ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

Candidates should take care to read each question carefully to ensure that they are giving the full answer as required. In questions involving trigonometric equations, particular care should be taken to identify whether an answer in radians or degrees is required. A question involving the use of calculus and trigonometry will always mean that radian measure is to be used.

Candidates should also ensure that they give their final answers, in the case of non-exact numerical answers, correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. Too many candidates do not take note of the rubric on the front page of the examination paper and thus lose accuracy marks needlessly. They should also be aware that working throughout a question should always be to a greater accuracy to that required in the final answer.

Candidates should be aware that if an answer is given in the first part of a multi-part question it is intended to help candidates make progress in subsequent parts and candidates should be advised to use the given result in the later parts.

General comments

Many candidates were able to make a good attempt at each of the questions and appeared to have no timing issues. However, it was very evident that many candidates had not prepared well for the examination, with papers showing no response to several questions. As there was no marked difference in the type of questions set in the past, it has to be assumed that these candidates had either not done enough revision or had not covered the syllabus objectives in great enough detail.

Comments on specific questions

Question 1

(a) It was intended that candidates make use of the space below the initial introduction to the question to draw a Venn diagram to help with the following demands. Many candidates did just this but initial errors in completing the Venn diagram often led to errors in the subsequent parts. Most candidates were familiar with the n notation and gave integer answers.

(b) (i) Many correct responses were seen, with candidates appreciating that Q was a subset of R.

(ii) Too many candidates included the integer 30 in the universal set. This subsequently led to many incorrect answers of {30} rather than the required empty set.

Answer: **(a)(i)** 10, **(ii)** 22, **(iii)** 4, **(b)(i)** $\subset$, **(ii)** $\varnothing$

Question 2

It was intended that candidates consider the terms of each of p, q and r separately and simplify them. Most candidates were able to do this and many candidates were successful in obtaining at least one correct answer.

Answer: $a = 1$, $b = -3$, $c = -1$

Question 3

Many candidates were able to write $\log_3 x^5$ as $5\log_3 x$ or as $5y$ and $\log_3 9$ as 2. Solving the resulting quadratic using factorisation or the formula was straightforward enough for most candidates. However, many candidates had difficulties with the correct use of base 3 logarithms. While many candidates were able to evaluate 3^{-2} correctly, many were unable to evaluate $3^{\frac{1}{3}}$ correctly. This was a question where some candidates lost accuracy marks by not giving their answers to the required level of accuracy.

Answer: 1.44, $\frac{1}{9}$

Question 4

(i) Many candidates struggled with this question although most had some idea, however weak, of the application of the binomial expansion. Some candidates wrote out the expansion accurately but did not simplify the terms.

A common error was to obtain a first term of $32x^7$ or $2x^7$. Other common errors included being unable to simplify a correct expression correctly. A few candidates lost accuracy by using rounded decimals. A significant number of candidates wasted time by attempting the whole expansion, with a small number of candidates reversing the order of the terms.

(ii) Many did more than the required work by considering more terms than necessary. It was expected that candidates make use of the appropriate terms from their answer to **part (i)**, so if a candidate was not able to obtain any terms in x^7, they should have been alerted to the fact that they had probably made an error and go back and check their previous work.

Answer: **(i)** $32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$ **(ii)** –48

Question 5

(i) Many fully correct responses were seen from the more able candidates. The main error which led to the majority of candidates only earning part marks arose from an inability to differentiate the given equation $y = \frac{1}{2}\ln(3x+2)$ correctly, with some candidates using the quotient rule which, although not being a problem, often led to the inclusion of an extra incorrect term. The majority of candidates who did differentiate correctly often found the correct coordinates of the point *P*. Despite many incorrect answers for the derivative, most candidates were able to apply a correct method and made a good attempt at the gradient of the normal and then proceeded to find a linear equation.

(ii) Despite many correct answers to **part (i)**, few correct answers were seen for **part (ii)**. Many candidates found the correct coordinates required to find the area of the triangle, with credit being given for those candidates who had obtained an incorrect linear equation in **part (i)**. A simple sketch was all that was required to help with the calculation of the area of the triangle as two of the points lay on the *y*-axis. Too many candidates made use of the determinant method used for areas, usually making errors in the evaluation. Candidates should be encouraged to always consider possible easier alternative methods to questions of this type.

Answer: **(i)** $y = -\frac{2}{3}\left(x + \frac{1}{3}\right)$ **(ii)** 0.0948

Question 6

This question was answered very well across the whole ability range, with very few candidates not earning some credit.

(a) Many candidates earned full marks giving both correct solutions. However, marks were often lost by giving extra incorrect solutions or only one correct solution, usually **XZ**. Many candidates gave **XY** rather than **YX**.

(b) (i) This part was answered very well with most candidates earning credit. The only real error that occurred quite often was a determinant of 10 rather than 18, due to an arithmetic slip.

(ii) Full marks were often earned by candidates who had correctly found the inverse of **A**. The main errors were when either candidates did not make use of pre-multiplication or candidates ignored the instruction 'Hence' and attempted solutions using simultaneous equations. Candidates are to be reminded of the implications of the word 'hence' when used in mathematical questions.

Answer: **(a) YX** and **XZ** **(b)(i)** $\frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$

Question 7

(i) Many candidates had their calculators in the incorrect mode which, when set to degrees gave $2\cos\left(-\frac{\pi}{6}\right)$ as 1.99. This meant that the response of (0, 2) was seen more often than the correct response. Candidates should always check that their calculator is in the correct mode, ensuring that radian mode is used for questions that involve angles in terms of π and/or involve calculus. Another very common response was (0, 1.7). Candidates should ensure that they give their answers to the degree of accuracy mentioned in the rubric.

(ii) Too many fortuitously correct responses were obtained from candidates with their calculators in the incorrect mode being used with an incorrect method of solution and these did not earn marks. It was intended that candidates make use of the fact that the maximum value that $\cos\left(x-\frac{\pi}{6}\right)$ can take is 1 and thus obtain the coordinates of the maximum point. Too many candidates chose to express $\cos\left(x-\frac{\pi}{6}\right)$ incorrectly as $\cos x-\cos\frac{\pi}{6}$. It was not intended that calculus be used for this part of the question and its use was not penalised if correct, but correct solutions were seldom seen.

(iii) As in the previous part, too many candidates expressed $\cos\left(x-\frac{\pi}{6}\right)$ incorrectly, rather than use a correct method to solve the equation $\cos\left(x-\frac{\pi}{6}\right)=0$. Again, the incorrect level of accuracy lost candidates the final mark when they chose to give their response for the *x*-coordinate as a decimal. Candidates should remember that angles in radians need to be given correct to 3 significant figures.

