MATHEMATICS

Paper 9709/12
Pure Mathematics

Key messages

1 **Angles in radians**. In this paper there were two questions (**Questions 4** and **5**) in which angles were given in radians and it was very clear that many candidates are not at all comfortable with the concept of radians. It was very common indeed to see candidates converting from radians to degrees before addressing the actual question. Not only is this inefficient but candidates very often obtained answers which were not accurate enough because of conversion and reconversion or made errors in the process. It also needs to be said that the concept of radians is one of the nine sections of the syllabus and as such is a concept which needs to be fully embraced by candidates.

2 **Accuracy**. Marks throughout the paper were lost by large numbers of candidates through failing in numerical questions to use sufficient accuracy. The required level of accuracy (unless a particular question requires otherwise) is three significant figures for answers – and there many occasions when answers were given to only two significant figures. But what candidates seemed to not appreciate was that in order for a given three-figure answer to be correct it is often the case that the intermediate steps need to be undertaken using at least four-figure accuracy.

3 Two questions on the paper (**Questions 1** and **8(iii)**) involved solving a two-term quadratic equation in which there was no constant term. A significant proportion of candidates seemed to forget that, in general, quadratic equations have two solutions and lost one of the solutions (zero) by dividing throughout by the variable.

4 It needs to be emphasised that a careful reading of the question is important. For example, in **Question 4(ii)** a number of candidates found the area of the shaded region instead of the perimeter.

General comments

The paper was generally well received by candidates and many good scripts were seen. Most candidates seemed to have sufficient time to finish the paper.

Comments on specific questions

Question 1

This question was intended as a straightforward start to the paper but the marks scored were surprisingly disappointing with the average being about two marks out of four. Most candidates calculated $b^2 - 4ac$ correctly but a surprising number put this equal to zero rather than greater than zero. This is a valid method for finding critical values of k providing the correct inequalities are inserted at the end. But many candidates forgot this final step. Other candidates seemed to assume that the critical region was automatically between the critical values rather than either side of them. Other candidates obtained only one critical value by dividing throughout by k. (See point 3 above.)

Answer: $k < 0$, $k > 8/9$

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Question 2

This was a successful question for the majority of candidates. Most candidates were able to identify the correct term from the expansion – although many wrote out the complete expansion. A few mistakes were made forgetting to cube *a* or square 2*a*.

Answer: $a = 8$.

Question 3

The majority of candidates answered **part (i)** correctly. In **part (ii)** those candidates who successfully sorted out the three variables involved (V, h and t) without using any others and who realised they needed to find $\mathrm{d}V/\mathrm{d}h$, were the most successful. A significant proportion of candidates applied the Chain Rule incorrectly but those who followed a systematic approach were usually successful.

Answer: **(i)** $\frac{1}{12}h^3$; **(ii)** 0.8.

Question 4

In **part (i)**, only an exact method was acceptable – which excluded those candidates who converted to degrees and then reconverted back to radians. It was noticeable, also, that weaker candidates often omitted **part (i)**. In **part (ii)**, a number of different ways of obtaining the length of *BC* were seen (e.g. Sine Rule or Cosine Rule). This was one of the places where candidates were not careful enough with accuracy and frequently 6.9 or even 7 was seen carried forward for the length of *BC*. It was, however, pleasing to see correct methods being used to find the perimeter of the shaded region – even if marks were frequently lost by an inaccurate answer at the end.

Answer: **(ii)** 28.1.

Question 5

This question was a little unusual and it was again noticeable that weaker candidates sometimes made little or no attempt. In **part (i)**, most candidates were able to make progress by attempting to solve the equation $\tan x = \sin x$ leading to $\sin^2 x + \sin x = 1$. But then, accuracy was often lost in solving this equation for sin*x* and in finding the value of *x* in radians. A proportion of candidates gave the answer in degrees. **Part (ii)** was even less successful and candidates struggled both with accuracy and radians, as well as in identifying a strategy for finding the coordinates of *B*.

Answers: **(i)** $x = 0.666$; **(ii)** $x = 2.48$,. $y = –0.786$ (or –0.787).

Question 6

In **part (i)**, the first step of finding vector $\overrightarrow{BA}$ was often omitted and a substantial number of candidates simply found the angle between the two given vectors, essentially finding angle *AOB* instead of angle *OAB*. At least half the remaining candidates found the scalar product $\overrightarrow{OA}.\overrightarrow{AB}$ or $\overrightarrow{AO}.\overrightarrow{BA}$ and gave the final answer as an acute angle rather than an obtuse angle. **Part (ii)** was far more successful and those candidates who had found angle *AOB* in **part (i)** were still able to obtain the correct answer in **part (ii)**.

Answers: **(i)** 95.1°; **(ii)** 16.8.

Question 7

This question was well-answered and many candidates obtained full marks. **Part (i)** was largely well done although some candidates forgot to multiply by the derivative of the bracket (4). **Part (ii)** attracted high marks in general and very often those candidates who had made mistakes in **part (i)** were still able to score four marks out of five.

Answers: **(i)** $6(4x+1)^{\frac{1}{2}}$; $12(4x+1)^{-\frac{1}{2}}$; **(ii)** $k = 3$.

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Question 8

Part (i) was very well attempted and most candidates scored both marks. In **part (ii)**, the majority of candidates showed that they knew how to find the inverse of a quadratic function although quite a few ignored the unsimplified form they had obtained in **part (i)** and started afresh by completing the square in their simplified form. Unfortunately this sometimes led to arithmetic errors. Finding the domain of the inverse function was often not attempted. When it was attempted a very common wrong answer was $x \geq 3$.

Answers: **(i)** $\mathrm{gf}(x) = 3(2x^2+3)+2 = 6x^2+11$, $\mathrm{fg}(x) = 2(3x+2)^2+3$; **(ii)** $(\mathrm{fg})^{-1}(x) = \frac{1}{3}\sqrt{(x-3)/2} - \frac{2}{3}$, domain is $x \geq 11$; **(iii)** $x = 0$ or 4.

