of their **five** best scores from the six puzzles.

Ahmed, Ben, Chloe, Darren, Ekta, Fleur, Gavin and Hazel are the eight competitors taking part in this year's tournament. The scoreboard after the times for the fifth puzzle had been entered is shown below. Due to a technical error, however, Fleur's time for puzzle 4, which she did complete within the time limit, is missing from the display.

	Puzzle 1	Puzzle 2	Puzzle 3	Puzzle 4	Puzzle 5	Puzzle 6	Total score
Time limit	30 mins	45 mins	55 mins	60 mins	75 mins	90 mins	
Ahmed	17 mins	38 mins	44 mins	47 mins	60 mins		77
Ben	22 mins	34 mins	37 mins	49 mins	59 mins		89
Chloe	18 mins	29 mins	40 mins	51 mins	62 mins		82
Darren	28 mins	33 mins	38 mins	53 mins	61 mins		68
Ekta	25 mins	36 mins	45 mins	48 mins	57 mins		71
Fleur	12 mins	43 mins	41 mins		63 mins		64
Gavin	23 mins	35 mins	43 mins	44 mins	65 mins		67
Hazel	14 mins	36 mins	45 mins	52 mins	64 mins		62

They have now all completed the final puzzle, with the following results:

Hazel	67 minutes
Darren	71 minutes
Gavin	72 minutes
Chloe	73 minutes
Ahmed	75 minutes
Ekta	75 minutes
Fleur	76 minutes
Ben	77 minutes

When these times are entered into the scoreboard, the total scores will automatically adjust to display the sum of the best five scores for each of the competitors.

- (a) (i) Who is the only competitor who took less time to complete one of the puzzles than the previous puzzle? [1]
- (ii) Which competitor's times have increased by the same amount, from one puzzle to the next, throughout the tournament? [1]
- (b) (i) Gavin was the first competitor to complete puzzle 4. What was his score for this puzzle? [2]
- (ii) Each puzzle was completed first by a different competitor. Which **two** competitors were **not** the first to complete any of the puzzles? [1]

Although Fleur's time for puzzle 4 is missing, the score of 64 for her first five puzzles is correct.

- (c) What time should be displayed on the scoreboard for Fleur's puzzle 4? [3]

After being the last to complete puzzle 6, Ben is worried that he has failed to stay in the lead. However, when the scoreboard is updated it will show that he has won the tournament, finishing just 1 point ahead of Darren.

- (d) (i) The score for which puzzle will **not** count towards Ben's final score? [2]
- (ii) What is Ben's final score? [1]
- (iii) Give the order in which the other competitors have finished behind Ben and Darren. [4]

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