(iv) Many correct responses were seen, but there was the occasional error in the sign of the response given. A few candidates attempted to integrate every individual part of the original function. Candidates should be encouraged to take note of the mark allocation. A question part which offers 1 mark implies that the response required takes very little work and can often be written down straightaway.

(v) For candidates who had correctly integrated in **part (iv)** or just made a sign error, method marks were often obtained. Marks were lost when candidates chose to mix radians and degrees when using their limits, reinforcing the fact that candidates need to be aware that trigonometry and calculus implies use of radians only. With errors in finding the coordinates of the point *C* common, completely correct responses were rare.

Answer: **(i)** $\left(0, \sqrt{3}\right)$, **(ii)** $\left(\frac{\pi}{6}, 2\right)$, **(iii)** $\left(\frac{2\pi}{3}, 0\right)$, **(iv)** $2\sin\left(x - \frac{\pi}{6}\right) + c$, **(v)** 3

Question 8

(i) The majority of candidates used a correct method to obtain $\theta = \frac{23}{12}$. Unfortunately, many candidates then just wrote down $\theta = 1.92$. Candidates need to realise that where a question requires a candidate to show an answer is correct to a specific number of decimal places, they need to be showing working to a greater degree of accuracy.

(ii) Most candidates answered this part correctly with the majority using the cosine rule. Most errors occurred through poor computation and lack of accuracy, particularly with many failing to deal correctly with the negative value of the cosine produced since the angle was obtuse.

(iii) There were many totally correct responses seen here, with candidates showing their ability at problem solving together with a good knowledge of the syllabus objectives.

Problems occurred when some candidates used degrees when working out the area of the triangle rather than radians, but these candidates were usually able to gain all the subsequent method marks, only losing the final accuracy mark. Another common error was for candidates to disregard the area of the triangle in their method.

Answer: **(ii)** 19.6 or 19.7 **(iii)** 18.1 or 18.0

Question 9

(i) Most candidates attempted to use $x = \frac{3}{2}$ in the given polynomial and obtain an equation in terms of *a* and *b*. A significant number attempted to use algebraic long division and divide the given polynomial by $2x - 3$. This method was rarely successful, often not resulting in an equation in terms of *a* and *b*. It was very evident that many candidates were not familiar with the notation $\mathrm{p}'(x)$ and did not realise that they had to differentiate $\mathrm{p}(x)$ with respect to *x* and use a further substitution of $x = \frac{3}{2}$. Some candidates mistakenly thought that they had to make a substitution of $x = \frac{2}{3}$ into the given polynomial. A significant number of candidates, unable to obtain a second equation, made use of the fact that they were given the value of *a* and substituted this into the linear equation they had obtained initially to get *b*, usually successfully. Completely correct solutions to this part of the question were rare. Candidates had been given the value of *a* so that it either provided a check on their working or so that they could make use of this if they were unable to make use of $\mathrm{p}'(x)$. Candidates should be encouraged to familiarise themselves with the different notation that may be used in this syllabus.

(ii) For candidates who had values for *a* and *b*, good attempts made at algebraic long division were seen. Credit was given to those candidates who had incorrect values for *b*, but employed a correct method. Some candidates lost marks by solving $\mathrm{p}(x) = 0$ using their calculator despite being instructed not to use a calculator. Candidates should ensure that they are meeting the demands of the question – factorisation of $\mathrm{p}(x)$ was asked for, not the solution of $\mathrm{p}(x) = 0$.

(iii) Very few candidates took heed of the word "hence" and made use of their factorised response to the previous part. Of those that did, a significant number were able to obtain two solutions, but 'lost' the third solution by cancelling the common factor of $x+2$. Many candidates unfortunately showed a lack of algebraic technique and completely correct solutions were rare. Again it was evident that candidates were making use of the polynomial solver on their calculator despite being instructed not to use a calculator. In these circumstances, candidates cannot be awarded any marks for the question.

Answer: **(i)** $b=-15$, **(ii)** $(x+2)(2x-3)^2$, **(iii)** 1, ± 2

Question 10

It was apparent that many candidates were unfamiliar with this area of the syllabus due to the number of 'no responses'. It is clearly a syllabus objective that needs to be covered in more detail.

(a) (i) The majority of candidates attempting this part of the question realised that they needed to find the area under the graph and equate it to 165; hence most scored 2 or more marks. Of these candidates, only about half of them were able to calculate the area correctly. Many candidates attempted to calculate the area of the five-sided shape as one trapezium. Too many candidates did not attempt to find the area, but simply incorrectly used 'velocity = distance / time'.

(ii) There was a good awareness among most candidates of the use of gradient for acceleration and the need for a negative sign. A few candidates made use of '$v=u+at$' correctly to obtain their answer. These candidates were awarded the marks, even though this method is not on the syllabus.

(b) (i) Most candidates made use of $t=0$ and obtained $v=-27$. However, most of these candidates did not appreciate the difference between velocity and speed and so did not give the required answer of 27.

(ii) Many correct solutions were seen, but there were cases of fortuitously correct answers obtained from incorrect cubing of terms in the equation. Most candidates were adept at dealing with the transition between exponentials and logarithms.

(iii) Although most candidates realised that differentiation was required, and made a reasonable attempt at the chain rule, completely correct derivatives were rare. Commonly, either the term $e^{\frac{t^2}{8}}$ or the term $2t$ was omitted from the final result. A few candidates correctly used a binomial expansion and then differentiated all terms perfectly, thus producing a correct numerical result.

Answer: **(a)(i)** 6 **(ii)** –0.3, **(b)(i)** 27 **(ii)** 3.33 **(iii)** 6.98

Question 11

(i) Completely correct solutions to this question were rare. This is clearly a syllabus area that needs to be looked at in more detail by both Centres and candidates. The key point that $\ln y=\ln A+x\ln b$ was missed by many candidates. Often base 10 logarithms were introduced rather than the required natural logarithms. Incorrect substitutions were made into either a logarithmic equation or the original equation showing a lack of basic understanding of graphs in straight line form. Many candidates were able to find the gradient of the graph correctly but were unsure what to do with it. Some candidates correctly gave the equation of the graph in its straight line form but did not actually answer the question as it was required that the value of *A* and of *b* should be found.

(ii),(iii) Many candidates were able to obtain credit for using a correct method with their values of *A* and *b*, but many candidates did not show an appreciation of the relationship using logarithms and the straight line graph form and so were unable to gain any credit.

