ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates are reminded of the importance of checking that they have fulfilled the requirements of the question they are answering. In this paper there were examples of not enough work having been done and also examples of too much work having been done. Whilst the latter case does not actually affect the marks available, it is time consuming and could therefore affect the time left to complete the paper.

There are still candidates who do not work to the correct level of accuracy throughout a question, giving an answer that is also not of the required level of accuracy. Checking the rubric on the front page of the question paper and also any differing demands in the question itself is essential.

In questions where either a calculator is not permitted or there is a demand for all working to be shown, it is important that candidates do not miss out stages in their working as this may affect the mark allocation.

General comments

There were many excellent scripts showing that candidates had been prepared well for the examination. Most candidates were able to produce solutions that were well set out and easy to follow. For those candidates who needed extra space for particular questions, it was pleasing to see the blank page at the end of the answer booklet being made use of as well as the addition of extra sheets. It is much better for a candidate to be able to set out their work clearly rather than try to fit it in a reduced space which may make it difficult to follow.

Comments on specific questions

Question 1

Many correct solutions were seen, with candidates showing a good understanding of both the remainder theorem and the factor theorem. Any errors were usually due to arithmetic slips in the simplification of the resulting equations or the solution of these equations simultaneously. For those candidates that misunderstood the relationship between the remainders, marks were available for the case $2p(-3) = p(2)$.

Answer: $a = 10$, $b = -12$

Question 2

(i) There were very few candidates who did not write down a completely correct derivative.

(ii) Most candidates produced a correct equation for the tangent to the given curve making use of their answer to part **(i)**. As a specific form of this equation had not been required, equations that were exact and unsimplified, exact and simplified, and simplified with decimals were all acceptable. A few candidates made errors in the interpretation of the required gradient, but most found the y-coordinate at the given point with ease.

Answers: **(i)** $15\cos 3x$ **(ii)** $y - 4 = -15\left(x - \frac{\pi}{3}\right)$

Question 3

As the use of a calculator was not permitted in this question, it was important that candidates showed each stage of their working in full. Candidates who did not do this were penalised.

(a) It was essential that an expansion of the brackets in the numerator of at least three terms was seen before simplification. Evidence of rationalisation together with at least three terms in the expansion of the brackets in the numerator was also essential for obtaining the method marks available. Some candidates chose to rationalise first and then expand out and simplify the terms in the numerator. This was equally acceptable provided sufficient terms were shown in this simplification. There were many completely correct solutions seen.

(b) Candidates were expected to make use of the cosine rule in this part. The information $\cos ABC = -\frac{1}{2}$ was given to candidates as the use of a calculator was not permitted. This did not stop some candidates working out that the angle was 120° and then finding the cosine of this angle again later in the question. It was essential that the cosine rule was written out with the lengths and the trigonometric ratio substituted in correctly. Evidence of the expansion and simplification of each of the terms in the cosine rule was expected, showing that any terms involving surds cancelled out. Some sign errors were made, mostly by candidates who had chosen to unnecessarily work out the angle in degrees or radians. A few candidates lost the final accuracy mark by not giving their answer in the correct form but leaving it as $\sqrt{120}$.

Answers: **(i)** $2+2\sqrt{5}$ **(ii)** $2\sqrt{30}$

Question 4

(i) Very few correct solutions were seen. Many candidates realised that y is not defined when $x = -2$ and were able to obtain a mark for this. Many candidates also realised that $4x^2 - 1 \neq 0$ and obtained $x = \pm\frac{1}{2}$ giving this as a solution. What most did not realise was that $4x^2 - 1$ could not be negative either and that $x = \pm\frac{1}{2}$ were the critical values required for an inequality. This part of the question was a good discriminator, with very few candidates obtaining both the required answers.

(ii) Nearly all candidates were awarded full marks for the differentiation of the quotient. Any errors tended to be in the differentiation of $\ln\left(4x^2 - 1\right)$ rather than anything else. Those that chose to differentiate using the product rule were equally successful. Simplification of the result was unnecessary.

