# MARK SCHEME for the May/June 2011 question paper for the guidance of teachers 

## 9694 THINKING SKILLS

9694/31 Paper 3 (Problem Analysis and Solution), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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1 (a) A different temple of four columns also has the heights 56 as seen from one 46 from another.

The arrangement must be equivalent to $\begin{array}{rllll}5 & 6 & 5 & 6 \\ ? & 4\end{array}$ or $\begin{aligned} & \text { a }\end{aligned}$.
(i) What is the greatest possible total number of blocks in this ruin?
[1]
The 'hidden' digit cannot be greater than 4 , giving a total of $5+6+4+4=\underline{19}: \begin{array}{ll}5 & 6 \\ 4 & 4\end{array}$
(ii) What is the smallest possible total number of blocks in this ruin?
[1]
The 'hidden' digit cannot be less than 0 , giving a total of $5+6+4+0=\underline{15}: \begin{array}{ll}5 & 6 \\ 4 & 0\end{array}$ OR

Candidates may assume that columns cannot have 0 blocks: $5+6+4+1=\underline{16}$
(b) Two pictures of another four-column ruin have been found.
(i) Draw a possible plan view of the ruin to show that these pictures could have been taken at the same time.

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$\begin{array}{ll}6 & 3 \\ 2 & 2\end{array}$ or equivalent. One of the 2 s could be 0 or 1.
(ii) A further picture of the same ruin was found.

From this it is now certain that the three pictures A, B and C were all taken at different times. Give the order in which they were taken, and draw a possible sequence of plan views showing the ruin at the three times.

1 mark for correct order of pictures $\mathrm{B}, \mathrm{A}, \mathrm{C}$.
A possible sequence of plans would be:

| 36 | 26 | 25 |
| :--- | :--- | :--- |
| 25 | 25 | 25 |

3 marks for a possible triplet of plans showing B, A and C.
If 3 marks cannot be awarded, award 1 mark if it is possible to move from the 1 st to the 2nd to the 3rd plan by just removing blocks/leaving them unchanged (they are 'internally consistent') AND award 1 mark if 2 of the plans could represent the views they purport to show.
(c) A larger ruin had nine columns arranged in a 3 by 3 square. Originally they cubits high. From one side it now appears as 545 and from another as 425 .
(i) What is the greatest number of blocks that can be there now?

The global maximum is reached when each element is the smaller value of the row and column maxima:

|  | 4 | 2 | 5 |
| ---: | ---: | ---: | ---: |
| 5 | $\mathbf{4}$ | $\mathbf{2}$ | $\mathbf{5}$ |
| 4 | $\mathbf{4}$ | $\mathbf{2}$ | $\mathbf{4}$ |
| 5 | $\mathbf{4}$ | $\mathbf{2}$ | $\mathbf{5}$ |

These sum to $\underline{32}$
(ii) What is the greatest number of blocks that can have been taken away?

There must be at least one of the max value in each row and column, but this local minimum is not sufficient to get the global minimum, e.g.

|  | 4 | 2 | 5 |
| ---: | ---: | ---: | ---: |
| 5 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{5}$ |
| 4 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{4}$ |
| 5 | $\mathbf{4}$ | $\mathbf{2}$ | $\mathbf{5}$ |

[1 mark for a solution such as this $(54-20=34)$
OR 1 mark for a similar solution in which the zeros are ones : 30]
By placing the 4 so as to provide both row and column, and given that the 5 s cannot be under anything smaller, we have a range of possible answers such as

|  | 4 | 2 | 5 |
| ---: | ---: | ---: | ---: |
| 5 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{5}$ |
| 4 | $\mathbf{4}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| 5 | $\mathbf{0}$ | $\mathbf{2}$ | $\mathbf{5}$ |

The 2 could be at any point in the column.
This sums to 16 , but working is required.
Originally $9 \times 6=54$ blocks, so the most taken away is $54-16=38$.
ALTERNATIVELY a solution in which the zeros are ones, giving $54-21=\underline{33}$
[2 marks for either underlined answer seen]
A correct diagram without a clear stated answer should be credited 1 mark.

## 2 (a) (i) What was Fred's number?

The last two digits of the year are split - into first and last (sixth) position, with the in 2 nd and 3 rd and the day in 4th and 5th.