Answer: **(i)** $A=1.30$, $b=0.985$ **(ii)** 1.19 **(iii)** 11

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

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Question 10

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Answer: **(a)(i)** 6 **(ii)** –0.3, **(b)(i)** 27 **(ii)** 3.33 **(iii)** 6.98

Question 11

(i) Completely correct solutions to this question were rare. This is clearly a syllabus area that needs to be looked at in more detail by both Centres and candidates. The key point that $\ln y=\ln A+x\ln b$ was missed by many candidates. Often base 10 logarithms were introduced rather than the required natural logarithms. Incorrect substitutions were made into either a logarithmic equation or the original equation showing a lack of basic understanding of graphs in straight line form. Many candidates were able to find the gradient of the graph correctly but were unsure what to do with it. Some candidates correctly gave the equation of the graph in its straight line form but did not actually answer the question as it was required that the value of *A* and of *b* should be found.

(ii),(iii) Many candidates were able to obtain credit for using a correct method with their values of *A* and *b*, but many candidates did not show an appreciation of the relationship using logarithms and the straight line graph form and so were unable to gain any credit.

Answer: **(i)** $A=1.30$, $b=0.985$ **(ii)** 1.19 **(iii)** 11

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should take care to read each question carefully so that they are clear what they are being asked to do. In questions involving trigonometric equations, particular care should be taken to identify whether an answer in radians or degrees is required. If an answer is required in radians they should check if numerical answers or answers in terms of pi are asked for. In coordinate geometry questions candidates should be clear whether they have been asked to find the equation of a curve, a tangent or a normal.

Candidates should be aware that if an answer is given in the first part of a multi-part question it is intended to help candidates make progress in subsequent parts and candidates should be advised to use the given result in the later parts.

General comments

Most candidates were able to attempt a high proportion of the questions and the standard of presentation was generally very good, but candidates should be aware that going over work done in pencil can make responses difficult to read as a double vision effect is produced. Nearly all candidates planned their responses carefully in order to fit the space provided. There was some carelessness in candidates' responses, particularly in miscopying their own figures and in rearranging algebraic expressions. Errors were also made in some simple calculations where a calculator had not been considered necessary.

Candidates should be aware that improper fractions that can be simplified to whole numbers should be expressed as whole numbers in their final answers.

It is often helpful to replace an expression with a single letter in order to form and solve an equation, but candidates should clearly state any substitutions made and should avoid using a letter already in the expression. It confuses candidates and Examiners if, for example, $x = \frac{1}{x}$ or $x = \log x$ is used. Care should also be taken to remember that, when a substitution has been made, they should go back to obtain values of the original variable.

Comments on specific questions

Question 1

A significant number of candidates did not appreciate that the graph of a modulus function had to lie entirely on or above the x-axis. A good number of candidates knew that the maxima on the curve lay on $y = 2$ but not all of those used an appropriate frequency. Although this was a sketch graph, some accuracy was expected in identifying the points on the x-axis and many candidates achieved this. Candidates who had otherwise correct graphs did not always realise that the modulus of a cosine function would have cusps rather than curves on the x-axis. However, many candidates earned full marks.

Question 2

The most successful candidates multiplied numerator and denominator by $\sqrt{m}$ (or similar) to obtain $\frac{4m^2-9}{2m+3}$ and then factorised the numerator and cancelled. Candidates who tried to multiply numerator and denominator by $2\sqrt{m}-\frac{3}{\sqrt{m}}$ were less successful and generally found it difficult to simplify the products correctly. Candidates would benefit from practice in simplifying expressions such as $4m\sqrt{m}\times 2\sqrt{m}$. Even candidates who obtained correct products using this method tended not to proceed further as factorising the cubic expression in the numerator was required and this proved difficult. There was some carelessness with signs in this question and the work of some candidates appeared to have been miscopied from rough work or work in pencil.

Answer: $2m-3$

Question 3

Candidates should be aware that the answer given in the first part was intended to help them progress in the second part.

(i) Many candidates took a correct approach to this question but a significant number did not fully understand that *a*, *b* and *c* substituted into b^2-4ac had to be coefficients of x^2 and x and the number term in a rearranged quadratic equation. Of the candidates who used the correct discriminant not all used $\geqslant 0$ throughout their working. Working had to be carefully checked as some candidates made sign errors on their way to a 'correct' answer. Not all candidates read this question carefully and some attempted to solve $p^2-3p-9=0$ in this part.

(ii) Candidates should be aware that the 'exact form' required in the question was for an answer expressed in surd form. Answers in decimal form did not score accuracy marks. Most candidates using the quadratic formula were successful but those who attempted a completing the square method were not as successful. Many candidates obtained critical values correctly in surd form but few went on to obtain a correct range of values of *p* in surd form; either omitting this requirement completely or giving answers in decimals.

Answer: **(ii)** $p\leqslant\frac{3-3\sqrt{5}}{2},\quad p\geqslant\frac{3+3\sqrt{5}}{2}$

Question 4

There were many good solutions to both parts of this question, but some were marred by carelessness with signs.

(i) All candidates knew what was required in this question. Many correct solutions were seen but some were left unsimplified. Sometimes the negative sign in front of the $\frac{x}{4}$ was forgotten completely and sometimes $\left(-\frac{x}{4}\right)^2$ was mistakenly simplified to $\frac{x^2}{4}$, $-\frac{x^2}{4}$ or $-\frac{x^2}{16}$.

(ii) Candidates should be aware that the term independent of *x* in an expansion is the number term. Candidates had to consider three products using their answer to the first part and many did this. Some reached the correct answer but many were prevented from doing so as they had had incorrect signs in the first part.

Answers: **(i)** $64-48x+15x^2$ **(ii)** 205

Question 5

(i) Most candidates were able to use $\log_3 xy = \log_3 x + \log_3 y$ or $\log_9 xy = \log_9 x + \log_9 y$ in their proof but the working for a change of base from base 9 to base 3 or from base 3 to base 9 was not always shown and Examiners had to assume that candidates not showing this step adequately were making use of the given answer. Similarly, candidates who used an alternative approach using powers of 9 often obtained $9^{\frac{5}{2}}$ but did not always show a conversion to a power of 3. Candidates should be encouraged to present proofs such as this as coherent arguments.

(ii) This proved to be one of the most challenging questions on the paper. Candidates should be aware that the result from the first part had to be used. Of the candidates who did use that result few successfully combined it with $\log_3 x \times \log_3 y = -6$ to form a quadratic equation. Candidates who formed the correct quadratic equation went on to solve it correctly and often proceeded to score full marks. However a full set of x and y values was not always obtained. This was because of a mistaken belief that $\log_3 x = -1$ had no solutions or they had not appreciated that both y and x had to be found.

