Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

3053484001

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

2 A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

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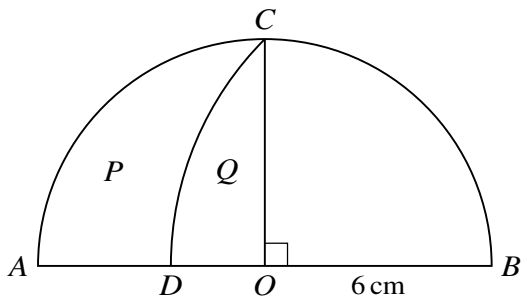
(ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

3 **(a)** Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

4



The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

(i) Calculate the length of the arc DC. [3]

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(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

5 **(i)** Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

6 **(a)** The function f, defined by $\mathrm{f} : x \mapsto a + b\sin x$ for $x \in \mathbb{R}$, is such that $\mathrm{f}\left(\frac{1}{6}\pi\right) = 4$ and $\mathrm{f}\left(\frac{1}{2}\pi\right) = 3$.

(i) Find the values of the constants a and b. [3]

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(ii) Evaluate $\mathrm{ff}(0)$. [2]

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(b) The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leqslant g(x) \leqslant 10$. Find the values of the constants c and d. [3]

7 Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

(ii) Find the set of values of m for which L does not meet the curve. [4]

8 A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

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(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

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(iii) Given that the curve passes through the point (6, 2), find the equation of the curve. [4]

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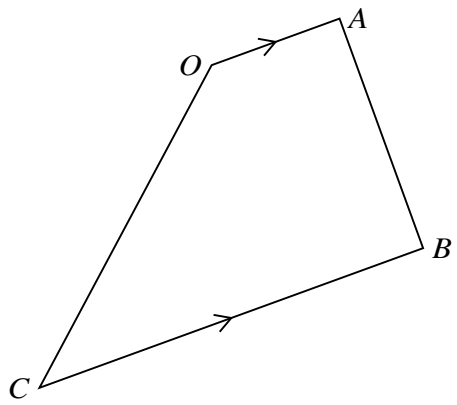

The diagram shows a trapezium $OABC$ in which OA is parallel to CB. The position vectors of A and B relative to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

(i) Show that angle OAB is $90°$. [3]

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The magnitude of $\overrightarrow{CB}$ is three times the magnitude of $\overrightarrow{OA}$.

(ii) Find the position vector of C. [3]

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(iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]

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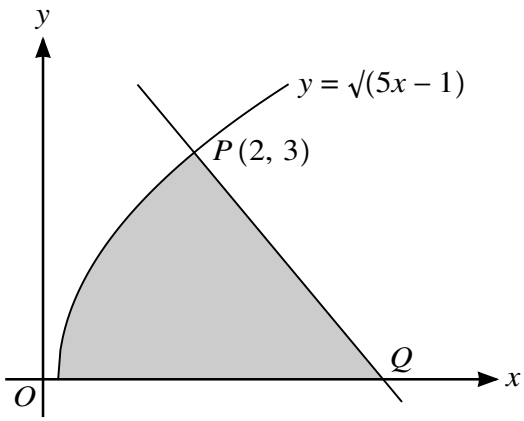

The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P\,(2,\ 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

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(ii) Find, showing all necessary working, the area of the shaded region. [7]

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