(iii) It was expected that candidates would substitute $x = 2$ into their answer to part **(ii)** and evaluate it prior to using the method of small changes. However, many candidates were not able to do this correctly in spite of having a correct answer for part **(ii)**. Too many candidates were not awarded the accuracy mark due to inaccuracy. A final answer of three significant figures is expected in this paper unless otherwise stated. Too many candidates gave final answers of 0.097h or 0.01h.

Answers: **(i)** -2, $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$ **(ii)** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+2)\frac{8x}{4x^2-1} - \ln\left(4x^2-1\right)}{(x+2)^2}$ **(iii)** $0.0974h$

Question 5

(i) Many candidates produced a completely correct solution for this part of the question, showing a good understanding of the binomial theorem. The main error was when candidates did not consider the negativity of the second term and thus had an incorrect answer of $a = \frac{1}{4}$. It was pleasing to see that most candidates dealt with a^2 when attempting to find b.

(ii) This part of the question was not completed as successfully as the first part, with some candidates still not understanding the meaning of the expression 'independent of x'. Most were able to expand $\left(x - \frac{1}{x}\right)^2$ correctly and then attempt to use their answer to part **(i)**. It was essential that candidates realised that it was necessary to consider two terms in order to get the final answer.

Answers: **(i)** $n = 10$ $a = -\frac{1}{4}$, $b = 720$ **(ii)** -1328

Question 6

(i) The great majority of candidates found the correct vector.

(ii) Many candidates were able to use the given ratio correctly and obtain the correct vector making use of their answer to part **(i)**. There were the occasional sign errors and errors in the fraction used from an incorrect consideration of the given ratio.

(iii) The great majority of candidates found the correct vector, having made the correct use of the given ratio.

(iv) It was expected that candidates make use of their answers to parts **(ii)** and **(iii)** by equating them and simplifying. Many candidates did just that and obtained a correct result. Some candidates did not appreciate that they had found the same vector, but in different terms, in the last two parts and made no use of them. Any alternative methods attempted met with little success.

(v) It was expected that candidates make use of their result from part **(iv)** and the vector $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ in order to obtain the required result. Many candidates were able to do just this, provided they had recognised the links between all the previous parts in this question.

Answers: **(i)** $\mathbf{c} - \mathbf{a}$ **(ii)** $\frac{2}{3}\mathbf{c} + \frac{1}{3}\mathbf{a}$ **(iii)** $\frac{3}{5}\mathbf{b}$ **(iv)** $9\mathbf{b}$ **(v)** $-\frac{4}{9}\mathbf{a} + \frac{10}{9}\mathbf{c}$

Question 7

(a) Most candidates were able to make use of the determinant correctly, obtain a quadratic equation and solve it thus obtaining full marks.

(b) (i) Nearly all candidates wrote down a correct inverse matrix.

(ii) The use of the word 'Hence' is an indication that candidates were expected to make use of their answer to part **(i)** in order to find the matrix **C**. Those candidates that attempted to use a simultaneous equation method did not score any marks. Most attempted a correct pre-multiplication using their inverse matrix. There were the occasional arithmetic slips in the evaluation of the matrix multiplication, but many correct responses were seen.

(c) This part of the question was intended to test the candidates' recognition of the zero matrix. It was evident that some candidates were unaware of this even though it is a syllabus requirement.

Answers: **(a)** $2,\ -\frac{3}{2}$ **(b)(i)** $\frac{1}{5}\begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$ **(ii)** $\frac{1}{5}\begin{pmatrix} 11 & -5 \\ -12 & 10 \end{pmatrix}$ **(c)** $\begin{pmatrix} -\frac{3}{4} & 0 \\ 0 & -\frac{3}{4} \end{pmatrix}$

Question 8

(i) Integration was attempted by most candidates. Very few candidates were unable to obtain $6e^{2t} - 24t^2$ but many did not consider the arbitrary constant and that they had enough information to find its value.