Some candidates substituted letters for $\log_3 x$ and/or $\log_3 y$ to form an equation of the correct form. This helped to clarify the question for some candidates but in some cases it was not always made clear that they had made a substitution, particularly if it appeared to be $x = \log_3 x$ and $y = \log_3 y$. Not all candidates remembered that they had used a simplified equation to find values of $\log_3 x$ or $\log_3 y$ and did not go on to use their solutions to obtain values of x or y.

A common misunderstanding was to confuse $(\log_3 x)^2$ with $\log_3 x^2$ and then not to use a quadratic equation. Another common misconception was that a law of logarithms could be used to simplify the product of two logarithms. Many candidates tried to use $xy = 243$, which was true but not helpful in leading to a solution in this question.

Answer: **(ii)** $x = \frac{1}{3}$, $y = 729$; $x = 729$, $y = \frac{1}{3}$

Question 6

(i) Candidates should be aware that they have to know derivatives of standard functions such as $\ln x$. Some candidates who knew the derivative of $\ln x$ did not always appreciate that here they were required to differentiate a composite function and the $6x$ was not always obtained.

(ii) Candidates were expected to recognise that integration was the reverse of differentiation and candidates who obtained a correct answer for the first part often obtained a correct answer in this part. However, there was some misunderstanding of the relationship between p and $6x$ and an answer of 6 was also seen. It was not always appreciated that p was a constant and not a function of x.

(iii) Candidates should be aware that the given result in **part (ii)** could help them with this part even if they had not succeeded with this question so far. However, candidates who did not use a constant for p could not earn the method marks available in this part. In fact, a significant number of candidates did not use $\ln(3x^2 - 11)$ at all.

Of those who substituted limits correctly, not all were able to deal with the logarithms correctly. Candidates who made simple arithmetic errors in the evaluation at $x = 2$ and those that did not realise that $\ln 1 = 0$ were at a disadvantage when it came to further manipulation and many candidates did not employ the correct order of operations required to isolate x.

Some candidates simplified $6(\ln 2)$ to $\ln(2^6)$ to obtain $\ln(3x^2 - 11) = \ln 64$, but many successful candidates correctly used their calculators to evaluate $e^{6\ln 2}$ to obtain 64. Candidates who evaluated $6\ln 2$ and then calculated e to the power of a rounded value lost accuracy.

Answers: **(i)** $\frac{6x}{3x^2 - 11}$ **(ii)** $\frac{1}{6}$ **(iii)** $a = 5$

Question 7

Candidates would benefit from practice in transforming relationships of the form $y = Ae^{\frac{b}{x}}$ into straight line form using natural logarithms. Candidates should be aware that the graph shows $\ln y$ plotted against $\frac{1}{x}$ and that to succeed in all parts of the question the roles of $\ln y$ and $\frac{1}{x}$ need to be thoroughly understood. In particular, the given coordinates are (a value of $\frac{1}{x}$, a value of $\ln y$). Candidates who tried to use base 10 logs were unable to progress with this question.

(i) The majority of successful candidates obtained the equation $\ln y = \ln A + \frac{b}{x}$ and used the gradient to find b and the y-intercept to find $\ln A$. Candidates who used the given points in simultaneous equations or found b and then used one point were not as successful as they did not always take into account that the given coordinates were (a value of $\frac{1}{x}$, a value of $\ln y$) and not (x, y) or (x, $\ln y$).

Some candidates obtained $\ln y = -\frac{0.8}{x} + 4.7$ or $Y = -0.8X + 4.7$ but their equations were not used to find b and A. Some candidates appeared to have a correct equation but had not fully understood it as they then found b in terms of x. (i.e. not a constant) or included $\ln e$ and evaluated it as something other than 1.

(ii) Candidates who used $\ln y = -\frac{0.8}{x} + 4.7$ or read values from the graph could succeed even if they had the first part incorrect and these were the most successful methods. A common source of error was to use 0.32 in an equation where they should have used 3.125 $\left(\text{i.e. } \frac{1}{0.32}\right)$ or to forget that they had in fact found $\ln y$ rather than y.

(iii) Candidates who used $\ln y = -\frac{0.8}{x} + 4.7$ or read values from the graph could succeed even if they had the first part incorrect and these were the most successful methods. A common source of error was to use x in an equation where they should have used $\frac{1}{x}$ or to use 20 where they should have used ln20.

Answers: **(i)** $b = -0.8$, $A = 110$ **(ii)** $y = 9$ **(iii)** $x = 0.47$

Question 8

(a) (i) Candidates should be aware that $\frac{1}{\sin A} = \operatorname{cosec} A$ and $\frac{1}{\cos A} = \sec A$ and that these relationships are not given in the list of formulae. Candidates who tried to use the relationships for $\sec^2 A$ and $\operatorname{cosec}^2 A$ given in the formulae list were not successful.

Many candidates substituted $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ but there were some who were then unable to simplify the denominator. Candidates who obtained $1 - \sin^2\theta$ in the denominator usually went on to correctly substitute $\cos^2\theta$ and complete the proof, but some candidates who tried working back from the final answer were trying to work towards $1 + \tan^2\theta$ which made their task more difficult.

Candidates should try to present trigonometric proofs as coherent arguments. In this case candidates' work became difficult to follow if they worked with the denominator in such a way that it was no longer clearly a denominator. Candidates should also use θ throughout and not lose it or replace it with another letter.

(ii) Most candidates obtained $\cos\theta = \frac{1}{2}$ or an equivalent expression but many candidates forgot that taking a square root would lead to both positive and negative values. Most candidates obtained at least two solutions, but the third and fourth were often lost through the lack of a negative root.

(b) Candidates should be aware that answers to this question should be in radians and those answers in radians had to be multiples of π (not decimals). Although a few candidates worked in degrees or tried to work with a mixture of degrees and radians most candidates obtained at least one solution or $-\frac{\pi}{12}$. Some candidates found extra solutions but the usual reason for loss of marks was omission of a solution, usually $\frac{23\pi}{12}$.

Answers: **(a)(ii)** $60°, 120°, 240°, 300°$ **(b)** $\frac{11\pi}{12}, \frac{23\pi}{12}$

Question 9

Candidates would benefit from practice in identifying questions where combinations should be used and those where permutations should be used. With the exception of **part (a)(ii)** a significant proportion of the responses in this question were using permutations instead of combinations or vice versa. Candidates should also be aware when it is appropriate to add combinations or permutations and when it is appropriate to multiply.

(a) (i) Many candidates used permutations instead of combinations.

(ii) This part was generally well answered. A good proportion of candidates formed a correct plan including the four different cases and executed it accurately. A few candidates however did not fully understand the question and included extra cases Candidates who tried the alternative approach of subtracting the number of all boy and all girl teams from the total number of different teams were not as successful and tended not to subtract both correct cases.

(b)(i) Many candidates used combinations rather than permutations.

