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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01

Paper 1

General comments

The majority of candidates were able to make some progress with almost all of the questions and there were few very poor scripts. The number of misreads was small. The quality of presentation varied greatly over Centres. Some work was clear and well set out, but some responses were entirely haphazard and almost impossible to read. Future candidates will enhance their prospects for this examination if they first became proficient in the formation of mathematical notation according to established norms.

Not all candidates argued in a logical way so that erroneous reasoning of the form $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ appeared in a significant number of scripts. Examinations of this type do require an ability to set out a proof in a well ordered way. A solution is, therefore, a lot more than a random assemblage of disconnected mathematical statements, even if correct. It must have a discernible structure and it must be clear to the reader what the objective actually is.

Topics which stood out as well understood were summation of series by use of the difference method, second order differentiation in the context of parametric representation, the relevant application of the scalar and vector products to 3-dimensional problems, sums of powers of roots of polynomial equations and reduction formulae. In contrast, systems of planes defined by a single parameter, induction, complex numbers and the sketching of a curve of the form y = a rational function of x appeared to be not at all well understood by the majority of candidates.

In general, the accuracy of the working was often deficient so that elementary errors were a frequent cause of loss of marks. There was also some rubric infringement in evidence, in that numerical results were not rounded off as required and occasionally responses to both alternatives of **Question 12** were handed in.

Comments on specific questions

Question 1

Almost all candidates produced some good work in response to this question. Common errors were the writing of S_N as $\sum_{n=1}^{N^2} \mathrm{f}(n) - \sum_{n=1}^{N} \mathrm{f}(n)$, where $\mathrm{f}(n) = \dfrac{1}{n(n+1)}$, or even simply as $\sum_{n=1}^{N} \mathrm{f}(n)$. However, the majority did work from $\sum_{n=1}^{N^2} \mathrm{f}(n) - \sum_{n=1}^{N-1} \mathrm{f}(n)$ to obtain the required sum function in terms of N.

The concept of a limit in this context appeared to be well understood by most candidates and the working here was generally accurate and complete.

Answers: $S_N = \dfrac{1}{N} - \dfrac{1}{N^2+1}$; $\lim_{N\to\infty} S_N = 0$.

Question 2

This question was very well answered by at least half of the candidature.

The initial result, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin t}{1-\cos t}$ appeared on almost all scripts. However, as was so often the case with questions of this type in previous papers; there was a persistent confusion of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ so that a significant minority of the candidature wrote, in effect, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\sin t}{1-\cos t}\right)$. Nevertheless, the majority did work accurately from $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ and showed sufficient working to establish the displayed result in a valid way.

Question 3

This question generated a lot of good work which showed that candidates generally understood the geometrical meanings of the scalar and vector products and could apply them in a coherent and relevant way. Some responses, however, were marred by elementary errors.

(i) The majority evaluated vector products such as (b**j** – a**i**) x (c**k** – a**i**), though sometimes there were sign errors in the result.

(ii) The perpendicular distance of the origin from Π, denoted here by p, can be determined immediately from $|\mathbf{a.n}|$, where **n** is the normalised form of the vector obtained in part **(i)**, if correct. Not all candidates worked along these lines, but instead, first began by obtaining the equation of the plane in the form **r.**[(bc)**i** + (ca)**j** + (ab)**k**] = abc. This was an unnecessary diversion, and moreover some went on from here to infer that p = abc. Comment must also be made on the responses which legally obtained minus the required result and stated this to be p. Evidently their originators were unfamiliar with the modulus notation.

Answers: **(i)** (bc)**i** + (ca)**j** + (ab)**k**; **(ii)** $\frac{abc}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$.

Question 4

This straightforward question generated a diversity of methods.

The majority of candidates worked from $S_3 + \lambda S_1 + 3 = 0$, where $S_n = \alpha^n + \beta^n + \gamma^n$ (n = 0, 1, 2, …), though in some cases, lack of an effective notation seriously inhibited progress.

At this stage it is essential to explain why $S_1 = 0$, especially as the result for S_3 is displayed in the question. Nevertheless, some candidates failed to do this.

Alternatively, there were those who used the substitution $x^{\frac{1}{3}} = y$ to obtain the equation whose roots are $\alpha^3, \beta^3, \gamma^3$. This turns out to be $y^3 + 3y^2 + (3 + \lambda^3)y + 1 = 0$ from which the result for S_3 is immediate.