17 March 1981 - 170381 would appear as 803171
(ii) Although they could make this prediction knowing the numbers and dates of birth of both lain and Jeremy, they could not be sure how the numbers were constructed by just looking at the number and date of birth of only one of them. Why not?

Each had two zeros, so couldn't be sure which one was moved to which position although might make a reasonable guess. (Because the repeats are in different places for the two people, the cases can be distinguished.) Repeated digit.
(iii) Give an example of a date of birth which would have been sufficient on its own to make this prediction with confidence.

Any date where all the six digits are distinct is necessary and sufficient. e.g. 23 Jan ' 45 or 17 March ' 82.
Condone 11 November 2011 (or 1911, 1811, ...) as a special case which would allow the particular birth date to be predicted, but no general pattern.
(b) (i) How much is added, and to which digit?

662126 would give a month 62 , and 752232 month 52 , so it must be 5 added to the tens of the month/the second digit of the six.
Award 1 mark for appreciating that the second digit is affected. Dependent on achieving the first mark, award 1 mark for " 5 " added.
(ii) What is Jocelyn's date of birth?

752232 would be (a female) born on 23 February 1972.
(c) Give an example of how this could have been done, within the six digits, without losing any of the existing information.

Add 4, 5 or 6 to the fourth digit OR add 2 or 3 to the second digit. (or 7 or $8 \bmod 10$.)
Other possible methods for distinguishing foreign-born citizens include " +31 to the date" (or any other number up to and including 69), "to the second digit: +3 for male foreign-born, +5 for female UK-born, +8 for female foreign-born".
[1 mark for any valid example]
(d) Given that a police officer can estimate a person's age to within ten years, chance that the deception would be noticed from looking at the person number on the driving licence?

As parents are at least 10 years older, the deception is certain to be noticed.
Award 1 mark for any clear indication of certain recognition by the police officer.
OR award 1 mark for an estimate of high probability (or alternatively, low probability of the deception succeeding) combined with a viable explanation of the high likelihood: likely to appeal to the 10+ year gap which (almost definitely) exists between parent and child.
(e) Using a random number for making a fake licence for a male, what is the probability that it would fail to give a valid month and date (ignoring the year)?

10000 possible numbers, of which only $366 / 365$ are valid. So (10000-366)/10000 = 96.34\% or 96.35\% o.e.

If 2 marks cannot be awarded, award 1 mark for
EITHER using 9999 instead of 10000
OR estimating the probability of specifying a valid date as $12 / 100 \times 31 / 100$ (giving a final answer of $96.28 \%$ ) or $12 / 100 \times 30 / 100$ (giving $96.4 \%$ )
OR using 99 instead of 100 in the method referred to above (giving 96.20\%)
AND/OR solutions giving the probability of succeeding, rather than failing (not subtracting from 1).

3 (a) Consider a block with dimensions $2 \mathrm{~m} \times 2 \mathrm{~m} \times 6 \mathrm{~m}$. Calculate the minimum possible time that it would take to roll the block through $360^{\circ}$.

Two orientations are possible.

- Along a $2 \times 2$ face : time $=1+1+1+1=4$ minutes
- Along a $6 \times 2$ face : time $=3+1 / 3+3+1 / 3=6$ minutes 40 seconds

So 4 minutes the minimum possible time.
(b) Consider a block with dimensions $1 \mathrm{~m} \times 4 \mathrm{~m} \times 6 \mathrm{~m}$. Calculate all the different possible distances that the block could travel in one $360^{\circ}$ revolution, according to the different initial orientations.

Three orientations are possible.

- Along the $1 \times 4$ face : distance $=1+4+1+4=10$ metres
- Along the $1 \times 6$ face : distance $=1+6+1+6=14$ metres
- Along the $6 \times 4$ face : distance $=6+4+6+4=20$ metres

Award no marks if extra distances are given.
(c) If a $24 \mathrm{~m}^{3}$ block is to travel at least 610 m , what is the smallest possible num turns that will be needed?