(ii) Most candidates differentiated correctly and attempted to equate their result to zero and solve. It was acceptable for a candidate to leave their answer in a simplified exact form as no form had been specified. However, those candidates that chose to use their calculators often lost the final accuracy mark by either not giving their answer to three significant figures or by incorrectly rounding their answers in their working.

(iii) Candidates were expected to substitute their value from part **(ii)** into the velocity equation. Most did attempt this, but there were many incorrect answers seen, some due to incorrect evaluation and some due to incorrect significant figures used in the final answer.

Answers: **(i)** $6e^{2t} - 24t^2 - 6$ **(ii)** 0.347 **(iii)** 7.36

Question 9

It was intended that candidates make use of natural logarithms throughout this question. There were a few candidates who chose to use base 10 logarithms and they were not penalised provided they had used a correct approach for base 10 logarithms.

(i) Most candidates were able to write down the correct straight line form.

(ii) Many correct graphs were seen, with axes labelled correctly and a straight line passing through all the points drawn. It should be noted that space is left under the given table so that candidates may add their own row(s) to help with the plotting of the graph. Inevitably there were candidates who just plotted the given points thus obtaining a curve. Such candidates were unable to score marks in part **(iii)** which required the use of the straight line graph obtained.

(iii) Many candidates chose to make use of the gradient of the graph and extrapolate to find the intercept on the *y*-axis. Many correct solutions were seen. It was also acceptable to form two simultaneous equations in terms of *A* and *b*, provided the values used were on the straight line graph.

(iv) and (v) Candidates either used their graphs to estimate the required values or made use of their values of *A* and *b* correctly in an appropriate equation. Most candidates were able to obtain a method mark for these parts even if they had incorrect values of *A* and/or *b*, provided a correct approach was taken.

Answers: **(i)** $\ln y = \ln A + bx$ **(ii)** $-0.45 \leqslant b \leqslant -0.55$ $1900 \leqslant A \leqslant 2100$ **(iii)** $2.2 \leqslant x \leqslant 3.0$ **(iv)** $155 \leqslant y \leqslant 175$

Question 10

(i) Candidates chose to either expand out the given brackets in the given equation and differentiate, or differentiate as a product. Each method was equally popular and equally successful. Many candidates obtained the correct *x*-coordinates. Some 'spotted' that $x = -2$ from the repeated factor in the equation. This was acceptable even though this is not on this syllabus.

It was here that many candidates made extra work for themselves by finding the *y*-coordinates of *A* and of *B*; this was not required. Many also went on and found the nature of each of these stationary points. Again, this had not been requested. The allocation of the correct *x*-coordinate to the correct point could be done by observation of the given diagram. Candidates must ensure that they are doing what is requested.

Many correct solutions were seen in spite of all the extra unnecessary work done.

(ii) A very straightforward exercise for which the great majority of candidates obtained both marks provided they gave their answers as coordinates.

(iii) Those candidates who had chosen to expand out the brackets of the equation in part **(i)** were at a slight advantage over those who had used the product rule in part **(i)** as they already knew what terms they had to integrate. Many correct solutions were seen with candidates showing all the necessary working to obtain the marks. It was expected that the integral of each term was seen and that it was clear that the correct limits had been substituted in, correctly, to obtain a second method mark. Those candidates who did not show this step and just gave a numerical answer were not awarded the final two marks as they had not shown all the necessary working. An exact answer was acceptable as well as a decimal answer, but some candidates who used decimals did not give their final answer to three significant figures.