Yet again, some responses began with an attempt to establish an identity such as $S_3 = S_1^3 - 3S_1\sum\alpha\beta + 3\alpha\beta\gamma\,(*)$ which as $S_1 = 0$ and $\alpha\beta\gamma = -1$, leads at once to $S_3 = 0$. As it happens, some erroneous versions of (*) will also lead to the correct result for S_3 and this reality no doubt led some candidates to infer that they had done better here than they actually had.

In the second half of this question, most candidates established the result $S_4 = 2\lambda^2$ by use of $S_4 + \lambda S_2 + S_1 = 0$ together with a correct evaluation of S_2 as -2λ. Some however, first transformed the given cubic by $x^{\frac{1}{2}} = y$. This leads to $y^3 + 2\lambda y^2 + \lambda^2 y - 1 = 0$ from which S_4 may be calculated as the sum of squares of the roots. Yet others took this strategy a stage further by arguing that S_4 is the sum of the roots of the polynomial z-equation which can be obtained by use of the substitution $y^{\frac{1}{2}} = z$. Working in this context was generally accurate though some candidates lost their way.

In most responses, the concluding argument was extremely hazy. In fact, all that was required was something like the following: $S_4 < 0 \Rightarrow \lambda^2 < 0 \Rightarrow \lambda \notin \mathbb{R}$. In contrast, a common distortion of this argument went along the lines $\lambda \notin \mathbb{R} \Rightarrow S_4 < 0$.

Question 5

Responses to part **(i)** were generally complete and correct. In part **(ii)**, however, there were quite a lot of errors, both primary and secondary, to be seen. Nevertheless, the majority of candidates showed a satisfactory level of technical expertise. Most responses began with a correct integral representation of s = length of C such as $s = \int_1^4 \left[\left(1 - 4t^{-1/2}\right)^2 + 16t^{-1/2}\right]^{\frac{1}{2}} \mathrm{d}t$.

The simplification $\left[\left(1 - 4t^{-\frac{1}{2}}\right)^2 + 16t^{-\frac{1}{2}}\right]^{\frac{1}{2}} = 1 + 4t^{-\frac{1}{2}}$ (Q) necessary for the evaluation of this integral, appeared in most responses. Subsequently, the majority continued to work accurately and so obtained the required result.

In part **(ii)** most responses started with $S = 2\pi \int_1^4 \frac{16}{3} t^{3/4} Q \,\mathrm{d}t$, or at least with the candidate's version of Q, so indicating a general understanding of the methodology needed for the evaluation of an area of a surface of revolution. Thereafter, there were errors, both at the integration stage and also with the concluding numerical work, so that only about half of all responses to **Question 5** were complete and correct in every respect.

Answer: **(ii)** 697.

Question 6

The majority of responses to this question were of a high standard, so indicating that polar coordinates, as a syllabus topic, had been given due attention. Unexpectedly, the graphical work in part **(i)** was markedly less satisfactory than what was produced in response to part **(ii)**.

(i) Most responses showed an approximately correct continuous curve, starting at the pole, finishing at the point, A, whose polar coordinates are $r = 1$, $\theta = \frac{\pi}{2}$ and entirely within the second quadrant. In contrast, common errors were a non-zero gradient of C at the pole, and a zero gradient at A. Actually, as $y = r\sin\theta = (\pi - \theta)\frac{\sin\theta}{\theta} \to \pi \times 1 = \pi$ as $\theta \to 0$, it can be inferred that C is asymptotic to the line $y = \pi$. Such considerations would indicate that C is climbing upwards as it crosses the line $\theta = \frac{\pi}{2}$, and so does not have a zero gradient at A.

(ii) Here, almost all responses worked with the correct integral, namely $\frac{1}{2}\int_{\pi/2}^{\pi} \frac{(\pi - \theta)^2}{\theta^2} \mathrm{d}\theta$. The integrand was then expanded into a suitable form and integrated correctly. The subsequent detail to establish the displayed result was usually complete and correct.

Question 7

This was one of the least well answered questions on the paper. There were few complete and correct responses.

The introductory part of this question proved to be beyond almost all candidates. Part of the difficulty here was failure to work with a simple and clear notation. In fact, all that was wanted was something like the following: $(x_1, y_1, z_1) \in \Pi_1 \cap \Pi_2 \Rightarrow x_1 + 2y_1 - 3z_1 + 4 = 0$ and $2x_1 + y_1 - 4z_1 - 3 = 0$ so that $x_1 + 2y_1 - 3z_1 + 4 + \lambda(2x_1 + y_1 - 4z_1 - 3) = 0 + \lambda \times 0 = 0$. Hence for all λ, (x_1, y_1, z_1) is in the plane $x + 2 - 3z + 4 + \lambda(2x + y - 4 - 3) = 0$ (*).