Question 9

Part (i) was very well answered with most candidates scoring full marks. **Parts (ii)** and **(iii)** were found progressively less successful with a substantial number of candidates making no attempt with **part (iii)**. Arithmetic errors in **part (iii)** accounted for a proportion of the marks which were lost and it was noticeable that most candidates seemed to be finding the intersection of the tangents by using a simultaneous equations approach rather than by simply writing $2x - 2 = -3x + 7/4$.

Answers: **(i)** $y = 2x - 2$; **(ii)** $\left(-\frac{1}{2}, 3\frac{1}{4}\right)$; **(iii)** $\left(\frac{3}{4}, -\frac{1}{2}\right)$

Question 10

Part (i) was done quite well although some candidates also gave – 1 as a possible solution. **Part (ii)** was also well attempted with a large proportion of correct answers. It is worth mentioning, however, that surprising numbers of answers were left as $x^2 - \frac{2}{-2x^2} - \frac{17}{4}$. It is reasonable to expect at this level that the middle term is simplified. **Part (iii)** was considerably less well done. Not all candidates realised that they had to multiply throughout by x^2 to obtain a quartic equation in x, then to treat this equation as a quadratic equation in x^2 and then to square root the resulting solutions. The integration in **part (iv)** was usually managed reasonably successfully with the limits being the solutions from **part (iii)**. The substitution of the limits gives a negative answer but the final answer, representing area, needs to be shown as a positive number.

Answers: **(i)** $x = 1$; **(ii)** $\mathrm{f}(x) = x^2 + \frac{1}{x^2} - \frac{17}{4}$; **(iii)** $x = \frac{1}{2}$, $x = 2$; **(iv)** $2\frac{1}{4}$

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MATHEMATICS

Paper 9709/22
Pure Mathematics

Key messages

Candidates are to be reminded of the importance of reading each question carefully and making sure that they have completed all the demands of the question and to the correct level of accuracy. It should also be noted that in questions with parts labelled **(i)**, **(ii)** and **(iii)**, that results either obtained or given in an earlier part, will often need to be used in a later part.

General comments

Only a very small number of candidates formed this year's cohort, making it difficult to generalise problem areas or questions that had been done well. Because of this, the report will focus more on the intended methods of solution.

Comments on specific questions

Question 1

Candidates were expected to use the basic laws of logarithms and through simplification, produce a quadratic equation without logarithms. This was usually done well but candidates failed to realise that x had to be positive only, so very often the last accuracy mark was lost as the solution $x = -1$ was included in the final result.

Answer: 15

Question 2

(i) It was intended that candidates make use of the identities $\cot\theta = \frac{1}{\tan\theta}$ and $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ in order to obtain the required result. However, most candidates chose to write the trigonometric ratios in terms of sine and cosine and make use of the corresponding double angle formula together with the identity $\tan^2\theta + 1 = \sec^2\theta$, in order to obtain the required result. Although this method was longer and hence more time consuming, candidates were equally successful as those who had chosen to work in terms of tangent only.

(ii) Correct solutions were few as candidates often failed to realise that the result from **part (i)** was to be used to solve the given equation in **part (ii)**. The second solution within the range was usually omitted. It should also be noted that in the case of angles in degrees, the answer should be given correct to one decimal place.

Answer: **(ii)** 40.9°, 139.1°

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Question 3

(i) Most candidates were able to obtain the correct critical values using a correct approach. However, correct ranges were often not obtained.

(ii) It was intended that candidates should attempt to find y by making use of $\ln y$ being equated to the upper limit of the answer to **part (i)**. It was evident that candidates did not appreciate the connection between this part of the question and the preceding part of the question.

Answer: **(i)** $\frac{2}{3} < x < 8$, **(ii)** 2980

Question 4

It was recognised by most candidates that implicit differentiation was needed. Most candidates made a very good attempt at this, but errors in simplification and substitution often meant that the correct answer was not obtained.

Answer: $-\frac{4}{3}$

Question 5

(i) Most candidates were able to integrate and apply the given limits correctly and through correct manipulation, obtain the given result.

(ii) Some candidates did not attempt this part at all, failing to see the connection with the first part of the question, even though they had been instructed to use it. Some candidates lost the accuracy mark for the final answer by not giving it to the required level of accuracy. Iterations were usually done to the required level of accuracy.

Answer: **(ii)** 1.835

Question 6

(i) This part of the question was done well by most candidates who appear to be adept at dealing with both the remainder and factor theorems. Only arithmetic slips prevented most candidates from gaining full marks for this part.

(ii) It was intended that candidates obtain a quadratic factor by either observation or use of algebraic long division. Most were able to do this and go on to obtain the correct factors. However, some candidates solved the cubic equation in such a way that the factorised form of p(x) was never seen. This highlights the importance of making sure that the question demands have actually been met.

(iii) It was intended that candidates make use of their linear factors from **part (ii)**, equate them to zero and deduce that if x is replaced by 2^y then there is only one possible result.

Answer: **(i)** $a = 6,\ b = 5$ **(ii)** $(x+2)(3x+1)(2x-3)$ **(iii)** one, with appropriate justification

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Question 7

(i) Most candidates were able to use the appropriate compound angle formula, but few were able to make use of the appropriate double angle formulae to obtain a result in the required form.

(ii) It was intended that candidates should attempt to find the intercept of the given curve with the x-axis, thus giving some limits that could be used in evaluating the integral to give an exact value for the required area.

Answer: **(i)** $\frac{\sqrt{3}}{2}(1+\cos 4x)-\frac{1}{2}\sin 4x$ **(ii)** $\frac{\sqrt{3}}{12}\pi$

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MATHEMATICS

Paper 9709/32
Pure Mathematics

General comments

The standard of work on this paper varied considerably. Hence a spread of marks from zero to the maximum. However, there were far too many candidates who were simply not prepared for this examination. Every question was successfully solved by a number of candidates, although this number was small in the case of **Question 5**, **Question 6(ii)**, **Question 8(ii)** and **Question 10(i)**, **(ii)**, **(iii)**.