The face with the largest perimeter is needed to minimise the number of turns.
For integer solutions, this can be seen to be the $24 \times 1$ face, whose perimeter is 50 metres.
$610 / 50=12$ revolutions with 10 metres remaining. Being able to choose whether the block is standing upright or lying down at the beginning allows us to choose which way it is standing at the end. Clearly, if upright it can pass the 610 finish line with one more $90^{\circ}$ turn: 49 turns in total.
3 marks for the correct answer [underlined above].
If 3 marks cannot be awarded, award 2 marks if the number of complete revolutions or half revolutions is seen for the $24 \times 1$ case OR if the number of quarter turns for a reasonable alternative orientation is given. $(12 \times 2,8 \times 3,6 \times 4,6 \times 2,4 \times 3,3 \times 2,2 \times 2,1 \times 1)$.
If 2 marks cannot be awarded, award 1 mark if the number of complete revolutions or half revolutions for the one of the suboptimal solutions (given in brackets above) is seen.
(d) What dimensions for a $24 \mathrm{~m}^{3}$ block will allow for the smallest possible time to move it 610 m ? State the time it will take, to the nearest minute.

The numerical space to explore is shown in the table below.
It can be seen that the face which is fastest to move along is that with dimensions $4 \mathrm{~m} \times 6 \mathrm{~m}$.
This is sufficiently clear-cut to short-circuit any need to consider any peculiarities that might be thrown up by the need for a whole number solution.
Fastest block's dimensions: $4 \mathrm{~m} \times 6 \mathrm{~m} \times 1 \mathrm{~m}$.
Time taken to complete 610 metres:
$610 /(6+4+6+4)=30$ complete revolutions, with 10 metres remaining.
Time taken $=\left(30 \times 4 \frac{1}{3}\right.$ minutes $)+3 / 2$ minutes $+2 / 3$ minutes
$=132$ minutes

| length | width | perimeter | $360^{\circ}$ time | $610 /$ <br> perimeter | approx. <br> time | Precise <br> time <br> [mins] | precise <br> time <br> runded <br> (mins) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 24 | 50 | 48.083 | 12.2 | 586.617 | 577.042 | 577 |
| 1 | 12 | 26 | 24.167 | 23.462 | 566.987 | 555.917 | 556 |
| 1 | 8 | 18 | 16.25 | 33.889 | 550.694 | 544.5 | 545 |
| 1 | 6 | 14 | 12.333 | 43.571 | 537.381 | 536.667 | 537 |
| 1 | 4 | 10 | 8.5 | 61 | 518.5 | 518.5 | 519 |
| 1 | 3 | 8 | 6.667 | 76.25 | 508.333 | 507 | 507 |
| 1 | 2 | 6 | 5 | 101.667 | 508.333 | 508 | 508 |
| 1 | 1 | 4 | 4 | 152.5 | 610 | 610 | 610 |
| 2 | 12 | 28 | 12.333 | 21.786 | $\mathbf{2 6 8 . 6 9 0}$ | $\mathbf{2 6 5 . 3 3 3}$ | $\mathbf{2 6 5}$ |
| 2 | 6 | 16 | 6.667 | 38.125 | $\mathbf{2 5 4 . 1 6 7}$ | $\mathbf{2 5 3 . 6 6 7}$ | $\mathbf{2 5 4}$ |
| 2 | 4 | 12 | 5 | 50.833 | $\mathbf{2 5 4 . 1 6 7}$ | $\mathbf{2 5 3}$ | $\mathbf{2 5 3}$ |
| 2 | 3 | 10 | 4.333 | 61 | $\mathbf{2 6 4 . 3 3 3}$ | $\mathbf{2 6 4 . 3 3 3}$ | $\mathbf{2 6 4}$ |
| 2 | 2 | 8 | 4 | 76.25 | 305 | 305 | 305 |
| 3 | 8 | 22 | 6.083 | 27.727 | $\mathbf{1 6 8 . 6 7 4}$ | $\mathbf{1 6 7 . 6 6 7}$ | $\mathbf{1 6 8}$ |
| 3 | 4 | 14 | 4.167 | 43.571 | $\mathbf{1 8 1 . 5 4 8}$ | $\mathbf{1 8 2}$ | $\mathbf{1 8 2}$ |
| 4 | 6 | 20 | 4.33 | 30.5 | $\mathbf{1 3 2 . 1 6 7}$ | $\mathbf{1 3 2 . 1 6 7}$ | $\mathbf{1 3 2}$ |

4 marks to be awarded if the underlined time is given [choice of block size is implic
If 4 marks cannot be awarded, award 3 marks for a suboptimal time less than 300 min see table above for the options [in bold].