(i) It is only necessary to set $x = 0$, $y = 0$, $z = a$ in (*) in order to determine the λ which defines the required plane, Π_3, say. This turns out to be $\frac{4-3a}{3+4a}$ from which the required form of the equation of Π_3 may be obtained immediately. Nevertheless, relatively few candidates argued in this way, but instead, used more protracted, and hence more error prone strategies. Thus most responses began by finding a vector parallel to $l = \Pi_1 \cap \Pi_2$, e.g., $5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, and continued with an attempt to find another vector in Π_3 not parallel to l. This could be effected by considering the line joining the point $(0, 0, a)$ to some point on l. The vector product of these 2 vectors could then be used appropriately to find the scalar equation of Π_3. However, such a strategy generated many errors, for most candidates did not possess the technical expertise to work accurately throughout its implementation.

(ii) In general, candidates perceived that the required value of a is given by $2p + q - 4r = 0$ where $px + qy + rz =$ constant is the equation of the plane obtained in part **(i)**. However, as the majority did not obtain a correct set of values for these coefficients, then only a small minority concluded this question successfully.

Answers: **(i)** $(11 - 2a)x + (10 + 5a)y - 25z + 25a = 0$; **(ii)** $a = -132$.

Question 8

Only a minority of responses showed understanding of all the concepts involved in this question. Moreover, a lot of the working was badly presented and, in some cases; was nearly illegible. In particular, few knew how to set out a proof by mathematical induction.

(i) A technically correct though difficult to read proof of the displayed reduction formula appeared in most responses. In most cases, the integration by parts rule was used, even though it is much easier to first differentiate $e^{-x}(1 - x)^n$, or $e^{-x}(1 - x)^{n+1}$, with respect to x and then go on to the required result. Few candidates took this more direct route.

(ii) The basic idea that $I_k = A_k + B_k e^{-1}$ for some integers A_k, B_k implies that $I_{k+1} = A_{k+1} + B_{k+1}e^{-1}$ for some other integers A_{k+1}, B_{k+1}, was comprehended, if only remotely, by the majority of candidates. However, few could fill in the technical detail in a convincing way. What was needed was a clear use of the reduction formula followed by the correct grouping of terms so as to make it clear that $A_{k+1} = 1 - (k + 1)A_k$ and $B_{k+1} = -(k + 1)B_k$. In some scripts it was also unclear what the starting value of n actually is, and no doubt this lack of coherence was due to an inability to work out I_0 or I_1 correctly.

(iii) About half of all candidates obtained a result of the form $B_n = \phi(n)n!$, but only a minority identified $\phi(n)$ as $(-1)^{n-1}$.

Answer: **(iii)** $B_n = (-1)^{n-1}n!$.

Question 9

This question generated a lot of good work. Most candidates understood all the concepts involved, but it was at the technical level that many responses went off the rails. Although the majority of candidates identified *a*, *b*, *c* as the eigenvalues of **M**, there were some who expanded det(**M** – λ**I**) = 0 as a cubic equation in λ and then after further effort found *a*, *b*, *c* as its roots. This strategy must have dissipated much examination time.

Essentially correct methodology for the determination of the eigenvectors was in evidence, but various inaccuracies led to only about half of all candidates obtaining a complete and correct set. Remarkably, some candidates even failed to obtain a correct eigenvector corresponding to *a*.

Most candidates understood that the matrix **M** – *k***I** has eigenvalues $a-k$, $b-k$, $c-k$.

They also comprehended that the eigenvectors obtained for **M** would serve as eigenvectors for **M** – *k***I**, and hence as the columns of **P**. Almost all responses exhibited a correct result for **D**.

Answers: Eigenvalues of **M** are *a*, *b*, *c*.

Corresponding eigenvectors are: $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\b-a\\0\end{pmatrix}, \begin{pmatrix}c-b-2\\a-c\\(a-c)(b-c)\end{pmatrix}$.

$$\mathbf{P} = \begin{pmatrix}1 & 2 & c-b-2\\0 & b-a & a-c\\0 & 0 & (a-c)(b-c)\end{pmatrix}, \mathbf{D} = \begin{pmatrix}(a-k)^n & 0 & 0\\0 & (b-k)^n & 0\\0 & 0 & (c-k)^n\end{pmatrix}.$$

Question 10

There were few complete and correct responses to this question so that one must infer that complex numbers is not a generally well understood syllabus topic.

(i) Most responses showed 12 complex numbers, all of which were roots of $\omega^{12}=1$. (*A*) However, there were frequent repetitions, for example, $\exp(\pi i)$, $\exp(-\pi i)$, and also some omissions.