In general the presentation of the work was acceptable and most candidates made some attempt at the majority of the questions. It was a concern to see that many candidates had not taken on board the comments in the recent reports. Namely that when attempting a question it is essential that sufficient working is shown, especially in **Question 8(i)** and **(ii)**, where despite the question stating, '**do not use a calculator and show all your working**', the z^2 and z^4 terms often had little detail of how their values were arrived at and the solution of the quadratic was clearly being undertaken on the calculator.

Candidates should realise that the notation **(i)**, **(ii)** and **(iii)** usually implies that there is a linkage between the various sections and this linkage needs to be addressed in order to solve the question by the most direct method, see the details of this in the various sections of **Question 10**.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

Many candidates produced a completely correct solution, however, some either failed to give their answer to 3 decimal places or rounded it incorrectly. The other basic error was the failure to take the logarithm of the RHS of $2^x = a$, so $x\ln 2 = a$ was common. Very weak candidates were never really able to remove successfully the ln term at the start of the question.

Answer: $x = 2.676$

Question 2

In this question most candidates were able to score the first 3 marks, irrespective of whether they solved 2 linear equations or a single quadratic equation, although a few candidates omitted to square the coefficient 2 on the RHS of the inequality. However, any errors in the quadratic equation usually meant that candidates did not score the method mark for a reasonable solution attempt at a 3–term quadratic since they produced their solution via the calculator, void of the working which is required to be clearly displayed to gain that mark (see previous reports). Most candidates opted for the values of x to be between the critical values as opposed to outside this interval. Rare to see $\le$ or $\ge$.

Answer: $x < -\frac{6}{5}$, $x > \frac{2}{7}$

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Question 3

(i) Many candidates chose to omit this straight forward question. Some produced a **single graph** for the combined function, despite the question requesting that **suitable graphs** were sketched. Too often even the quadratic graph was incorrect, or the graphs were restricted to the region $x > 0$, something that the question itself implies is incorrect. The few candidates who did produce the correct graphs usually then failed to indicate that there was a negative and a positive root by projecting the points of intersection down onto the x-axis. The very least required to gain the accuracy mark was some indication that the roots of the equation were at the crossing of the curves, but usually no points were marked and there was no mention of the crossing of the graphs being related to the roots.

(ii) This required the numerical evaluation of $f(x) = 4 - x^2 - e^{-x/2}$ at $x = -1$ and at $x = -1.5$, preferably to 3 significant figures, but if given to less figures then the rounding needs to be undertaken correctly. Following this it is necessary to justify the statement by indicating one value is positive and one is negative and hence there is a change of sign, resulting in a root in the interval. The evaluation at these values of $f_1(x) = 4 - x^2$ and $f_2(x) = e^{-x/2}$ is an alternatively valid approach BUT requires a much more rigorous conclusion which has nothing to do with one positive value and one negative value, but the fact that at one value of x, $f_1 > f_2$, and at the other value of x, $f_1 < f_2$, so a crossover of the functions occurs. This approach when attempted was usually muddled with the one positive and one negative idea.

(iii) Most candidates scored highly on this section, however some believed after correctly showing convergence that their answer to 2 decimal places was 1.40 instead of the correct value of 1.41. Unfortunately many candidates muddled the sign of their iteration, so instead of obtaining the correct negative root they obtained the correct positive root. Since no single iteration was performed correctly no marks were possible.

Answer: 1.41

Question 4

(i) The solutions to this question were usually correct, especially the value of R. The common errors seen were the negative sign within the expression for α, so $\alpha = \tan^{-1}(-15/8)$, or a muddled $\tan^{-1}$, so $\alpha = \tan^{-1}(8/15)$ or the failure to give the value of α to 2 decimal places.

Answer: $R = 17$, $\alpha = 61.93°$

(ii) Many candidates successfully found the solution $x = 7.2°$, however, few candidates were able to find the second solution. In fact most candidates didn't realise that there was a possible second solution. The usual errors that occurred were the use of x instead of $2x$, the use of $2x + 2\alpha$ or some algebraic/arithmetical error.

Answer: 7.2°, 110.8°

Question 5

Most candidates were unable to progress far with this question, although usually the correct product rule was applied apart from some weak attempts at the differentiation of e^{-ax}. A few of the more able candidates managed to replace $\sec^2 x$ by $1 + \tan^2 x$, but their attempts usually ceased at the correct quadratic equation. Even fewer candidates realised that it was necessary to set the discriminant to zero in order to determine the value of a for there to be a repeated root. Some candidates simply guessed that $a = 2$, but produced no working to substantiate this claim. Something along the lines that for a repeated root, (only one point), an expanded $(\tan x - b)^2 = 0$, would produce $b = 1$, and hence $a = 2$, would have sufficed. One clever method, avoiding the standard approach, was to simplify the $\tan x$ and $\sec^2 x$ terms and introduce the double angle formula. This resulted in $\sin 2x = 2/a$ and only when $\sin 2x = 1$ is it possible for there to be a single solution (shown via a sketch) lead immediately to $a = 2$ and hence $x = \pi/4$.

Answer: $a = 2$, $x = \pi/4$

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Question 6

(i) Most candidates scored full marks, although a few candidates only showed that a single point was in the plane or that the direction vector of the line was parallel to the plane.

(ii) The working in this section was often incorrect as candidates assumed that the normal to the equation of the required plane was perpendicular to the direction vector of the line in **(i)** and the position vector 3**i** – **j** + 2**k** in **(ii)**, instead of being perpendicular to the direction vector of the line in **(i)** and the normal to the plane from **(i)**. Fortunately, the correct point (3, – 1, 2) was used by most candidates to try to establish the value of the constant *d*.

Answer: $4x + 13y + 5z = 9$

Question 7

(i) Too many candidates had little or no strategy in trying to establish the given expression. The successful candidates clearly stated what the expressions d*V*/d*h* and d*V*/d*t* were before stating d*h*/d*t* = (d*V*/d*t*)/(d*V*/d*h*) and applying separation of variables to acquire the given answer.