Answers: **(i)** -2, $-\frac{4}{9}$ **(ii)** $C = (0, 4)$ and $D = \left(\frac{1}{3}, 0\right)$ **(iii)** $\frac{241}{324}$ or 0.744

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To succeed in this examination, candidates need to offer logical and complete solutions and show sufficient method so that marks can be awarded. Candidates need to be aware of instructions in questions such as 'Showing all your working...' Such instructions mean that when a solution is incomplete a significant loss of marks will result. Candidates should ensure that their answers are given to no less than the accuracy demanded in a question. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. Careful attention should be given to the accuracy required for angles in degrees, which is different to the accuracy required for angles in radians.

General comments

The majority of candidates were very well prepared for this examination and many excellent scripts were seen. Candidates showed good knowledge of techniques and were able to successfully combine skills to solve problems when needed. Most candidates were careful about the presentation of their work and their logic was clear and easy to follow. Some candidates made good use of the blank pages at the end of the paper. This was very sensible when they made several attempts, such as when answering **Question 5** and **Question 9(ii)**. It ensured that their work was readable and could be marked. Most candidates who did this also added a note to indicate that their answer was written, or continued, elsewhere. This was very helpful. Candidates whose presentation was more randomly organised often made more errors in their thinking as their logic was not at all clear.

Many candidates gave fully worked solutions, showing a complete method. Other candidates could improve by showing **all** the key steps of their method. This is even more important if they make an error. Showing clear and full method is essential if a question asks candidates to 'Show that…' This instruction indicates that the answer has been given and that the marks will be awarded for showing how that answer is found. The need for this was highlighted in **Question 11(a)(i)** in this examination.

When a question demands that candidates 'Explain why' something is valid or correct, it is important that any explanation is not contradictory or does not contain incorrect statements. This was required in **Question 10(a)(i)** in this paper.

In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. This avoids a premature approximation error. This was evident in **Question 5**, **Question 11** and occasionally in **Question 7** in this paper.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

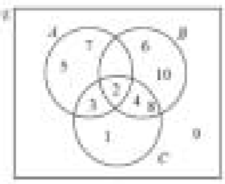
Question 1

This question proved to be an accessible start to the paper for most candidates.

(a) This part was well answered by the majority of candidates. The expression needed to be written using correct set notation. This meant that the expression also needed to be well-defined, which included the correct use of brackets.

(b) (i) Again, this part was very well answered, with a high proportion of candidates earning full marks. Some candidates thought that 1 was a prime number and misplaced it. A small number of candidates would have improved if they had crossed through the numbers of the universal set they had used as they placed them. This may have avoided the doubling of some elements in the Venn diagram or the omission of the element 9. Other candidates may have improved if they had read the question more carefully as they completed the Venn diagram with the number of elements in each set instead of the actual elements, as required.

(ii) Almost all candidates interpreted the notation correctly and stated the number of elements in the required set. A small number of candidates stated the element of the set rather than the number of elements.

Answers: **(a)** $(P \cup Q) \cap R'$ **(b)(i)** 

(ii) 1

Question 2

Again, almost all candidates considered the correct condition, $b^2 - 4ac < 0$, and were able to complete the solution to the problem correctly. Candidates made good use of sketches of number lines or curves to determine the correct form of the inequality. Very occasionally, candidates attempted to use $b^2 - 4ac \leqslant 0$ or $b^2 - 4ac \geqslant 0$. A small number of candidates may have improved by writing $(2k-3)^2$ as $(2k-3)(2k-3)$ and then expanding, as occasional slips were made in finding the cross-term, $-12k$, which was sometimes given as $-6k$.

Answer: $-0.5 < k < 1.5$

Question 3

(i) Candidates found this part of the question the most challenging. A reasonable number offered fully correct solutions. These candidates often wrote down their interpretation as '2 men and 1 woman or 3 men' and went on to correctly find ${}^3P_2 \times {}^3P_1$ or $3 \times 2 \times 3$. Some candidates found the number of ways in which the chairperson and treasurer were both men and the secretary was a woman. This was a slight misinterpretation of the question. Order was important in this question and ${}^3C_2 \times {}^3C_1$, therefore, was not appropriate. Some candidates needed to interpret the information more carefully as this, along with ${}^3C_2 \times {}^2C_1$ were fairly common answers. Weaker candidates attempted incorrect operations, adding or adding and multiplying their permutations.