(ii) Many candidates multiplied by 4 and by 3 but some used an incorrect permutation or used a combination instead of a permutation.

(iii) Candidates found this part difficult. Some calculated the number of ways with one symbol only. Many others identified that there were three different cases to consider but either used combinations or did not fully understand what had to be calculated for each case. Successful candidates tended to be those who used the subtraction method.

Answers: **(a)(i)** 8568 **(ii)** 8260 **(b)(i)** 151 200 **(ii)** 20 160 **(iii)**146 160

Question 10

Both parts of this question required careful reading.

(i) A high proportion of candidates knew they had to integrate and most of those integrated at least one term correctly. Candidates would benefit from practice in integrating expressions such as $8e^{2x}$. Some candidates omitted a constant of integration and so could not progress further. Other candidates used the *x*- and *y*-coordinates the wrong way around when finding the constant of integration or produced an incorrect constant without working. A significant number of candidates tried to find the equation of a tangent or normal rather than the equation of the curve. The work of those candidates who tried several approaches was poorly set out and difficult to mark.

(ii) There were some good solutions seen where candidates correctly obtained the equation of the normal to the curve and went on to find a correct area. Candidates should be aware that this question had to be read carefully in conjunction with the first part. A significant proportion of candidates did not relate this part to the information given in the first part and found a normal to $y = 2 - 3x$. Others used a second derivative of f(x) to obtain the gradient of the normal not realising that they had already been given a gradient function. Some candidates found the equation of the tangent rather than the normal and candidates should be aware of the difference between these. Others tried to find the intersection of a curve with $y = 2 - 3x$ and seemed not to be aware that the equation of a straight line was required for a normal.

Candidates who had obtained a value of x often went on to correctly find the area of their triangle with those using the determinant method usually being most successful apart from the occasional loss of the $\frac{1}{x}$. Candidates who employed lengthy Pythagoras calculations had clearly not realised that x was the height of a triangle with a base of length 3.

Answers: **(i)** $f(x) = 3x^2 - 4e^{2x} + 1$ **(ii)** 2.4

Question 11

(i) Many correct solutions were seen with most candidates realising that differentiation was required. There were occasional careless mistakes in the evaluation where candidates had forgotten their 3 times table or had subtracted 3 rather than 8.

(ii) Many correct solutions were seen with very few errors in factorising or in obtaining solutions from factors.

(iii) Many candidates knew that they had to integrate the expression for v with respect to t in order to obtain an expression for displacement. Of those who integrated correctly many substituted 1.5 correctly but mistakes in mental arithmetic meant that a significant proportion of these did not obtain an answer of zero. Other candidates substituted but gave answers including a constant of integration.

Few candidates realised that their answers to the previous part indicated that the particle had turned around when $t = \frac{1}{2}$. This fact combined with the zero displacement at $t = 1.5$ led just a few candidates to correctly conclude that the distance travelled was twice that travelled before the particle turned around at $t = \frac{1}{2}$.

Candidates should be aware that in this question it was important to show values substituted in a correct integral rather than just giving numerical answers.

Answers: **(i)** 16 **(ii)** $\frac{1}{2}$, $\frac{3}{2}$ **(iii)** $\frac{4}{3}$

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

In order to do well in this paper, candidates need to have covered the full syllabus and be confident in their use of algebra. Candidates are reminded that, in questions which have a 'proof' or 'show that' instruction, written work must be thorough and complete in order to obtain full marks. Correct work is often spoilt in questions of this nature by omission of brackets in one or more lines of the working. This, together with a lack of care with regard for signs, are the most typical causes of credit being withheld in this type of question.

General comments

The complete range of marks were seen. The majority of candidates found **Question 10** difficult, with even the most able candidates making little progress.

Many candidates did not possess the skill of realising when their work is not making progress. They often continued to produce a considerable amount of worthless expressions. This was a particular problem in **Question 6(i)**. In general, candidates should take note of the number of marks available for each part question and the amount of working space provided, as a guidance as to the amount of work required.

Comments on specific questions

Question 1

This question was very straightforward for those who knew the method, but many did not and gave $x = 1$ as the only solution. Solutions using $4x - 3 = \pm x$ were nearly always correct, but those using $\pm(4x - 3) = x$ often led to sign errors.

Many candidates attempted to square, but this often led to a variety of errors and misunderstandings. It was common to see only one side of the equation squared or to see the terms squared individually. Some thought that the modulus sign and squaring cancelled each other out.

Answers: $x = 1$ and $x = 0.6$

Question 2

This was well completed by many but there were also a considerable number who, after taking a common denominator or multiplying throughout by $(\sqrt{3} - 1)(\sqrt{3} + 1)$, came to a halt not knowing how to proceed.

There were a number of candidates who got to this stage and continued to produce more lines of working without any clear idea of where they were going. Many candidates did not appear to appreciate that the next step was to form a pair of equations in a and b by equating the coefficients of $\sqrt{3}$ terms and constant terms.

Answers: $a = 4$ and $b = -2$

Question 3

This question was found difficult by a large number of candidates who appeared not to remember the laws of logarithms. Candidates were awarded marks for knowing $2\lg x = \lg x^2$ and $1 = \lg 10$. When candidates were attempting to obtain an equation to solve, the $\lg\left(\frac{x+10}{2}\right)$ term was often not combined with another lg term but the lg was just removed from each of the three terms. Others attempted to multiply the whole equation by the 2 from the denominator.

A number of otherwise good solutions lost the last mark because they did not identify that $x = -5$ was not a valid solution.

Answer: $x = 10$

Question 4

(i) The correct calculation was seen in most cases but a considerable proportion of candidates did not realise that, due to the context, they should give an integer solution.

(ii) The majority of candidates managed to set *N* equal to 7500 and rearranged to make $e^{-0.05t}$ the subject. There were a number, however, who attempted to take logs at an early stage and this resulted in incorrect rules being applied and no further credit being available. A few attempted to use logs to the base 10.

(iii) Those who realised that $\frac{dn}{dt}$ was required proceeded well, apart from the candidates who applied an extra multiplier of *t* either as part of their differential or as an extra multiple at the end of their response. Others who did not appreciate that the differential was required calculated values of *N* at $t = 0$ and $t = 8$ and found the average of these values.

Answers: **(i)** 8213 **(ii)** 27.7 **(iii)** 67

Question 5

(i) Most candidates realised that they had to differentiate the cubic equation to obtain the correct quadratic. A few then attempted to solve by setting this quadratic to zero rather than finding the gradient at $x = 2$. Of those who did the correct method, a number made a sign error getting a gradient of 13 rather than -3. Most who got a gradient of -3 found the correct equation, but some found the equation of the normal. Others started by assuming that the gradient was -7, presumably from the coefficient of *x* in the original equation.

