UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Level

## THINKING SKILLS

9694／32
Paper 3 Problem Analysis and Solution
October／November 2012
1 hour 30 minutes
Additional Materials：Answer Booklet／Paper

## Electronic Calculator

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet，follow the instructions on the front cover of the booklet．
Write your Centre number，candidate number and name on all the work you hand in．
Write in dark blue or black pen．
Do not use staples，paper clips，highlighters，glue or correction fluid．
DO NOT WRITE ON ANY BARCODES．
Calculators should be used where appropriate．

Answer all the questions．
Start each question on a new answer sheet．

At the end of the examination，fasten all your work securely together．
The number of marks is given in brackets［ ］at the end of each question or part question．

1 Study the information below and answer the questions. Show your working.
A cognitive scientist is investigating how information can be transmitted across the system of 'nodes' shown below. Most of the nodes have two pathways leading away from them, and contain a switch which alternates between sending signals down the left-hand and the right-hand pathways. Every time a signal is passed through a node, its switch changes direction (from left to right or vice versa). As can be seen, some nodes only have one pathway leading away from them (node 11 for example).

Initially, all the nodes which have two pathways leading away from them are set to send the signal to the left (as seen on the page). When the first signal is introduced at the top of the network, it will therefore travel to the following nodes: $1,2,4,7,11,16,20,23,25$. All of those nodes which the signal passes then switch to the right (if they have two pathways leading away from them).

(a) Describe the path of the second signal that passes across the network.
(b) Describe the path of the signal that is the first to travel through node 21.
(c) How many signals does it take for the first three rows of switches (nodes 1 to 6 ) to be returned to their initial positions?
(d) Which will be the last node to receive a signal, and how many signals does it take for it to be reached?
(e) After the 64th signal, all the switches have returned to their initial positions. Nodes 1 and 25 have received all the signals. Which nodes have received exactly half of the signals?

2 Study the information below and answer the questions. Show your working.
This is a list of the 'serial numbers' of the Euro banknotes in my wallet. I withdrew most of them from the cash machine yesterday and those formed a sequence of consecutively printed notes.

| €5 | €10 | €20 |
| :--- | :--- | :--- |
| P06773001949 | X51362867927 | X17261971652 |
| P06773001958 | X51362867909 | X17261971679 |
| P06773001967 | X51362867891 |  |
| P06773001985 | X51362867882 |  |
| P06773001994 |  |  |
| V07474957348 |  |  |

They do not appear to have consecutive numbers because the last digit is a 'check digit', which is calculated from the other characters in the serial number. If we add up all the digits (including the check digit) and the value for the letter, we always get a multiple of 9 . The value used for $\mathbf{P}$ is 80 , $\mathbf{V}$ is 86 , and $\mathbf{X}$ is 88 .

The letter indicates the country which commissioned the printing. For example, $\mathbf{P}$ denotes The Netherlands, V Spain, and X Germany. There are at least 12 countries issuing Euro notes.
(a) This morning I spent one of the $€ 5$ notes. What was the serial number?

There is a number which never appears as the check digit on Euro notes.
(b) What must it be and why?

A suggestion as to why there is a check digit system is that, most of the time, it can detect if the number has been mistyped. Missing a keystroke is easily detected as the number would be too short, and pressing a key twice would result in a number that was too long. Other simple typing mistakes are pressing the wrong key (e.g. 6 instead of 5 ) or switching the position of adjacent digits (e.g. 27 instead of 72).
(c) (i) In the case that the last two digits are switched, does the system detect an error always, sometimes or never? Explain your answer.
(ii) In the case that any other pair of adjacent digits is switched, does the system detect an error always, sometimes or never? Explain your answer.
(d) Does the system detect a single instance of typing the wrong digit always, sometimes, or never? Explain your answer.
(e) When does the system not detect a mistyped letter?

3 Study the information below and answer the questions. Show your working.
The small Bolandian Army has a secure communication system with just 3 stations: Westenders (W), Headquarters (HQ), and East Point (E). There are two secure links from Headquarters, one each to $W$ and $E$.

The current equipment is old (and paid for), but would last for only another 5 years. Each end of each link requires a supply of paper tape. This costs $\$ 50$ per year for each of the four ends of the two links: a total of $\$ 100$ for the HQ and $\$ 50$ for each outstation.

Two possible types of replacement equipment are available which have a new electronic system not needing paper tape, with lower annual operating costs, and each would last for 20 years. The equipment can be purchased outright; however, the suppliers are aware that there are limitations on the budgets, and offer to provide equipment and spread the payment over a number of (consecutive) years.

| Equipment type | Purchase price | Spread payment offer | Annual operating <br> cost |
| :--- | :---: | :--- | :---: |
| Securité Blue | $\$ 200$ | $\$ 70$ a year for 3 years | $\$ 10$ |
| Sicher Red | $\$ 230$ | $\$ 50$ a year for 5 years | $\$ 20$ |

Unfortunately, these two types of equipment are not interoperable; the same type is needed at both ends of any particular link. HQ needs separate equipment for each of the two links, even if they are both of the same type.