(ii) This was the best-answered part of this question. Almost all candidates found the correct number of seating arrangements. Many candidates drew diagrams indicating the number of ways of filling each seat, which proved to be very helpful to them.

(iii) A good number of candidates gave solutions that were fully correct. Some candidates omitted one key part of the method – commonly this was that Alice and Carl could arrange themselves in 2! ways or that there were 4 ways that the Alice/Carl pair and 3 others could position themselves. This resulted in many answers of 24 or 12.

Answers: **(i)** 18 **(ii)** 24 **(iii)** 48

Question 4

(a) Candidates almost always recalled the meaning of the terminology in this part of the question. A very high proportion of candidates earned both marks. On the rare occasion when errors were made, the amplitude was given as 2 or –5 and the period was given as one of the numbers 15, 2 or –5.

(b) (i) Many candidates earned one mark for $\tan x$ and a good number of these also understood that another possible function was $-\tan x$. Common incorrect functions offered were $\cot x$ or $-\cot x$. A small number of candidates gave answers of the form $\tan kx$ or $k\tan x$, where $k > 0$. These candidates may have improved if they had considered the scale given and understood the significance of the point (45, 1), for example. This may have helped candidates to correct their functions.

(ii) Most candidates correctly interpreted the question and drew the line $y = 1$ or marked crosses at the 4 relevant points on the given diagram. A few candidates showed confusion about the meaning of the absolute value and stated the answer as 2, discarding the two points with negative x-coordinates. A small number of candidates needed to read the question more carefully as they listed the solutions rather than stating the number of solutions.

Answers: **(a)(i)** 15 **(ii)** 180° or π radians **(b)(i)** $\tan x$, $-\tan x$ **(ii)** 4

Question 5

The most efficient method of solution was to find $\frac{104}{1.6}$, giving the time in seconds. This then needed to be multiplied by the speed $0.5\,\text{ms}^{-1}$ to give the distance in metres. Good candidates understood this and earned four straightforward marks. Some candidates found the speed from *A* to *C* using Pythagoras' theorem and the distance *AC* using trigonometry and used these. While this was not incorrect, it was unnecessary. It often resulted in premature approximation errors as candidates often used 1.68 as their working value in trigonometric calculations to find angles. These angles were sometimes incorrect because of the approximation and this resulted in more inaccuracies. Candidates must understand that working values should be correct to more than the three significant figures required for the answer, to be confident that the final answer is acceptable. Some candidates gave their final answers to more than three significant figures having used 1.68, or similarly rounded values, in their calculations. This is poor practice and should be discouraged. Some candidates would have done well to reread the question as they only found the time it took and forgot to find the distance. Other candidates thought that the distance *AC* was what was required. These candidates needed to read the question more carefully as it was clear in the question that the distance *BC* was what was needed. A small number of candidates incorrectly thought that the speed at which the woman swam from *A* to *C* was $2.1\,\text{ms}^{-1}$, misinterpreting the given information. A few candidates drew incorrect triangles, again misinterpreting the given information, and marked the speed from *A* to *C* as $1.6\,\text{ms}^{-1}$. Some candidates found the time from *A* to *B* as 65 seconds and then continued to use trigonometry to find a time of 68.1 seconds from *A* to *C*. These candidates may have improved if they had understood the similarity of the triangles for distance and speed and drawn more than one diagram.

Answer: 65 seconds, 32.5 metres

Question 6

(i) This part of the question was very well answered, with a very high proportion of candidates earning full marks. Very occasionally, candidates misunderstood the chain rule. In these cases, for example, the answer was given as $\frac{x}{3}\sec^2\left(\frac{x}{3}\right)$ or as $\frac{1}{3}\sec^2 x$. These candidates may have benefitted from a more formal consideration of the chain rule, rather than the mental 'cover up' approach.

