

CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
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FURTHER MATHEMATICS

9231/21

Paper 2

October/November 2017

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be 10 m s^{-2} .

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

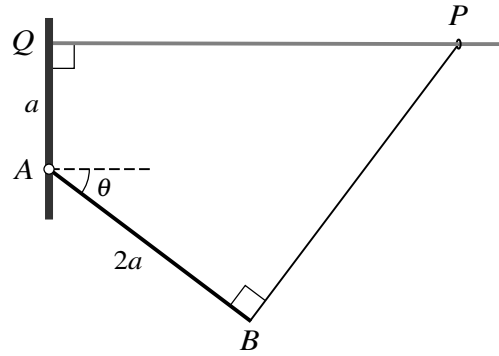
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **23** printed pages and **1** blank page.





A small ring P of weight W is free to slide on a rough horizontal wire, one end of which is attached to a vertical wall at Q . The end A of a thin uniform rod AB of length $2a$ and weight $\frac{5}{2}W$ is freely hinged to the wall at the point A which is a distance a vertically below Q . A light elastic string of natural length $2a$ has one end attached to the ring P and the other end attached to the rod at B . The string is at right angles to the rod and A, B, P and Q lie in a vertical plane. The system is in limiting equilibrium with AB making an angle θ with the horizontal, where $\sin \theta = \frac{3}{5}$ (see diagram).

- (i) Find the tension in the string in terms of W . [2]

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- (ii) Find the coefficient of friction between the ring and the wire. [2]

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- (iii) Find the magnitude of the resultant force on the rod at the hinge in terms of W . [3]

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(ii) Show that small oscillations of the system about this axis are approximately simple harmonic and state their period. [5]

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6 A pair of fair dice is thrown repeatedly until a pair of sixes is obtained. The number of throws taken is denoted by the random variable X .

(i) Find the mean value of X . [2]

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(ii) Find the probability that exactly 12 throws are required to obtain a pair of sixes. [2]

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(iii) Find the probability that more than 12 throws are required to obtain a pair of sixes. [2]

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7 The random variable X has probability density function f given by

$$f(x) = \begin{cases} 0.2e^{-0.2x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X . [2]

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(ii) Find $P(X > 2)$. [2]

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(iii) Find the median of X . [3]

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- 9 The land areas x (in suitable units) and populations y (in millions) for a sample of 8 randomly chosen cities are given in the following table.

Land area (x)	1.0	4.5	2.4	1.6	3.8	8.6	7.5	6.5
Population (y)	0.8	8.4	4.2	1.6	2.2	10.2	4.2	5.2

$[\Sigma x = 35.9, \Sigma x^2 = 216.47, \Sigma y = 36.8, \Sigma y^2 = 244.96, \Sigma xy = 212.62.]$

- (i) Find, showing all necessary working, the value of the product moment correlation coefficient for this sample. [3]

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- (ii) Using a 1% significance level, test whether there is positive correlation between land area and population of cities. [4]

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The land areas and populations for another randomly chosen sample of cities, this time of size n , give a product moment correlation coefficient of 0.651. Using a test at the 1% significance level, there is evidence of non-zero correlation between the variables.

(iii) Find the least possible value of n , justifying your answer. [2]

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