Annual budget limits (which can not be carried into the following year):

| Station | W | HQ | E |
| :--- | :---: | :---: | :---: |
| Limit | $\$ 80$ | $\$ 125$ | $\$ 75$ |

(a) (i) The budget constraints mean that no equipment can be purchased outright. Why is it not possible, even taking advantage of the spread payment offers, for both W and E to replace their equipment in the same year?
(ii) Explain why W can not replace its equipment with Securité Blue in the first year.
(iii) Although the cheapest option in the first year is to not replace any of the equipment, this would mean that it would not be possible to operate both links in the sixth year. Explain why not.
(iv) In order for it to be possible to operate both links in the sixth year, what must be the total annual cost for each of the first five years?
(b) Consider just the first eight years, assuming no change in the annual budget limits. Which station should purchase what equipment and in which year, in order to keep the total cost over this period to a minimum?
(c) (i) Looking instead over the first twenty years, assuming no change in the annual budget limits, what purchasing strategy will keep the total cost to a minimum?
(ii) What is this minimum total cost?
(d) It has been suggested that greater savings could be made over the first twenty years if there were not such strict budget limits.
(i) If the annual budget limits were amended to be simply a total of $\$ 280$ for all three stations, what would be the minimum total cost over the first twenty years?
(ii) If the annual budget limits were removed, what would be the minimum total cost over twenty years?

4 Study the information below and answer the questions. Show your working.
There are two branches of the furniture store Menestor on the island of Unitsi. One is at Candela and the other at Kelvin. Both branches will deliver purchases, if required, at the following rates:

| Total Value of Goods | Distance from Store to Delivery Address |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Local | Up to 40 km | 41 km to <br> 80 km | Greater <br> than <br> 80 km |
| Up to $\$ 300$ | $\$ 29$ | $\$ 36$ | $\$ 66$ | $\$ 167$ |
| $\$ 300.01$ to $\$ 600$ | $\$ 48$ | $\$ 53$ | $\$ 87$ | $\$ 182$ |
| $\$ 600.01$ to $\$ 900$ | $\$ 62$ | $\$ 68$ | $\$ 111$ | $\$ 198$ |
| $\$ 900.01$ to $\$ 1200$ | $\$ 87$ | $\$ 95$ | $\$ 152$ | $\$ 214$ |
| $\$ 1200.01$ to $\$ 1500$ | $\$ 118$ | $\$ 128$ | $\$ 198$ | $\$ 313$ |
| Greater than $\$ 1500$ | $\$ 147$ | $\$ 157$ | $\$ 237$ | $\$ 417$ |

The following table shows the distances (in km) between the towns on Unitsi, and is used by Menestor when quoting delivery charges.

|  |  | From |  |
| :---: | :---: | :---: | :---: |
|  |  | Candela | Kelvin |
| To | Bequerel | 118 | 63 |
|  | Candela | Local | 110 |
|  | Coulomb | 93 | 55 |
|  | Farad | 22 | 103 |
|  | Gray | 93 | 33 |
|  | Hertz | 54 | 59 |
|  | Joule | 121 | 102 |
|  | Katal | 93 | 154 |
|  | Kelvin | 110 | Local |
|  | Lux | 118 | 28 |
|  | Ohm | 55 | 113 |
|  | Newton | 116 | 117 |
|  | Pascal | 127 | 40 |
|  | Radian | 178 | 121 |
|  | Sievert | 23 | 135 |
|  | Tesla | 66 | 74 |
|  | Watt | 71 | 82 |
|  | Weber | 176 | 79 |

(a) List all the towns to which Menestor's delivery charges are the same from both stores.
(b) Megan has bought a table and six dining chairs from the Kelvin store. The table cost $\$ 450$ and the chairs were $\$ 80$ each.

How much will Menestor charge for delivery to her home in Coulomb?
Wheelduit is a company based in Candela that offers competitive rates for delivering goods from the nearby Menestor store. They charge $\$ 7$ per 10 km (or part of 10 km ) distance from the store to the delivery address, plus $4 \%$ of the value of the goods.
(c) Hector has bought bedroom furniture for a total of $\$ 950$ from the Candela store.
(i) How much could Hector save by asking Wheelduit to deliver his furniture to his home in Sievert instead of using Menestor's own delivery service?
(ii) What is the value of goods for which Menestor and Wheelduit charge exactly the same to deliver to Sievert (from Candela)?

Millie lives in Katal. She is planning to buy a sofa from the Candela store, which she knows will cost $\$ 640$.

She is considering the possibility of hiring a van from the Katal branch of Go-4-It to drive to Candela and bring the sofa back. Go-4-lt offers two options: to hire for 4 hours at a cost of $\$ 66$ (with 60 km of travel included); or to hire for a whole day at a cost of $\$ 82$ (with 160 km of travel included). In both cases, each extra kilometre travelled is charged at a rate of $\$ 0.30$.
(d) Which of the four delivery options available is the cheapest for Millie? Support your conclusion with calculations.

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