(ii) The substitution was often applied successfully within the integrand, although the limits were often reversed and then fudged to avoid a negative final answer. Unfortunately a common error was the omission of the constant 10, despite the integration producing a ln*u* and a *u* term. A few candidates omitted to change the limits, which is perfectly acceptable if one reverts back to the variable *h*, but not if the expression to be substituted is still a function of *u*.

Answer: 11.1

Question 8

(i) Most candidates substituted the root $u = -1 + \mathrm{i}$ into the polynomial in order to show that p(–1+i) was zero. The question clearly stated that calculator use was not permitted and that ALL working had to be shown. The reason for requesting that ALL working is shown is to prevent candidates simply entering – 1 + i into their calculator and multiplying by – 1 + i to produce – 2i. Then multiplying by – 2i to produce – 4. The latter step can be undertaken mentally so it is only by showing $(-1 + \mathrm{i})^2 = 1 - 2\mathrm{i} + \mathrm{i}^2 = 1 - 2\mathrm{i} - 1 = -2\mathrm{i}$ in detail that a candidate's knowledge of multiplying complex numbers can be tested. Failure to show this basic detail will result in the loss of these marks, despite the fact that some candidates may feel that they can do the calculation mentally.

(ii) Candidates found this section difficult, since many failed to use the fact that another root of p(*z*) was the conjugate of *u*. The product of these two linear factors, not the product of their roots, producing a quadratic factor of p(*z*). Again the working of p(*z*) divided by this quadratic factor, either by long division or by inspection, must be clearly presented in detail, for the same reason as in **(i)**. For example $(z^2 + Az + B)(z^2 + Cz + D)$ leading to equations which are then correctly solved. Finally the solution of the new quadratic factor equal to zero must be undertaken using either the formula approach or completing the square, since otherwise it is a button pressing exercise on the calculator for which no credit will be given.

Answer: −1 – i, 1 + 2i, 1 – 2i

Question 9

(i) The partial fractions for f(*x*) was nearly always completely correct. Any errors that did occur were usually either from choosing just a constant for the numerator of the term involving $(x^2 + 4)$ or from the omission of a bracket when undertaking a relevant method to determine a constant.

Answer: $\frac{A}{2+x} + \frac{Bx+C}{4+x^2}$, $A = -2$, $B = 1$, $C = 4$

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(ii) The expansions of $(2 + x)^{-1}$ and $(4 + x^2)^{-1}$ were usually correct but for the extraction of the constants 2 and 4, which were nearly always to be found in the numerator as opposed to their correct position in the denominator.

Answer: $\frac{3}{4}x - \frac{1}{2}x^2$

Question 10

(i) Too often the derivative of y had either the omission of a $\ln x$ or a $1/x$ from the chain rule. Most candidates knew how to find the x-coordinate of Q, however often used the gradient of the tangent, or the gradient of the normal as a function of x, or with the reciprocal of their derivative at $x = \mathrm{e}$ instead of the negative of their reciprocal at $x = \mathrm{e}$, for the gradient of their normal.

Answer: $e + \frac{2}{e}$

(ii) There was a given answer and hence this required the full details regarding integration by parts. So to gain the single mark everything had to be correct and there had to be a clear indication of the term x/x cancelling to unity followed by the integral of this becoming x. Simply showing the integral of unity to be x, is insufficient since where does the integral of unity originate from? Most candidates were simply writing down the given answer and as a result failed to gain the mark.

(iii) There is a clear linkage between **(i)**, **(ii)** and **(iii)**. In **(ii)** it is highlighting how one integrates $\ln x$ by using integration by parts where unity is the term being integrated and $\ln x$ the term that is being differentiated. This should have been taken over into **(iii)** where unity is still the term being integrated and $(\ln x)^2$ is the term being differentiated, and this is readily available from **(i)**. The other basic errors seen included not giving an exact answer as the question requests and integrating under the curve from $x = 1$ to the x– coordinate at Q, instead of to the x– coordinate at P.

Answer: $e - 2 + \frac{1}{e}$

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MATHEMATICS

Paper 9709/42
Mechanics

General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to three significant figures as requested. Marks were also lost due to prematurely approximating within their calculations leading to the final answer. This was particularly noticeable in **Questions 2** and **4**.

Students should also be reminded that if an answer is required to three significant figures, as is the rubric on this paper for non-exact answers, then their working should be performed to at least four significant figures.

In questions where an answer is given, candidates should be particularly careful to show all of their working in order to justify their proof of a given answer

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases, such as in **Question 6**, it is impossible to achieve the correct given answer unless this value is used.

Comments on specific questions

Question 1

(i) This was found to be a very straightforward question and well done by almost all candidates. It involved using the definition of kinetic energy as $\text{KE} = \frac{1}{2}mv^2$ and substituting the given values into this expression. The only error which was rarely seen was the use of v rather than v^2 in the definition of KE

Answer: Initial kinetic energy of the particle = 28.8 J

(ii) Here the question asked for the use of an energy method and full marks were only available for candidates who adopted this approach. Most candidates correctly evaluated the gain in potential energy as the particle moved up the plane either in terms of the height gained or the distance moved up the plane. This expression is then equated to the initial kinetic energy found in **part (i)** and an expression for the required distance moved by the particle can be found. One error that was frequently seen was where candidates found the height gained by the particle rather than the distance moved up the plane, these differing by a factor of sin 30 which was sometimes omitted. Candidates who applied Newton's second law to the particle rather than the required energy approach were penalised to a maximum mark of one mark out of three.

Answer: The distance the particle moves up the plane before coming to rest is 14.4 m

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Question 2

Most candidates performed well on this question. The two main approaches used were to resolve forces horizontally and vertically or to use Lami's theorem. A few candidates applied the triangle of forces approach. Those who resolved forces should have equated the horizontal components of each tension and in the vertical equate the weight to the sum of the two vertical components of each tension. The two equations can now be solved simultaneously to find the tension in each string. Those who used Lami's theorem needed to look first at the angles between the forces which are in fact 110°, 120° and 130°. Once these angles have been found the two Lami equations give the results immediately. One error that was seen was when candidates wrongly assumed that the two tensions were the same in each string. Some candidates who resolved forces either made errors in solving the equations or prematurely approximated within their calculations and did not achieve the values of the two tensions correct to three significant figures.