(ii) Many candidates argued that as the equation (*), as given in the question, has 12 roots of which only ± 1 are real, then (*) has 10 non-real roots. This argument is clearly invalid for 1 is manifestly not a root as $(1+2)^{12} \neq 1^{12}$. Instead, it should first be shown that (*) is a polynomial equation of degree 11 for $(z+2)^{12} = z^{12}$ implies that $24z^{11} + \ldots = 0$. Moreover, –1 is a root since $(-1+2)^{12} = (-1)^{12}$. The remaining 10 (non-real) roots can be obtained by observing that (*) implies $\frac{z+2}{z} = e^{ik\pi/6}$ for $k = \pm1, \pm2, \pm3, \pm4, \pm5$ from which it follows that $z = \frac{2(e^{-ik\pi/6}-1)}{2-2\cos(k\pi/6)} = \ldots = -1 - i\cot(k\pi/12)$, as required by the question. However, only a minority of candidates could make their way through this detail.

(iii) The necessary work here depends essentially on the elementary trigonometric identity $1+\cot^2\theta = \operatorname{cosec}^2\theta$. Most candidates realised this and produced a simple and convincing verification of the displayed identity.

(iv) Consideration of the product of the 11 roots of (*) shows that $(-1)^{11} \times \prod_{k=1}^{10}[1+i\cot(k\pi/12)] = -\frac{512}{3}$ and the required result is then almost immediate. However, failure in part **(ii)** to recognise –1 as a root of (*) inevitably led to the incorrect statement that $\prod_{k=1}^{10}[1+i\cot(k\pi/12)] = -\frac{512}{3}$ and hence to $\prod_{k=1}^{5}\sin^2(k\pi/12) = -\frac{3}{512}$, which is clearly impossible.

Answer: **(i)** $\omega = e^{ik\pi/6}$ for $k = 0, 1, \ldots 11$; **(iv)** $\frac{3}{512}$.

Question 11

Not all candidates attempted to answer the four parts of this question in a systematic way and so good responses were very much in a minority. In fact, there was a lot of poorly presented work in evidence together with a number of elementary errors. Because of the sequential nature of the reduction of a matrix to the echelon form, it is essential that the working be checked at each stage, especially as conclusions appertaining to the system of equations depend in an obvious way on the final form of the reduction.

About half of all candidates reduced the matrix **A** to $\begin{pmatrix} 1 & 3 & 2 \\ 0 & -4 & -3 \\ 0 & 0 & \theta-1 \end{pmatrix}$ and so could determine r(**A**), the rank of **A**, immediately, both for $\theta \neq 1$ and for the special case $\theta = 1$. The rest arrived at a correct conclusion for at least one of the above cases, even though there were errors in the working.

For the first part of part **(i)**, it is only necessary to point out that the system has an unique solution if r(**A**) = 3 which was proved previously to be the case for any $\theta \neq 1$. However, most candidates ignored their previous working and started again, or began with the unnecessary evaluation of det **A**.

Reduction of the augmented matrix to the form (R): $\begin{pmatrix} 1 & 3 & 2 & 1 \\ 0 & -4 & -3 & -1 \\ 0 & 0 & \theta-1 & 3\theta+\phi-3 \end{pmatrix}$ together with setting $\phi = 0$ will lead immediately to the required solution. Nevertheless, only a minority of candidates systematised their working in this way.

Both parts **(ii)** and **(iii)** can be answered expeditiously using R, but again most candidates ignored their earlier working, even if relevant to the problems here. A common error was to argue that [det(**A**) $\neq 0 \Rightarrow$ system has a unique solution] $\Rightarrow$ [det **A** = 0 $\Rightarrow$ system has an infinite number of solutions.]

Answers: rank of **A** = 3 when $\theta \neq 1$, rank of **A** = 2 when $\theta = 1$; **(i)** $x = 1$, $y = -2$, $z = 3$.

Question 12 EITHER

This generated more responses than the alternative for **Question 12**. Generally, most of the ideas involved were understood, but elementary errors did some damage and much of the graphical work was of poor quality.

Most responses showed correct values for *a*, *p* and *q*, even when the underlying reasoning was invalid.

(i) Not a few responses began with a statement such as:

‘$\dfrac{dy}{dx} = (4x+b)(x^2-5x+4) - (2x-5)(2x^2+bx+c)$’ which as an application of the quotient rule is manifestly incorrect. Setting the right hand side of the above to zero together with $x = 2$, will, of course, lead to the correct value of *c*. Nevertheless, it must be emphasised here, as previously, that correct answers obtained from fundamentally erroneous working cannot gain full credit or, in some situations, any credit at all.