If 3 marks cannot be awarded, award 2 marks for a suboptimal time more than 300 minutes: see table above for the options [not in bold].

If 2 marks cannot be awarded, award 1 mark for a solution showing a method in which the speed for one revolution (shown in the fourth column in the table) is multiplied by the number of revolutions (shown in the fifth column), allowing for numerical errors.
(e) Show that it is possible for him to move exactly $\mathbf{6 1} \mathrm{m}^{3}$ of stone in less than 500 minutes.

There are two ways to achieve a time under 500 minutes:

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20 + 20 + 21 = (139 mins 24 secs) + 5 mins + (139 mins 24 secs) + 5 mins + (168 mins 29
secs) = 457 mins 17 secs
21+24+16 = (168 mins 29 secs) + 5 mins + (132 mins 10 secs) + 5 mins + (153 mins) =
463 mins 39 secs
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4 marks for either of the above solutions, with rounding condoned [e.g. $168+5+132+5+$ $153=463 \mathrm{mins}]$.

If 4 marks cannot be awarded, 3 marks for any correctly calculated answers which add up to $61 \mathrm{~m}^{3}$ [allowing for rounding]. Allow for rounding errors mid-calculation [i.e. make no deductions] throughout this question, provided it is supported by clear working.
For instance:
$24+24+13=(132$ mins 10 secs $)+5$ mins $+(132$ mins 10 secs $)+5+(562$ mins 23 secs $)=$ 836 mins 43 secs
$22+21+18=(267 \mathrm{mins} 3 \mathrm{secs})+5 \mathrm{mins}+(168 \mathrm{mins} 29 \mathrm{secs})+5 \mathrm{mins}+(170 \mathrm{mins})=615$ mins 32 secs
$15+15+15+16=(172$ mins 52 secs $)+5$ mins $+(172$ mins 52 secs $)+5$ mins $+(172$ mins 52 secs $)+5$ mins $+(153 \mathrm{mins})=686$ mins 36 secs
$24+22+15=(132$ mins 10 secs $)+5 \mathrm{mins}+(267 \mathrm{mins} 3 \mathrm{secs})+5 \mathrm{mins}+(172 \mathrm{mins} 52$ secs) $=582$ mins 5 secs

If 3 marks cannot be awarded, award 1 mark for a choice of block sizes (each less than $24 \mathrm{~m}^{3}$ ) which add up to $61 \mathrm{~m}^{3}$, AND 1 mark for a calculation of the time taken to move a block 610 m , so long as the calculation has not been credited in (d).

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| length | width | perimeter | $360^{\circ}$ time | 610/perimet er | approx time | precise time (mins) | roun (mi |
| 1 | 13 | 28 | 26.154 | 21.786 | 569.780 | 562.385 | 562 |
| 1 | 14 | 30 | 28.143 | 20.333 | 572.238 | 562.929 | 563 |
| 2 | 7 | 18 | 7.571 | 33.889 | 256.587 | 253.929 | 254 |
| 1 | 15 | 32 | 30.133 | 19.063 | 574.417 | 572.6 | 573 |
| 3 | 5 | 16 | 4.533 | 38.125 | 172.833 | 172.867 | 173 |
| 4 | 4 | 16 | 4 | 38.125 | 152.5 | 153 | 153 |
| 1 | 20 | 42 | 40.1 | 14.524 | 582.405 | 581.5 | 582 |
| 2 | 10 | 24 | 10.4 | 25.417 | 264.333 | 265 | 265 |
| 4 | 5 | 18 | 4.1 | 33.889 | 138.944 | 139.4 | 139 |
| 2 | 11 | 26 | 11.364 | 23.462 | 266.608 | 267.045 | 267 |
| 3 | 6 | 18 | 5 | 33.889 | 169.444 | 170 | 170 |
| 3 | 7 | 20 | 5.524 | 30.5 | 168.476 | 168.476 | 168 |

(f) What set of block sizes should he move to minimise his total time?