Answer: The tension in string *AP* is 14.2 N and the tension in string *BP* is 17.4 N (both correct to three significant figures)

Question 3

The majority of candidates scored well on this question. There are two extreme cases to consider, namely when the particle is about to slip down the plane which gives the least possible value of P and the case when the particle is about to slip up the plane which gives the greatest possible value of P. A number of candidates only considered one or other of these cases and hence lost marks. In both cases the frictional force needed is $F = \mu R$ where $\mu = 0.3$ and $R = 0.6g \cos 21$. In the case of the least possible value of P, the component of the weight down the plane is balanced by $P + F$, while in the case of the greatest possible value of P, the component of the weight down the plane is balanced by $P – F$. Solving for P in each case gives the two required values for P. One error which was occasionally seen was the incorrect use of R as $0.6g$ rather than to use the component of the weight perpendicular to the plane.

Answers: Least possible value of P is 0.470 (given) Greatest possible value of P is 3.83

Question 4

(i) Most candidates found this part to be straightforward. As the car is travelling at a constant speed, the driving force F on the car must balance the resistance and hence $F = 800$. The speed of the car can be found simply by using the equation for the power as $P = Fv$ and using $P = 36\,000$ and $F = 800$. Once the constant speed has been found the distance travelled in 120 seconds is given by $120v$. An alternative approach is to use the definition that P is the rate of doing work by F and in this problem this leads to the equation $P = Fd/120$ where d is the distance travelled in 120 seconds. By substituting $P = 36\,000$ and $F = 800$ this gives the value of d. The speed can now be found as $d/120$.

Answer: The speed of the car is 45 ms^{-1} The distance *AB* is 5400 m

(ii) In this part there is no driving force and the only force acting is the resistance of 800 N. One approach is to use Newton's second law to find the deceleration over this stage *BC*. Once this has been found the constant acceleration formulae can be used to determine the speed of the car at *C*. An alternative approach is to equate the loss in kinetic energy to the work done against friction as the car moves from *B* to *C*. Either of these methods is perfectly acceptable. Some candidates correctly found the magnitude of the deceleration as 8/9 but when using it they forgot that it was a negative quantity. An error seen in the energy approach was to forget to include the initial kinetic energy in the calculations.

Answer: The speed of the car at *C* is 35 ms^{-1}

(iii) In this part many candidates did not realise that it is first necessary to find the distance *CD* from the given information and wrongly attempted to use the value given for *AD* to determine the deceleration in *CD*. Once the value of the distance *CD* has been found, the constant acceleration formulae can be used to find the required deceleration as the car comes to rest. Application of Newton's second law of motion using the deceleration in *CD,* where the only force acting is the resistance P N, enables the value of P to be determined.

Answer: The deceleration of the car is 7/9 ms^{-2} = 0.778 ms^{-2} The value of P is 700

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Question 5

(i) In this question the answers are given. It is particularly important to realise that in such cases, it is vital to show all of your working. The expression for v over the period from $t = 15$ to $t = 35$ involves the values of a and b. The method of approach is to match the velocities at $t = 15$ for the two adjoining definitions of v and to equate the velocity at $t = 35$ to zero. This produces two simultaneous equations in a and b which need to be solved. Many used their calculators to perform this solution but did not show any working and hence scored few marks as the answers were given. Full details must be shown to produce a convincing proof of the two values found.

Answers: $a = 49$ and $b = –0.04$ (both given)

(ii) In this part many candidates thought that the three regions were represented by straight lines. However the two end regions are quadratic curves and only the central portion is a straight line. The values are given on the t-axis but correct values must be shown on the vertical v-axis.

Answer: The curve from $t = 0$ to $t = 5$ should be represented by a quadratic curve which is concave upwards starting at (0, 0) and ending at (5, 20). The part from $t = 5$ to $t = 15$ is a straight line joining (5, 20) to (15, 40). The final section, involving the values of a and b, from $t = 15$ to $t = 35$, is a quadratic curve, concave downwards from (15, 40) to (35, 0). To complete the velocity-time graph the values of 20 and 40 should be correctly shown on the vertical v-axis.

(iii) In this question the region needs to be treated as three separate parts. The aim is to find the total distance travelled and this is represented by the area under the curve. From $t = 0$ to $t = 5$ this involves integrating the expression $4t^2/5$ between the limits of $t = 0$ and $t = 5$. The area under the curve between $t = 5$ and $t = 15$ can either be found geometrically using the area of the trapezium or by integration of $2t + 10$ between the correct limits. Finally the region from $t = 15$ to $t = 35$ must be evaluated by integration of $at^2 + b$ between the correct limits and using the given values of a and b. Because some candidates had assumed that the v-t graph was composed solely of three straight lines, the area was often seen incorrectly as a sum of triangles and trapezia with no integration seen.

Answer: The total distance travelled by P given to three significant figures is
$\frac{100}{3} + 300 + \frac{1360}{3} = 33.33 + 300 + 453.33 = 787\text{m}$

Question 6

(i) This question was well done by most candidates. The majority wrote down Newton's second law applied to each particle, involving the acceleration of the particles, a, and the tension, T, in the string as $1.2g – T = 1.2a$ and $T – 0.8g = 0.8a$. Some found the value of a by writing down the combined equation of motion along the string by quoting the formula $(m_1 – m_2)g = (m_1 + m_2)a$. However, if this method was used then one of the Newton equations also had to be used in order to find the required tension in the string. Very few candidates failed to score on this question

Answer: The acceleration a is $2\,\text{ms}^{-2}$ (given) The tension in the string is 9.6 N

(ii) The first part of this question is to determine the time taken before the 1.2 kg particle reaches the ground. This can be achieved by using the given value of $a = 2$ in the constant acceleration formulae. In addition it is necessary to find the speed of the particles as this point is reached. It can be shown that the time taken for the particle to reach the ground is 0.8 seconds and that the speed of the particles at this instant is $1.6\,\text{ms}^{-1}$. In the second stage of the motion, the 0.8 kg particle has this value of $1.6\,\text{ms}^{-1}$ as its initial upwards velocity as it continues to move upwards under gravity with $a = –10$. Many candidates reached this stage but wrongly assumed that the particle would continue moving upwards for the final 0.2 seconds. However, it can be shown that the 0.8 kg particle comes to rest after a further 0.16 seconds and that during this time it has risen a further 0.128 m and both of these values can be found using constant acceleration formulae. In the final stage which uses the remaining 0.04 seconds, the 0.8 kg particle falls under gravity and it can be shown that the distance fallen in this time is 0.008 m.

