UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9694 THINKING SKILLS

9694/33 Paper 3 (Problem Analysis and Solution), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

1 (a) Give an example of an arrangement of villas that would maximise his profit.

The constraint only demands that cycles are of length at least 3, and it pays to have more of the most profitable villas. We do not need to know the actual profits, just the relative order.

Only **R P N R P N R** or its reverse has three **R**s and attains the maximum.

The regulation requiring a villa not to be the same as those next door or adjacent to them could be misread as demanding that any cycle would have to be at least 5 long.

2 marks for fully correct
1 mark for a sequence such as **R P L M N R P** with two **R**s and two **P**s, or three **R**s

(b) (i) If reversed designs are permitted, how many of each type of villa will there be in the most profitable combination? [1]

The use of **R*** as well as **R** would provide for <u>five **R** or **R*** and two **P** or **P***</u>.

(ii) Give two examples of how these might be arranged. [2]

Any valid sequences e.g. **R P R* R P R* R** or **R* R P R* R P* R***.

2 marks; 1 mark for each valid example, deduct 1 mark for each additional incorrect answer

(c) (i) How many *Lismore* villas (or mirror images) will he need to include? [1]

Having nine villas means that each villa counts for 11%, so two would pass the 20% minimum, and any more would reduce profits. Hence must be <u>exactly 2</u>.

(ii) How many of each type of villa will there now be in the most profitable combination? [1]

Two **L**/**L***, one **P**/**P***, three **R**, and three **R***.

(iii) Give one example of how these might be arranged. [1]

Any valid sequence, for example, **R R* L R R* L* R R* P**.

(d) Give three examples of arrangements for the nine plots that would maximise the developer's profit. [2]

Any three correct arrangements, including reversing the order or rotating the sequence.
e.g. **R P L R P N R P L**

2 marks for all three, 1 mark for two correct

2 (a) **How many gaps are there between the bars?**

There are 30 bars and thus 30–1 gaps.

(b) **Given this information about 0, identify a pattern of two bars and two spaces, starting with B, that could never represent a digit on the right-hand side. Explain why it could not be found on the right-hand side. [3]**

The patterns on the left are constructed by starting with white not black and having a pattern of changes that is in the same or in the opposite order.
Patterns can be inverted or reversed, so **BBBWWBW** becomes **WWWBBWB** or **WBWWBBB**

Answer: The inverted reversed pattern **BWBBWWW (0)** or **BWWBBWW (1)**
cannot be used on the right hand side because it would be confused with the ways of writing a **0/1** on the left.

2 marks for a correct pattern, 1 mark for reason

(c) **What are the two forms that the pattern for the digit 1 could take on the left-hand side? [2]**

BBWWBBW can be inverted or reversed, so becomes either **WWBBWWB** or **WBBWWBB**.

2 marks – 1 mark each

(d) **What is the total number of such combinations that have not yet been allocated, but would be safe to use because they would not lead to possible ambiguity in reading the barcode? [4]**

There are 2*2*2*2*2*2 possible directions, but one of them would be indistinguishable from the right hand side written backwards, so 64–1=63, of which 10 are already allocated to digits, so <u>53</u> in all.

1 mark for attempting to use powers of 2, 1 mark for 64, 1 mark for –10, 1 mark for –1

3 **(a) Calculate the percentage of the lower logs that will be submerged in a “3+2”**

Percentage of the five logs submerged = 0.59 x 5 = 2.95 logs.

Percentage of the lower logs submerged = $\frac{2.95}{3} \times 100 = 98.3\%$

2 marks for the correct answer given: 98.3%.
1 mark for 2.95 or 2.05 seen.

(b) Considering only two-layer rafts, what is the smallest total number of logs which could result in submerging the entire bottom row? Justify your answer. **[3]**

“4+3” will put the entire bottom layer under water
Total volume = 7 units
Amount above water = 0.41 × 7 = 2.87 units
Number of logs above water = 3

Algebraically:
Let x = the bottom row. Top row = x – 1.
(*Justify x – 1 consider odd and even cases separately)*
Amount above water = 0.41 (2x – 1) = 0.82x – 0.41
Amount of bottom row above water = 0.82x – 0.41 – (x – 1) = 0.59 – 0.18x
When is this negative? 0.59 – 0.18x < 0
0.59 < 0.18x
3.277.. < x so x = 4 or more

1 mark for a correct calculation of the amount submerged for an “(n) + (n – 1)” raft.
1 mark for calculating the proportion submerged in a “4+3” or “4.13 logs submerged” seen.
1 mark for an explicit attempt to justify the answer's minimality.