The quickest way of transporting between 61 and $70 \mathrm{~m}^{3}$ of stone is to move two $24 \mathrm{~m}^{3}$ blocks and one $20 \mathrm{~m}^{3}$ block:
$24+24+20=(132 \mathrm{mins} 10 \mathrm{secs})+5 \mathrm{mins}+(132 \mathrm{mins} 10 \mathrm{secs})+5 \mathrm{mins}+(139 \mathrm{mins}$ 24 secs) $=413$ mins 44 secs.
2 marks for identifying the correct combination of blocks: $24+24+20$.
If 2 marks cannot be awarded, award 1 mark for $24+20+20$ or $24+24+16$.

4 (a) After which heat did the Sharks first take the overall lead in the match?
Answer: Heat 7 OR Bears 52 - Sharks 53 [team must be identified in some way] If 2 marks cannot be awarded, award 1 mark for evidence of correct assessments of the situation after at least two of the heats between Heat 2 and Heat 6 inclusive:
i.e. Heat 2: 17-13 or Bears lead / Sharks trail by 4 points.

Heat 3: 23-22 or Bears lead / Sharks trail by 1 point.
Heat 4: 33-27 or Bears lead / Sharks trail by 6 points.
Heat 5: 39-36 or Bears lead / Sharks trail by 3 points.
Heat 6: 46-44 or Bears lead / Sharks trail by 2 points.
(b) (i) What is the maximum total distance (in metres) that one skater could possibly skate during an eightskate match?

Answer: 10800 m (accept: 10.8 km )
If 2 marks cannot be awarded, award 1 mark for appreciation that a skater may compete in a maximum of 6 heats, or an answer of 36 laps, or an answer of 1800 m .
(ii) What is the total distance (in metres) that Edward Ursa has skated so this match?

Answer: 5700 m (accept: 5.7 km )
If 2 marks cannot be awarded, award 1 mark for appreciation of at least two of the following:
5 laps ( 1500 m ) in Heat 2.
6 laps ( 1800 m ) in Heat 4.
2 laps $(600 \mathrm{~m})$ in Heat 7.
6 laps ( 1800 m ) in Heat 10.
Alternatively, award 1 mark for an answer of 19 laps or 950 m .
(c) Select a possible line-up of three skaters to compete in Heat 12 for
(i) the Bears,

Answer: Brown, Honey and Polar or Brown, Kodiak and Polar or Brown, Polar and Ursa or Honey, Polar and Ursa or Kodiak, Polar and Ursa

Polar has only appeared in three heats, so must skate.
Black, Grizzly and Malay cannot appear in consecutive heats.
Honey, Kodiak and Polar appeared together in Heat 3.
If 2 marks cannot be awarded, award 1 mark for any line-up that contains Polar, or a line-up that consists of any three of Brown, Honey, Kodiak and Ursa (except Brown, Kodiak and Ursa, who appeared together in Heat 10).
(ii) the Sharks.

Answer: Angel, Hammer and Head or Angel, Head and Horn or Hammer, Head and Horn

Head has only appeared in three heats, so must skate.
Basking, Nurse and White cannot appear in consecutive heats.
Angel, Head and Tiger appeared together in Heat 7.
Head, Horn and Tiger appeared together in Heat 10.
Hammer, Head and Tiger appeared together in Heat 4.
If 2 marks cannot be awarded, award 1 mark for any line-up that contains Head, or a line-up that consists of any three of Angel, Hammer, Horn and Tiger.
(d) (i) Assuming that no skater drops out, is it possible for any eightskate ma in a draw? Explain your answer.

Answer: It is not possible because:
total points in a match is an odd number.
Deduct 1 mark if the odd number given is incorrect (not equal to 189).
Deduct 1 mark if the justification is given, but no clear conclusion.
If 2 marks cannot be awarded, award 1 mark if the candidate explains the impossibility of a draw by referring to a particular game (such as the one given in the question, or one in which a draw is attained after 10 matches).
(ii) Rupert, a keen Bears fan, fears that his team can only win the match in progress if they secure first position in Heat 12. Is he right? Explain your answer.

Answer: No, he is not right. Award 1 mark for reference to EITHER the current score is $82-83 /$ the Bears are 1 point behind OR they need 13 points to win the match / 2 more than the Sharks OR the final score is $95-94$ / the Bears are 1 point ahead.

Award 2 marks for evidence that this can be achieved with 2nd, 3rd and 4th positions ( $7+4+2$ points).

A correct conclusion must be seen for the candidate to be awarded 3 marks.