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Some candidates attempted to use work/energy methods. This is perfectly acceptable provided the system of two particles is considered before the 1.2 kg particle reaches the ground. If only one particle is considered in isolation then the work done by the tension in the string must also be included.

Answer: The total distance travelled by the 0.8 kg particle during the first second after the particles are released is 0.64 + 0.128 + 0.008 = 0.776 m

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MATHEMATICS

Paper 9709/52
Mechanics

General comments

Most candidates work was neat and well presented.

$g = 10$ is now being used by most candidates as instructed on the front cover of the examination paper.

Candidates should be reminded that if an answer is required to three significant figures then their working should be performed to at least four significant figures.

Occasionally a candidate uses an incorrect formula. A formula booklet is provided so it is a good idea to use it.

The easier questions proved to be **2**, **4(ii)**, **6(i)** and **7(i)**.

Comments on specific questions

Question 1

Quite a number of candidates found the time for the total flight instead of the time to reach the highest point. Some candidates only found the horizontal distance. It was necessary to find both the horizontal and vertical distances and then to use Pythagoras's theorem to calculate the required distance.

Answer: 12.9 m

Question 2

This question was generally well done.

Answers: **(i)** 0.405 m **(ii)** 0.494

Question 3

(i) Sign errors often occurred when the candidate tried to set up the required quadratic equation. The candidate needed to use $s = ut + \frac{1}{2}at^2$ with $a = +10$ not -10.

(ii) If the wrong time from **part (i)** was used then incorrect answers were found. This part could be done without using the time. The use of $v^2 = u^2 + 2as$ vertically gives $v^2 = (20\sin 60)^2 + 2g \times 30$. This leads to a vertical velocity of 30 ms^{-1} at the ground. The horizontal velocity is $20\cos 60 = 10$ ms^{-1}. With these two values the required results can be found.

Answers: **(i)** 1.27 s **(ii)** 31.6 ms^{-1} 71.6° to the horizontal

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Question 4

(i) Some candidates had difficulty finding the centre of mass of the triangle from both *AB* and *BC*. Most candidates attempted to take moments about *AB* and *BC*, failing to get the correct answers because wrong distances were used.

(ii) Most candidates knew that $\tan\theta = \bar{y}/\bar{x}$ where θ is the required angle and $\bar{x}$ and $\bar{y}$ are the distances from *AB* and *BC* respectively.

Answers: **(i)** 0.38 m 0.35 m **(ii)** 42.6°

Question 5

(i) Many candidates resolved vertically for particle *P* in order to find the tension in the string *AP*. Unfortunately some candidates omitted to use the 1.5 N force. Newton's Second Law was then applied horizontally to find the value of ω .

(ii) This part of the question required the candidates to use the same approach as was used in **part (i)** for the particle *Q*.

Answers: **(i)** 11 N 7.42 **(ii)** 0.15 3.57 N

Question 6

(i) This part was generally done well by most candidates.

(ii) This part of the question proved to be very difficult for most candidates. When integrating too many candidates had v and not $v^2/2$ and also they could not integrate $2 / x$. The integration should have given $v^2 / 2 = -4x - 2\ln x + c$. $v = U$, $x = 1$ should then have been substituted to give a value for c in terms of U. By using $v = 0$, $x = 2$ and $v = 0$, $x = 2.1$ the range of values of U could be found.

Answers: **(i)** $v\mathrm{d}v / \mathrm{d}x = -4 - 2/x$ **(ii)** $3.28 < U < 3.43$

Question 7

(i) This part of the question was usually well done.

(ii) Few candidates managed to do this part of the question. Candidates knew that an energy equation was required. Generally only three terms were seen and not four as required.
The equation should be $0.4\times5^2/2 + 0.4gd + 24\times0.1^2/(2\times0.6) = 24(0.1+d)^2/(2\times0.6)$
where d is the required distance.

(iii) In this part of the question the new extension for the equilibrium position was needed. It can be found by using $T = \lambda x / L$. Again a four term energy equation was required. Often only three terms were seen.
The equation should be $24\times0.6^2/(2\times0.6) = 0.8v^2/2 + 24\times0.2^2(2\times0.6) + 0.8g\times0.4$ where v is the required velocity.

Answers: **(i)** 0.1 m **(ii)** 0.5 m **(iii)** 2.83 ms^{-1}

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MATHEMATICS

Paper 9709/62
Probability and Statistics

Key messages

Candidates should be aware of the need to work to at least four significant figures to achieve the required degree of accuracy. Efficient use of a calculator is expected, but candidates should be encouraged to show sufficient workings in all questions to communicate their reasoning.

Candidates would be well advised to read the question again after completing their solution to ensure they have included all the relevant details.

General comments

It was pleasing that many candidates seem to have prepared well for the examination and many good solutions were seen. There did not appear to be any issue with the accessibility of the questions, nor the time available for the paper.

Candidates are reminded that the instruction 'hence' as in **Question 1** means that they must use their previous work in order to obtain the required answers if full credit is to be gained.

Many candidates would benefit from the advice to read the question again after completing their solution to ensure that they have answered fully. This is particularly important when candidates need to interpret the information to solve the problem, as in **Question 2**.

Answers to **Questions 3**, **6** and **7** were generally stronger than other questions.