(ii) Most candidates correctly set their linear equation equal to the original cubic and correctly simplified and equated it to zero. Some then did not know what to do with this cubic even though they should have realised, from previous work, that there was a repeated root of $x = -2$. It was not uncommon for those who successfully factorised the cubic and obtained the solutions of $x = -2$ and $x = 2$ not to realise which of these solutions they required. Some candidates omitted to find the *y*-coordinate of the point or thought that it was zero.

Answers: **(i)** $y = -3x + 10$ **(ii)** $(2, 4)$

Question 6

(i) This part was found very difficult, with many candidates not changing the terms into $\sin x$ and $\cos x$ and attempting to proceed with a common denominator of $(1+\tan x)(1+\cot x)$. This often led directly to the required answer by some dubious cancelling or the denominator was incorrectly replaced by $\sec x \operatorname{cosec} x$. Candidates should be aware that Examiners check working carefully in this type of question and such solutions do not gain marks. Others who had proceeded well as far as $\frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$ arrived at the answer by cancelling $\cos x$ and $\sin x$ rather than factorising the numerator. Many candidates found themselves with a large amount of complicated trigonometric algebra. In such situations, candidates would be well-advised to consider whether there may be an easier method of solving the problem.

(ii) There were many errors in collecting the terms in $\sin x$ and $\cos x$ with some candidates even starting again, without reference to **part (i)**, perhaps not realising the significance of the word 'hence'. However, a large number of candidates achieved three of the four marks. The last mark was frequently lost because the candidate did not realise that a negative angle was required, often giving the answer in the third quadrant.

Answers: **(ii)** 51.3° and -128.7°

Question 7

(i) There were many well explained solutions to this question. However, there was also evidence of poor arithmetic, a lack of brackets and careless use of square and square root signs spoiling many solutions. A common error was to replace $\frac{14+x}{2}$ by $7+x$.

(ii) Most candidates attempted to use the product rule, although many struggled to differentiate $\sqrt{9-x^2}$ correctly. The minus sign and sometimes x was missing and often correct separate working was not transcribed correctly when combining the complete expression for the differential. Most candidates knew to put $\frac{dA}{dx}$ equal to zero but poor algebra plus earlier errors meant that very few correct answers were obtained. Those who did get $x = 1$ often forgot to obtain the corresponding value of A. Candidates are advised to always re-read the question to ensure they have fully answered all parts.

Answers: **(ii)** $A = 22.6$

Question 8

(i) The quotient rule was usually correctly applied but errors in simplification, often caused by omission of brackets, led to a sign error giving a value of $k = 6$ rather than 12. It was also fairly common to see $x^2 \times x^3 = x^6$. Weaker candidates were confused by the notation and attempted to find the inverse function.

(ii) Those candidates who recognised the connection to **part (i)** and used their value of k were able to produce accurate and concise solutions. However, the vast majority did not see the connection and attempted the integration from scratch, often integrating the numerator and denominator separately. Others simply inserted the limits into the expression without attempting to integrate.

(iii) Finding the inverse function was generally tackled well and although there were some errors in manipulation these were in a minority. Some candidates omitted to change x and y and many could not determine the range of values required for the domain.

Answers: **(i)** $k = 12$ **(ii)** $\frac{7}{54}$ **(iii)** $f^{-1}(x) = \sqrt[3]{\frac{x+1}{3-x}}$ Domain : $-1 \leqslant x \leqslant 2\frac{6}{7}$

Question 9

(i) Many candidates found this question difficult. Most made an attempt to eliminate *y*, although many were unable to simplify the resulting equation accurately, often getting $(k+1)$ rather than (k^2+1) for the coefficient of x^2. Those who realised that they should then use the discriminant of their quadratic usually did so successfully and gave just the positive solution. Many early errors prevented a large number of candidates from getting a correct value of k.

(ii) The vast majority of candidates inserted their value of k into their quadratic equation and attempted to solve, not realising that as they had used the fact that the discriminant was zero they could go straight to the value of as $\frac{-b}{2a}$. Only the very best candidates avoided making algebraic errors and obtained the correct point.

(iii) This part was attempted by only a minority of candidates. Those who did make an attempt did not always know the correct formula for finding the distance and, of those who did, many made sign errors when dealing with negative coordinate values.

Answers: **(i)** $k=\frac{4}{3}$ **(ii)** $(2.4, -0.8)$ **(iii)** 4

Question 10

The vast majority of candidates omitted this question and those who did attempt it appeared to have little idea of how to proceed. It was hoped that by giving assistance in the question that candidates would follow the lead and obtain some correct components. This was not the case as the angles used were invariably incorrect. In the second part there were three method marks which could have been obtained from incorrect answers but unfortunately they were rarely awarded. More work on this topic would be beneficial to candidates.

Answers: **(i)** $r_j=\begin{pmatrix}5000\\1000p\end{pmatrix}+\begin{pmatrix}-2\cos40\\2\cos50\end{pmatrix}t$ **(ii)** $T=2095$ and $p=2.23$.

Question 11

Many candidates, including those who had struggled with earlier questions, scored well here. However, there were a number of errors, with two in particular being common. A number of candidates added the results of their correct numbers of men and women rather than multiplying them, so answers of 25 and 14 for **parts (i)** and **(ii)** were often seen. The other mistake was to use permutations rather than combinations. In **part (iii)**, a common error was to select from 6 and 5 rather than 5 and 4.

Answers: **(i)** 150 **(ii)** 40 **(iii)** 80

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

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Question 8

(i) The quotient rule was usually correctly applied but errors in simplification, often caused by omission of brackets, led to a sign error giving a value of $k=6$ rather than 12. It was also fairly common to see $x^2 \times x^3 = x^6$. Weaker candidates were confused by the notation and attempted to find the inverse function.

(ii) Those candidates who recognised the connection to **part (i)** and used their value of k were able to produce accurate and concise solutions. However, the vast majority did not see the connection and attempted the integration from scratch, often integrating the numerator and denominator separately. Others simply inserted the limits into the expression without attempting to integrate.

(iii) Finding the inverse function was generally tackled well and although there were some errors in manipulation these were in a minority. Some candidates omitted to change x and y and many could not determine the range of values required for the domain.

Answers: **(i)** $k=12$ **(ii)** $\dfrac{7}{54}$ **(iii)** $\mathrm{f}^{-1}(x) = \sqrt[3]{\dfrac{x+1}{3-x}}$ Domain : $-1 \leqslant x \leqslant 2\dfrac{6}{7}$

Question 9

(i) Many candidates found this question difficult. Most made an attempt to eliminate *y*, although many were unable to simplify the resulting equation accurately, often getting $(k+1)$ rather than (k^2+1) for the coefficient of x^2. Those who realised that they should then use the discriminant of their quadratic usually did so successfully and gave just the positive solution. Many early errors prevented a large number of candidates from getting a correct value of k.