(c) If a certain journey requires rafts to be no more than 10 logs wide and two layers high, what is the largest number of dry logs (ones that are completely above the waterline) that can be transported? Justify your answer. **[3]**

“10+7” : 17 in total
Amount above water = 0.41 x 17 = 6.97m^2
Number of logs in top layer = 7

“10+6” : 16 in total
Amount above water = 0.41 x 16 = 6.56m^2
Number of logs in top layer = 6

(“9+6” also supports 6 dry logs)

So the largest number is 6.

Algebraically:
Let x = the number of the top layer. Total logs = 10 + x
Amount above water = 0.41 (10 + x) = 4.1 + 0.41x
Amount of logs above water = x
$4.1 + 0.41x > x$
$4.1 > 0.59x$
$6.949 > x$ so the maximum integer x can be is 6.

1 mark for correct answer (6).
1 mark for demonstrating that this combination is possible.
1 mark for demonstrating that 7 is impossible.

(d) In how many ways can 8 logs be transported, if rafts can be of any height (but still must obey the four constraints given at the beginning)? State the arrangement of each of the different ways it can be done. [3]

Four ways: 8 or 6+2 (two ways) or 5+2+1

3 marks for all four correct. 2 marks for three correct. 1 mark for two correct.
Deduct 1 mark for each incorrect arrangement [minimum 0].

(e) What is the narrowest raft that can transport exactly 8 dry logs, if the rafts can be of any height? You must state the arrangement of logs in your answer. [4]

Possible arrangements that candidates may consider:

8 + 7 + 6 + 5 + 4 + 3 + 2: 60% in bottom three layers: **14 dry**
7 + 6 + 5 + 4: 59% in bottom two layers: **9 dry**
<u>7 + 6 + 5 + 2 + 1: 61% in bottom 2 layers: **8 dry**</u>
7 + 6 + 5 + 4 + 3: 52% in bottom two layers: 7 dry
6 + 5 + 4 + 3 + 2 + 1: 11/21 = 52%: 3rd layer submerged – only **6 dry**
6 + 5 + 4 + 3 + 2: 55%: 3rd layer submerged – only **5 dry**
6 + 5 + 4 + 3: 11/18 = 61%: **7 dry**
5 + 4 + 3 + 2 + 1: 60% in bottom 2 layers: **6 dry**

4 marks for underlined answer.
If 4 marks cannot be awarded:
award 1 mark for a solution involving less than 12 logs [e.g. 8+7+6+2, 8+7+5+4+3+1, 9+8+5+2+1, 10+9+8, 11+10+8], irrespective of symmetry considerations;
award 1 mark for any raft that will transport at least 8 dry logs;
award 1 mark for improving on a previous example, or calculations showing the number of dry logs for any new arrangement of logs.

4 (a) How long does the journey from Quince to Starveling take

(i) on the Titania – Oberon line?

Answer: 23 minutes

(ii) on the Pyramus – Thisbe line? [1]

Answer: 37 minutes

(b) How many trams passed through Snout station while Nicola was waiting for Frances? [2]

Answer: 8
(11:01, 11:21 and 11:41 towards Theseus
11:08 and 11:28 towards Hippolyta
11:03 and 11:33 towards Pyramus
11:23 towards Thisbe)

If 2 marks cannot be awarded, award 1 mark for evidence of
either
- 5 trams on the Theseus – Hippolyta line

or
- 3 trams on the Pyramus – Thisbe line

(c) Peter has boarded the tram that will leave Thisbe at 06:45. His destination is Lysander, and he intends to change lines at Starveling and Snug.

(i) What time will he arrive at Lysander (assuming he will take the first available tram at both Starveling and Snug)? [3]

Answer: 07:41

If 3 marks cannot be awarded, award 1 mark each (to a maximum of 2 marks) for evidence of any of the following:
- arrival at Starveling at 06:52
- departure from Starveling at 07:06
- arrival at Snug at 07:14
- departure from Snug at 07:31

(ii) How much will his three tickets cost altogether? [1]

Answer: 155 cents (or $1.55)

(d) It is 15:25 and Robin has just arrived at Cobweb station. He needs to ge before 16:30.

(i) How many different routes are available to Robin? Give the arrival times at Sn for all the routes you consider. [5]

Answer: 3 routes (stated or by implication)

- (change at Snout to Theseus –Hippolyta line) arrival at 16:11
- (change at Quince to Titania – Oberon line) arrival at 16:24
- (change at Starveling to Titania – Oberon line) arrival at 16:14
- evidence of rejection of changing at Quince to the Theseus – Hippolyta line (arrival at 16:38)

(ii) How much will it cost him if he takes the cheapest option? [2]

Answer: 190 cents (or $1.90)
(40 + 2 × 75; 16:24 arrival)

- correct calculation of at least one of the other options:
  195 cents (3 × 40 + 75; 16:14 arrival)
  230 cents (2 × 40 + 2 x 75; 16:11 arrival)