Comments on specific questions

Question 1

Most candidates attempted this question but many did not recognise that the coded mean and standard deviation was expected as the method. The best solutions had a table listing the difference from 1760, which allowed them to easily evaluate the coded mean and then the values required for the standard deviation. A considerable number of solutions never identified the coded mean but did use it to calculate the mean of the stated values. Where the coded standard deviation was stated, many did not realise that this value did not alter when using the original values, and attempted to either adjust the value, often by adding 1760. A great number of solutions either simply calculated the mean and standard deviation of the original values or, having done so, calculated the coded mean. Neither of these approaches could gain full credit because of the circular reasoning.

Answer: $\mu = 1761.23$, $\sigma = 1.39$

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Question 2

Candidates who used the ‘combinations’ approach to probability were often more successful as the method assumes that items are not replaced. A few solutions simply considered choosing exactly three green balloons only, rather than three green balloons in a total of seven balloons. Although some good solutions were seen using the more common ‘multiplication’ approach, many candidates assumed that the balloons were replaced or did not consider that there were different arrangements that needed to be considered for the ordering of the balloons, hence did not multiply by ${}^7C_3$ or ${}^7C_4$.

Answer: 0.242

Question 3

Almost all candidates recognised that the normal distribution was an appropriate approximation to use, with the best solutions using the calculation check np<5, nq<5 to confirm. The majority of candidates calculated the mean and variance accurately, although occasionally the standard deviation was not used to an appropriate degree of accuracy. The standardisation formula was applied accurately, although occasional errors were seen where the continuity correction was used inaccurately or omitted. A few solutions calculated the incorrect probability by considering the ‘wrong’ area under the normal distribution curve. Candidates are well advised to sketch this curve to clarify their considerations.

Answer: 0.346

Question 4

Although attempted by most candidates, there were many careless errors in calculating the median or quartiles. Candidates are reminded that the techniques expected as prior knowledge are applied accurately.

(i) Most candidates used a scale that enabled their values for the median and quartiles to be plotted accurately. A few candidates had a scale that was either so small, e.g. 2 cm to 0.05 kg, or where it was not possible to plot any value accurately, e.g. 2 cm to 0.003 kg, which resulted in their final box-and-whisker plot been a poor representation of the data. The most common error was for candidates not to label the scale fully with units.

(ii) Although many candidates correctly identified that the normal distribution was a suitable model, few gave a full justification, which needed to include the fact that the data was approximately symmetrical and peaked around the median.

Answer: **(i)** LQ = 0.7495, Median = 0.7507, UQ = 0.7517

Question 5

(i) Many candidates clearly identified the possible combinations of cakes that could fulfil the criteria, often within a table. However, many candidates did not recognise that this was a ‘without replacement’ situation, and so only needed to consider the combinations for either Alex or James. A common error was to add the separate totals together, or to multiply the respective combinations for choosing from 12 cakes.

(ii) The best solutions considered the ordering of the cup cakes and then the different places that the brownies could be inserted into the line. Unfortunately, many candidates attempted to consider the number of ways all 11 cakes could be ordered and then subtract the ways that do not fulfil the conditions. This is a much more complex approach, with few candidates realising that there is more than one group of arrangements that did not meet the criteria. Candidates are reminded that exact answers are expected to be fully stated.

(iii) Good solutions recognised that the 4 chocolate biscuits always been together effectively made a single ‘biscuit’ so that the question could be modelled with 9 items. The majority of solutions did attempt to remove the repeated arrangements, although a few also divided by 4! to remove the chocolate biscuit repeats that were not in their solution.

Answer: **(i)** 2048 **(ii)** 8 467 200 **(iii)** 252

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Question 6

(i) Candidates are reminded that when the answer is given within the question, then work and reasoning has to be clear to justify their solution. Many candidates identified the separate probabilities required and stated clearly the multiplication required, showing the appropriate cancellation of their answer. Weaker candidates often used a probability space diagram to identify all possible outcomes and then used this as clear justification of the result, which was also acceptable. Candidates should be reminded that if they place their working elsewhere on the paper rather than on additional lined paper, they should indicate its position as many probability space diagrams were within **part (ii)** without being referred to in this part.

(ii) Many good attempts at this question were seen. Candidates who had used a probability space diagram for **part (i)** completed the probability distribution table efficiently. The more common approach was to calculate the separate probabilities and place the results in the table. The majority of probability distribution tables only included possible values, but where values that could not be obtained were included, a probability of zero was stated. A few solutions did not include 5 as a possible outcome.

(iii) Good candidates recognised that this was a question on conditional probability. Many fully correct solutions were seen, with the reasoning clearly stated. A common error was to consider that the $\text{P}(\text{Sum} = 3)$ should be used to calculate $\text{P}(1 \text{ from A} \cap \text{Sum} = 3)$, and hence led to a final answer of $\frac{1}{2}$. A few simple arithmetical errors were noted in this question.

Answer: **(ii)**

x	0	1	2	3	5
P(X=x)	2/30	5/30	4/30	13/30	6/30

(iii) 10/13

Question 7

Many candidates would have benefitted from requesting additional lined paper as their solution to **part (i)** was often less than clear because of the limitations of space.

(i) Many good attempts were seen to this question. The best solutions used the critical values stated in the tables as expected and provided clear solutions of the simultaneous equations that were generated. A sketch of the normal distribution curve often assisted in identifying whether the z-value was positive or negative. A number of solutions only stated the standard deviation to two decimal places as the mean was recorded to two decimal places. This resulted in a final answer that was not correct to three significant figures.

(ii) Again, many good attempts were seen, with few calculating the incorrect probability for having a middle finger less than 8.2 cm. The best solutions then used the binomial distribution to calculate the required final probability. A number of good solutions failed to achieve the required degree of accuracy with their final answer because of premature approximation to three significant figures of their earlier probability.

(iii) All but the weakest candidates attempted this question. Good solutions stated the standardisation formula, replaced the required value with 1.5μ, and then substituted to obtain one unknown variable. Strong candidates often used a different variable than stated for clarity. A few candidates did not state the final probability for *z*.