(ii) The vast majority of candidates inserted their value of k into their quadratic equation and attempted to solve, not realising that as they had used the fact that the discriminant was zero they could go straight to the value of as $\frac{-b}{2a}$. Only the very best candidates avoided making algebraic errors and obtained the correct point.

(iii) This part was attempted by only a minority of candidates. Those who did make an attempt did not always know the correct formula for finding the distance and, of those who did, many made sign errors when dealing with negative coordinate values.

Answers: **(i)** $k=\frac{4}{3}$ **(ii)** $(2.4, -0.8)$ **(iii)** 4

Question 10

The vast majority of candidates omitted this question and those who did attempt it appeared to have little idea of how to proceed. It was hoped that by giving assistance in the question that candidates would follow the lead and obtain some correct components. This was not the case as the angles used were invariably incorrect. In the second part there were three method marks which could have been obtained from incorrect answers but unfortunately they were rarely awarded. More work on this topic would be beneficial to candidates.

Answers: **(i)** $r_j=\begin{pmatrix}5000\\1000p\end{pmatrix}+\begin{pmatrix}-2\cos 40\\2\cos 50\end{pmatrix}t$ **(ii)** $T=2095$ and $p=2.23$.

Question 11

Many candidates, including those who had struggled with earlier questions, scored well here. However, there were a number of errors, with two in particular being common. A number of candidates added the results of their correct numbers of men and women rather than multiplying them, so answers of 25 and 14 for **parts (i)** and **(ii)** were often seen. The other mistake was to use permutations rather than combinations. In **part (iii)**, a common error was to select from 6 and 5 rather than 5 and 4.

Answers: **(i)** 150 **(ii)** 40 **(iii)** 80

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key Messages

Working should always be shown so that marks for method can be awarded, even when an answer is incorrect. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

In 'show that...' questions, every step leading to the given answer must be shown clearly.

In 'without using a calculator...' questions, every step in the working should be shown. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited.

When evaluating trigonometric expressions in calculus applications, angles must be input in radians, not degrees.

General comments

There was a very wide variation in the quality of work produced, with total marks over the whole of the range. Some candidates produced very clear answers which displayed sound mathematical knowledge and skills, whilst others clearly had inadequate knowledge for an examination of this standard.

It will seem superfluous to point out that a question should always be read carefully and any given instruction followed precisely, but there were instances in this paper where this was clearly not done. For example, if, in answer to a question which says 'without using a calculator...', a candidate produces a non-exact numerical value expressed to several places of decimals, it is obvious that the instruction has been ignored. Marks are inevitably lost in such situations.

It has been stated regularly in these reports that in solving quadratic equations the method should always be shown. If the equation is incorrect, a method mark can then be awarded. Method marks are not awarded for solutions to an incorrect equation taken directly from a calculator (see **Question 7(ii)** below).

The physical clarity of answers on scripts showed improvement this year, with fewer first attempts at solutions overwritten with second attempts, something which makes the work presented very difficult to read. But if a first attempt is subsequently corrected by a candidate reviewing their work, any change must be made absolutely clear. This is especially true of sign changes, where overwriting often makes it impossible for the Examiner to know what sign is intended (see **Question 3(i)** below).

Comments on specific questions

Question 1

The process of rationalising an expression of the form given was well understood and there were many fully correct answers to the question. As this was a 'without using a calculator...' question, it was necessary for more than two terms to be seen in the numerator in the working to convince Examiners that a calculator had not been used.

Answer: $k = 15$

Question 2

As often seems to be the case with questions involving indices and logarithms, there was a wide range in the quality of answers to this question. There were short accurate answers showing good understanding of the laws of indices and logarithms, and other longer attempts containing little valid mathematics. One approach, which was frequently successful, was to set another variable, y say, equal to e^x, and solve firstly for y. Another, that of taking logarithms of both sides at the start, whilst being a perfectly good method, often resulted in errors: a common one was the equating of $\ln 6e^x$ with $x\ln 6e$.

Answer: $x = 0.896$

Question 3

In **part (i)**, good clear use of the quotient rule (and occasionally the product rule) was seen frequently, with all the necessary steps being shown to arrive at the given answer. In some cases, where a sign error had been made and the result could not be achieved, a candidate might overwrite earlier work to an extent that what was intended was far from clear. Sometimes also, it was not easy for Examiners to distinguish between corrections, and cancelling. It is especially vital, in a question where the answer is given, that any corrections are made absolutely clearly if credit is to be earned.

In **part (ii)**, many candidates saw the link to **part (i)**, so were able to state the integrand correctly. However, a very common error here was to substitute the values of the limits in degrees instead of radians. This resulted in a numerical answer which was exceedingly small, a matter which seems to have caused no concern to candidates making this error. A brief pause to inspect the given diagram would have indicated that the area should be somewhere between 1 and 2 square units.

Answer: **(ii)** 1.56

Question 4

This question was very well done and a good source of marks for many candidates. Sound knowledge of the factor and remainder theorems was regularly shown in **part (i)**, with the equations being properly set up and solved. In answers to **part (ii)**, it was clear that there was general familiarity with the techniques for solving a cubic equation, but occasionally it was clear also that the question had not been read properly. It asks for $p(x)$ first to be factorised, and "hence" for the equation to be solved. Some candidates aimed directly for the solutions from the outset, and in doing so omitted one, or even two, of the factors, with consequent mark loss.

Answers: **(i)** $a = 5$, $b = -2$ **(ii)** $(x - 2)(x + 3)(x + 4)$, $x = 2, -3$ or -4

Question 5

It has to be repeated, and emphasised, that in 'without using a calculator...' and 'show that...' questions, every step in the working must be shown clearly, even if it seems obvious to the candidate, to demonstrate that the instruction has been followed, and the given answer properly proved.

In **part (i)**, the cosine rule was often correctly stated and simplified. However, in the evaluation of the squared brackets it was necessary for the Examiner to see sufficient terms to be convinced that a calculator had not been used.

Many correct answers to **parts (ii)** and **(iii)** were seen, although regularly also a mark or marks would be lost. In **part (ii)**, it was not enough simply to state the sine rule (or cosine rule if another method was used); a mathematically correct explicit expression for $\sin A$ (or $\cos A$) had to be given to obtain both marks. In **part (iii)**, an answer of 0.866 automatically indicated the use of a calculator.

Only a few very limited answers assumed the given triangle to be right angled.