(ii) Most candidates demonstrated that they understood how the previous part of the question enabled them to find the given integral. A good starting point, often seen, was $\int \frac{1}{3}\sec^2\left(\frac{x}{3}\right)dx = 1 + \tan\left(\frac{x}{3}\right)$. Many of these candidates earned a mark for an answer of $3\tan\left(\frac{x}{3}\right)+3$. Better candidates earned full marks for understanding that they still needed to add the constant of integration and that $3\tan\left(\frac{x}{3}\right)+3$ was only a particular case. Weaker candidates did not deal with the $\frac{1}{3}$ correctly and attempted to integrate a product $\int \frac{1}{3}dx \times \int \sec^2\left(\frac{x}{3}\right)dx$.

Answer: **(i)** $\frac{1}{3}\sec^2\left(\frac{x}{3}\right)$ **(ii)** $3\tan\left(\frac{x}{3}\right)+c$

Question 7

(i) Almost all candidates found the correct angle as a fraction or exact decimal, using the correct formula for the area of a sector for an angle in radians. A small number of candidates rounded their decimal to two decimal places. This was not sufficiently accurate, although it was ignored if a more accurate answer had already been stated. Almost all candidates worked in radians. The very few that worked in degrees and then converted the answer to radians usually gave inaccurate answers. This should be discouraged as the working is generally much simpler when the formulae for angles in radians are applied.

(ii) Again, almost all candidates were able to apply the formula for an arc length for an angle in radians and find the correct arc length. As with **part (i)**, candidates who worked in degrees gave inaccurate answers.

(iii) A very high proportion of candidates used the most efficient method of solution, finding the difference between the area of the sector and the area of the triangle using $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$. Almost all of these candidates earned full marks. Candidates who converted 1.4 radians to degrees or used less efficient methods to find the area of the triangle sometimes made premature approximation errors resulting in inaccurate answers.

Answers: **(i)** 0.625 radians **(ii)** 5 cm **(iii)** 13.3 cm^2

Question 8

(a) (i) Almost all candidates identified the correct order of operations and were successful. Most candidates divided by 5 before taking logarithms of both sides. This resulted in the easiest method of solution. The small number of candidates who took logarithms of both sides before dealing with the 5 made occasional errors when separating the terms of $\ln(5e^{3x+4})$. Commonly, these candidates either wrote $\ln 5 \times (3x + 4)$ or misinterpreted $5e^{3x+4}$ as $(5e)^{3x+4}$. These candidates may have benefitted from more practice at manipulating simple algebraic expressions involving indices.

(ii) Again, candidates found this straightforward with very many finding and solving the correct quadratic equation in y. Some candidates did not discard the negative solution. Candidates should be encouraged to check that the solutions they have found to equations such as this, where the original functions are logarithms, are valid for the original form. It is important that any solutions that are not valid are discarded or deleted or simply not written down.

(b) Candidates were clearly well practised in the use of log laws and usually had no difficulty in applying the correct log law to the numerator and changing the base of the denominator in order to be able to simplify the expression. Most candidates gave excellent, clear and detailed working to support their answer. There were one or two slips in changing the base and, on occasion, the numerator was not written as a single logarithm.

Answers: **(a)(i)** –0.990 **(ii)** $y = 4.5$ **(b)** $\log_2\left(\frac{p}{q}\right)$

Question 9

(i) This was very well answered. Candidates clearly knew how to find the gradient and perpendicular gradient of a line. Some candidates would have benefitted from rereading the question before finding the equation. The required line needed to pass through *P*. Some candidates found the equation of the line through *Q* or through the mid-point of *PQ*.