Answers: **(i)** $\mu = 8.34$, $\sigma = 0.684$ **(ii)** 0.261 **(iii)** 0.692

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MATHEMATICS

Paper 9709/72
Probability and Statistics

Key messages

Where questions ask for answers 'in context', candidates should take care that their answers give a clear statement in the context of the question and not just text book definitions.

The required level of accuracy, unless otherwise stated, is three significant figures. Candidates should take care not to prematurely round figures during their calculations.

General comments

In general, candidates scored well on **Questions 2(i)**, **3(i)**, **5(a)(i)** and **7(ii)(a)** whilst **Questions 2(ii)(iii)**, **3(ii)**, **5(a)(ii)** and **7(i)** proved more demanding. Candidates found **Question 6**, on probability density functions, more demanding than has often been the case on this type of question in the past. Candidates were largely able to demonstrate and apply their knowledge in the situations presented, though explanations 'in the context of the question' were not always well answered (see comments on **Questions 3**, **4** and **7** below). There was a complete range of scripts from good ones to poor ones.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy. This was particularly seen on **Question 7(b)** where many candidates rounded a value prematurely and therefore their final answer was not to the required three significant figures accuracy (see comments below).

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

This question was not always well attempted. A confidence interval for the population proportion was required; however, for some candidates there appeared to be confusion with this and with a confidence interval for the population mean. Those who did interpret the question correctly were usually successful in reaching the required interval, but common errors included using an incorrect *z* value. The answer was required in interval form.

Answer: 0.231 to 0.369

Question 2

Part **(i)** was generally well attempted with only a few candidates giving an incomplete or incorrect statement ($H_1 \neq 6.4$ being a common incomplete statement).

Parts **(ii)** and **(iii)** were not so well attempted. Some candidates remembered to show a valid and correct comparison with a *z*–value (or an equivalent valid comparison of areas), but there were cases where candidates drew conclusions without supporting evidence. Other errors included incorrect *z* values or invalid comparisons between *z* values and areas. It was important that a conclusion was reached in context rather than merely stating 'accept H_0'.

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Candidates were required to give an explanation of when a Type I error, rather than Type II, would be used. Many candidates did not answer in full.

Answer: **(i)** $\mu \neq 6.4$ **(ii)** No evidence that μ is not 6.4 **(iii)** Testing for an increase in μ, or for a decrease in μ, rather than for a change.

Question 3

Part **(i)** was a well attempted question, with most candidates realising the need to standardise both 51 and 53. Common errors included an incorrect application of a continuity correction. Finding the area between 51 and 53 was then required; some candidates did not do this and attempted an incorrect multiplication of probabilities.

Answers to part **(ii)** often illustrated a lack of understanding of the Central Limit Theorem. There were answers which showed some understanding, but were not precise enough; for example, statements such as 'it was not normally distributed' or 'it needs to be normally distributed' were not acceptable.

Answer: **(i)** 0.844 **(ii)** Need to assume $\overline{X}$ is approximately normally distributed.

Question 4

This was a reasonably well answered question, with many candidates scoring fully in part **(i)**. Common errors included use of an incorrect value of λ or the inclusion/exclusion of terms in the required Poisson calculation.

As is often the case, candidates struggled to explain meanings 'in context'. In part **(ii)** a full and clear statement in the context of the question (as below, for example) was required. A text book definition, even if correct, is not accepted when the question specifies 'in this context'.

In part **(iii)**, candidates made a good attempt with many realising that $\mathrm{P}(X \geq 3)$ was required. The calculation of $\mathrm{P}(X > 3)$ was a common error.

Answer: **(i)** 0.174 **(ii)** Accept reduction in the mean number of missed appointments although untrue. **(iii)** 0.0803

Question 5

This question proved to be more challenging than previous questions on probability density functions. Many candidates realised that k was equal to 1 in part **(a)(i)**, but few were able to give clear reasons why the functions f_2 and f_3 were not probability density functions. There were cases where candidates did not show any clear understanding of the properties of a pdf, and also cases where candidates had some understanding but lacked clarity in their explanations.

In part **(b)** the integration caused problems for a large number of candidates. In particular, $\int a^2 \mathrm{d}x$, which should have been a^2x (as 'a' was a constant), was often incorrectly integrated with respect to a and given as $\frac{a^3}{3}$.

In **(b)(ii)** some candidates correctly stated E(X)=0; however, many candidates found this answer by calculation, thereby losing time. Part **(b)(ii)** was generally well attempted.

Answer: **(a)(i)** $k = 1$ **(ii)** f_2: area>1, f_3: includes negative values of f_3 **(b)(ii)** 0 **(iii)** 0.05

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Question 6

This question was generally well attempted by candidates, though in part **(i)** very few candidates were able to state a necessary assumption, and some candidates failed to note that the final answer was required as a percentage; it is important that candidates read the question carefully to ensure they have answered the question fully, and as required. The calculation of E(*T*) was done well, though errors in the calculation of Var(*T*) commonly involved finding $8^2 \times 12.6^2 + 3^2 \times 10^2$ rather than $8 \times 12.6^2 + 3 \times 10^2$.

Part **(ii)** was particularly well attempted, with many candidates correctly attempting P(*F*–*S* > 0) or equivalent.

Answer: **(i)** Assume cartons are random sample, 1.74% **(ii)** 0.148

Question 7

Most of this question was well attempted, though part **(i)**, which required two assumptions for the Poisson model to be valid, was not. Once again, the context of the question was required, so text book definitions which had not been contextualised were not acceptable.

The calculation in **(ii)(a)** was generally done well, with the correct value of λ (1.3) used. Similarly **(ii)(b)** was attempted well, though accuracy errors were often seen due to premature approximation of the value for λ. As λ was $\frac{52}{15}$ many candidates rounded this to 3.47, which meant their final answer was not accurate to the required 3.s.f.

Many candidates used a suitable approximation (N(52,52)) and a correct calculation for part **(iii)**, though omission of a continuity correction, or an incorrect continuity correction, was often seen.

Answer: **(i)** Planes arrive at a constant mean rate. Planes arrive at random. **(ii)(a)** 0.330 **(b)** 0.456
(iii) 0.881

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