Answer: **(iii)** $\frac{\sqrt{3}}{2}$

Question 6

In **part (i)**, most candidates knew the method for finding the equation of the tangent to a curve. Where many made errors was in the computation of the values of the component parts of the equation: in the value of $\sec^2 x$, and of the y-coordinate of the point. It was very common to see $\frac{\pi}{4}$ substituted as a value in degrees instead of radians. In **part (ii)**, success was usually achieved by those who used the identity given on page 2 of the paper for $\sec^2 x$, and obtained a quadratic equation in $\tan x$. Those rewriting in terms of $\sin x$ and $\cos x$ were almost always unsuccessful. No marks were earned by those who used numerical values for y of 8, or $\sec^2 x$ of 2, from **part (i)**.

Answers: **(i)** $y = 2x + 6.43$ **(ii)** 1.25, 2.03

Question 7

In **part (i)** most candidates were able to use Pythagoras' Theorem correctly to prove the given relationship, although occasionally a mark was lost because of a missed step. Once more it must be emphasised that every step must be shown, even if it might seem obvious to the candidate, when a given result is to be proved.

Candidates who then used the relationship from **part (i)** in **part (ii)** to obtain an expression for V in terms of h only, differentiated, and set $\frac{dV}{dh}$ equal to zero, were usually successful, provided algebraic slips were avoided. Any invalid attempt to differentiate V in its standard form containing both r and h inevitably resulted in the loss of all subsequent marks. Candidates need to be aware that, at this level at least, it is not possible to differentiate a function of two variables.

In solving the quadratic equation arising from $\frac{dV}{dh} = 0$, a method mark could still be earned even if the equation itself was incorrect, provided method was shown. However, solutions to an incorrect equation taken directly from a calculator earned no marks. As this question related to an actual physical object, it was necessary to reject the negative root from the solution of $\frac{dV}{dh} = 0$.

The method most commonly used in **part (iii)** was that based on the sign of the second derivative at the stationary point. The best answers showed clear substitution of the answer from **part (ii)** into $\frac{d^2V}{dh^2}$, demonstration of the sign of its value, and a conclusion of 'maximum'. Other methods were acceptable, but more rarely seen.

Answers: **(ii)** 2.49 **(iii)** maximum (justified by a fully correct method)

Question 8

The mathematics presented in answers to **part (i)** was often correct, but often also had limitations. One such, which resulted in mark loss, occurred when an answer was given in degrees, instead of radians. Another was the immediate rounding of the radian answer to two significant figures, and sometimes even to just one significant figure. Such approximation in the first part of the question had obvious consequences for accuracy in the next two parts, which used this angle. Candidates need to be aware of the dangers of premature approximation, especially in longer structured questions. A limitation which did not result in mark loss, but which resulted in time wasted, occurred when extended methods were employed unnecessarily: for example, the unknown side in triangle OAT might be found, then the sine rule used (in a right-angled triangle) to find the required angle in degrees, then this angle converted into radians.

The best answers to **part (ii)** explained (sometimes with small diagrams) how the required area was being divided up, most simply as the kite $OATC$ and the major sector of the circle, and showed clearly the area calculations for the component parts. It is important in such a question as this for the candidate to explain their working, especially as the method employed becomes more complex, perhaps involving (many) more component parts. Examiners were sometimes presented with the whole answer space filled with several pieces of working, but no explanation as to which angle, length, or part of the diagram, each referred. When presented with such responses it is very difficult for Examiners to be able to award method marks when the answer to the question is incorrect.

A serious error made by some candidates was to attempt to find the area of the quadrilateral *TABCT*. This is actually impossible, as *B* is not a defined point.

Candidates who were successful in **parts (i)** and **(ii)** were generally successful also in **part (iii)**. However, a mark was often lost because of the approximation referred to above in **part (i)**.

Answers: **(i)** 0.927 **(ii)** 128 cm^2 **(iii)** 42.6 cm

Question 9

There was only a limited number of fully correct answers to this question. Many candidates made no attempt at some parts and appeared to be unprepared on the topic.

In **part (i)** few candidates were able to find *p* from the magnitude of the velocity vector and given speed of *A*. In **part (ii)** little general understanding was shown of a direction of motion as a bearing.

Those achieving most success with the remaining parts of the question were those who clearly understood, for **parts (iii)** and **(iv)**, how the position vector of a moving particle can be expressed. However, it was common for answers to these parts to contain only one vector, or for *t* to be completely absent. Such candidates were able to earn no further marks.

Answers: **(i)** 4 **(ii)** 108 **(iii)** $\begin{pmatrix}1\\5\end{pmatrix}+t\begin{pmatrix}4\\-3\end{pmatrix}$ **(iv)** $\begin{pmatrix}q\\-15\end{pmatrix}+t\begin{pmatrix}3\\-1\end{pmatrix}$ **(v)** 10 seconds **(vi)** $\begin{pmatrix}41\\-25\end{pmatrix}$ **(vii)** 11

Question 10

Good understanding was shown in **parts (i)** and **(ii)** of how to combine the functions, and in **part (iii)** of how to find the inverse of a function. For **part (iv)** there were also many correct answers, the exceptions occurring when f(4) was evaluated instead of the given equation being solved.

It was in **part (v)** where most errors tended to occur. The composite function was almost always formed correctly, and the equation set up properly. It was in the solving of this equation, involving the laws of indices and logarithms, where incorrect mathematics was seen frequently. The common error (as that noted for **Question 2** above) was to equate an expression of the form $\ln AB^p$ with $p\ln AB$.

Answers: **(i)** $2 + \ln(2e^x + 3)$ **(ii)** $2 + \ln(2 + \ln x)$ **(iii)** $\ln\left(\frac{x-3}{2}\right)$ **(iv)** 7.39 **(v)** $x = 1.15$

Question 11

In **part (i)**, almost all candidates knew how to find $\mathbf{A}^2$ properly, but fewer were able to find the required relationship. Of those who understood the method, many did not complete it fully.

In mathematics, candidates need to be advised to make their writing clear enough for the Examiner to follow, and also so that they themselves do not misread their own work. There were many instances in this question of the candidate copying a ‘9’ they had written in an early part of their working, as ‘*q*’ subsequently. This sometimes had serious mark loss consequences for the candidate in **part (ii)**.

Provided the relationship had been found in **part (i)**, and a correct expression for the determinant of **A** was stated, full marks were usually obtained in **part (ii)**. A fairly common error here was to set the determinant equal to $\frac{1}{6-pq}$, and a few candidates presented much fruitless work involving the inverse of **A**.

Answers: **(i)** $pq = 8$ **(ii)** $p = \frac{2}{3}$, $q = 12$