(ii) There were many good solutions seen to this part of the question. A variety of methods were used. Many started using the shoelace method, found two equations in *x* and *y* and solved simultaneously with their answer to **part (i)**. Other candidates used the shoelace method and made the substitution using their equation from **part (i)** at this stage to form two equations in *x* which they then solved. Many other candidates found the length of *PQ* and used it to form an area equation which they solved to find the length of *PR.* Some candidates did not seem to know what to do at this point. Better candidates either used the gradient of the line *L* and vector methods or formed a quadratic equation by substituting the equation from **part (i)** into the length equation they had formed using *PR*. Some candidates only found one of the points correctly, even though the question indicated clearly that there were 2. These candidates may have benefitted from rereading the question when they had completed their solution. Candidates who took care over their presentation and who made sketches to help them were often the most successful. Candidates whose presentation was not as neat often made arithmetic or sign slips when rearranging or simplifying expressions.

Answers: **(i)** $y-2=-\frac{3}{4}(x-8)$ **(ii)** (4, 5), (12, –1)

Question 10

(a) (i) As the given graph had a turning point, the simplest explanation was that this indicated that the function was not one-one. This was rarely observed. Many candidates did reference the graph failing the horizontal line test and this was allowed. Candidates who stated only that the function was one-one and made no reference at all to the graph were not credited, as reference to the graph was a requirement of the question. Weaker candidates often commented that no inverse existed as f could not be less than 1 meaning that negative values were not possible or said that the graph could not be reflected in the line $y = x$. These candidates had not identified the correct criterion for the inverse to exist. Some comments were partially correct. However, incorrect parts resulted in the whole comment not being acceptable. Candidates who talked about many-to-one functions often confused them with one-to-many functions and vice versa.

(ii) A good number of candidates formed the correct composite function and simplified their expression correctly. A number of candidates misinterpreted the notation and attempted to find the product $\left(\sqrt{1+x^2}\right)^2$ rather than composing the functions. Some candidates needed to take more care with powers as $\sqrt{1+\left(\sqrt{1+x}\right)^2}$ or $\sqrt{1+\left(\sqrt{1+x^2}\right)}$ were seen on more than one occasion.

(b) (i) A very good number of candidates gave a correct answer. Some candidates were giving a range of values for *k*, such as $k \geqslant 0$, when a single value was clearly required. These candidates may have corrected this error if they had reread the question. A few candidates stated the answer $k = \sqrt{-1}$ having solved the equation $1+x^2>0$. These candidates had not made the connection between the two parts of the question. It was simpler to use the graph from **part (a)** to answer this part of the question as function g was simply a restricted form of function f.

(ii) An excellent number of fully correct answers were given. Candidates were clearly well-practised at finding inverse functions. Almost all candidates gave their final answer as a function of *x*. Candidates who gave the answer as $g^{-1}(x)=\pm\sqrt{x^2-1}$ did not understand the relevance of the answers to part **(a)(i)** and part **(b)(i)**. These answers should have resulted in the negative square root being discarded. Candidates need to understand that they should use the domain to help determine which sign is correct.

(c) Generally, only the best candidates tended to earn all 4 marks in this part. These candidates considered the asymptotes and were careful to ensure that their graphs tended to the correct line. A good number of candidates earned three marks. Often this was for a sufficiently correct pair of graphs which tended to the axes rather than tending to $y = 2$ and $x = 2$. Some candidates were able to draw the graph of $y = 4e^x + 2$ correctly and marked the point of intersection with the y-axis. It was expected that candidates would reflect this curve in the line $y = x$ rather than finding the rule for the inverse function which was not needed. Many candidates attempted to reflect the graph of h, although many other candidates did try to find the inverse function. Candidates who did not use a square scale, with equally proportioned scales on the two axes, made the inverse much more difficult to draw. Other candidates clearly plotted points rather than making sketches and this often resulted in skewed scales which were not helpful when sketching the inverse function. Often in these cases the line $y = x$ was drawn very close to the x-axis. This commonly resulted in candidates reflecting h in the x-axis.

Answers: **(a)(ii)** $f^2(x) = \sqrt{2 + x^2}$ **(b)(i)** Any value greater than or equal to 0 **(ii)** $g^{-1}(x) = \sqrt{x^2 - 1}$

Question 11

(a)(i) A very high proportion of candidates gave a correct answer to this part. Candidates worked correctly from left to right and mostly showed sufficient working to earn full marks.

(ii) Most candidates earned the first mark for a correct statement using $\tan 3x$. Most candidates also found either 63.4 or 21.1 to earn the second mark. A good number of these candidates solved the equation correctly and gave their angles to the required accuracy for angles in degrees, one decimal place. Some candidates were penalised for making premature approximations when finding the triple angles which resulted in their final set of angles being slightly inaccurate. A few candidates gave only the two smaller angles rather than the three required. This was because they considered triple angles between 0° and 360° only. Many good candidates avoided these errors. They achieved this by writing down that $0 \leqslant x \leqslant 180$ means that $0 \leqslant 3x \leqslant 540$ and/or by drawing a quadrant (CAST) diagram or a simple tangent curve.

(b) Candidates who knew or used the correct trigonometric identity from the formula page usually earned four or five marks. Some candidates used $\tan^2 y = \frac{\sin^2 y}{\cos^2 y}$ and $\sec y = \frac{1}{\cos y}$. These candidates were also often successful. The extra steps involved, however, resulted in some of these candidates making errors which they were unlikely to have made should they have used the simpler substitution $\tan^2 y = \sec^2 y - 1$. Sometimes candidates made errors in the simplification of their equation to three terms. Those candidates who did so and showed a method of solution for their equation earned the second mark. Candidates who solved an incorrect equation using their calculator and showed no method did not earn the second mark. Candidates need to know that it is important to show all key method steps in solutions in order that marks can be awarded if they make an error. Some candidates used a base angle of 0 when finding the inverse cosine of –1. Although this was usually well done, a few candidates using this approach incorrectly included 0 and, occasionally, 2π as an answer. Very few candidates showed any evidence of checking their angles in the original equation. A check such as this would help candidates who have erroneous angles in their solutions to discard them.

Answers: **(a)(ii)** 21.1°, 81.1°, 141.1° **(b)** π, 0.430, 5.85 radians

Question 12

This question was well-answered. Occasionally, candidates incorrectly described their derivatives and this caused them some confusion when trying to construct the correct calculation. It is important that candidates have a clear understanding of what derivatives such as $\frac{dV}{dt}$ represent. When candidates are unsure, they should be encouraged to consider the units given. The volume was increasing at a rate of 200 cm^3 per second, a relationship involving the ratio of volume and time, for example.

(i) Most candidates understood how to use the chain rule to find related rates of change. They stated the relationships they were using and provided clear and fully correct solutions. A small number of candidates misinterpreted their final answer as $\frac{1}{2}\pi$ rather than $\frac{1}{2\pi}$. As an exact answer was not required, this may have been avoided if these candidates had given a decimal answer. A small number of candidates found the volume when the radius was 10, added 200, found the radius when the sphere had achieved this new volume and then subtracted 10. This use of an average rate of change did not give the actual rate of change of the radius at the instant when the radius was 10 as it assumed that the rate of change of the radius is constant, which was not the case. While this method resulted in an approximation of the answer, this was not what was asked for and was therefore not acceptable. A few other candidates rearranged the given formula for the volume of a sphere to make *r* the subject and hence find $\frac{dr}{dV}$. While this was not incorrect, it was unnecessary and resulted in more work and a few unnecessary errors from some candidates.

(ii) Candidates who understood the application of calculus to **part (i)** usually earned full marks in this part also. The candidates using the approximate average rate of change approach in **part (i)** applied the same approximate, unacceptable method in this part.

Answers: **(i)** 0.159 **(ii)** 40