On the other hand the statement ‘$\dfrac{dy}{dx} = 0 \Rightarrow (4xb)(x^2-5x+4) - (2x-5)(2x^2+bx+c) = 0$’ (*) is correct and some candidates began part **(i)** in this way.

(ii) The substitution $c = 8$ in (*) leads to $(-10 - b)x^2 + 40 + 4b = 0$, or some equivalent, and it was here that most candidates failed to see the relevance of the condition $b \neq -10$ to this result and so did not make any further progress.

A few candidates resolved *y* into partial fractions, all of *a*, *c*, *p*, *q* now having numerical values, set the derivative of this form to zero, cancelled the factor $b + 10$, usually without giving a reason, and so obtained the equation $4(x-1)^2 = (x-4)^2$. From there it was easy to establish the required conclusion.

(iii) Most sketch graphs showed the 3 asymptotes correctly placed in relation to the axes. Also, they usually showed Γ to have 3 branches. However, apart from the inclusion of these basic aspects, most sketch graphs were deficient in some way. The main errors were as follows:

- the left hand branch did not have a minimum at a point where $x = -2$, and below the horizontal asymptote, and/or was not asymptotic to both $x = 1$ and $y = 2$
- the maximum point of the middle branch was not located below the x-axis at point where $x = 2$ and the asymptotic approach to at least one of $x = 1$ and $x = 4$ was carelessly drawn
- the asymptotic behaviour of the right hand branch was unclear particularly as $x \to +\infty$.

Answers: $a = 2$, $p = -5$, $q = 4$; **(i)** $c = 8$.

Question 12 OR

Most responses showed understanding of the concepts involved but lack of a reliable technique for this type of problem undermined much of the working in the later part of the question.

The AQE, $m^2 + (2a - 1)m + a^2 - a = 0$ appeared in almost all responses, but the solution of it, sometimes involving the formula for solution of a quadratic equation, was not always correct. The obtaining of the complementary function was usually based on a sound methodology, but of course, errors in the solution of the AQE led to an incorrect version of it.

Most responses led to the required particular integral and also showed the methodologically correct construction of the general solution. Likewise, application of the initial conditions was correct in principle, but elementary errors in previous working precluded any possibility of obtaining a completely correct solution in a legal way. Finally, the working to show the asymptotic form of y for large positive x, was frequently deficient in that no proper explanation of the relevance of the condition $a > 1$ was provided.

Less than half of all responses showed the particular integral of the given x-z equation to be $\dfrac{e^x}{a + a^2}$ and few went on to exhibit a correct result for $\lim_{x\to\infty} e^{-x}z$.

Answers: $y = x + e^{-ax} - e^{(1-a)x}$; $\lim_{x\to\infty} e^{-x}z = \dfrac{1}{a + a^2}$.

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Paper 2

General comments

The tendency of many candidates to begin by attempting the questions on Statistics suggested that they preferred these to the Mechanics questions, and this was also evident as usual in the single question offering alternative choices, namely **Question 11**. However, there were probably rather more successful attempts at the Mechanics alternative than in previous years, with several candidates providing completely correct answers, and over the whole paper there was no great difference in performance between Mechanics and Statistics. As is also customary, the great majority of candidates attempted all the questions on the paper, indicating that the time pressure was not unduly great.

As indicated in more detail below, there was considerable variation between questions in the level of candidates' success. On the one hand **Questions 3**, **6**, **7** and **9** were often well done, while parts of **Questions 2**, **5**, **8** and **10** seemed to present more difficulty.

Comments on specific questions

Question 1

While the great majority of candidates appreciated that the change in momentum and hence the impulse involves the product of mass and change in velocity, some took the difference in the magnitudes of the velocities before and after the ball being struck by the racquet instead of adding them. The second part presented no great problem in principle, requiring the product of mass and change in velocity to be divided by the time. However, the answers to both parts were sometimes in error by a factor of 1000 or some other power of 10, usually due to not converting the mass of the ball to kg.

Answers: 2.8 Ns; 7 N.

Question 2

The angular speed of the wheel may be found by noting that the angular acceleration is $400 \times \frac{0.3}{600}$, so that after 30 seconds from rest the angular speed has reached 30×0.2 in appropriate units. Some candidates instead applied a possibly invalid formula for the moment of inertia of a wheel in order to find its mass, and then treated the question as if it related to the linear motion of a particle of this mass under the action of a force $T_1 - T_2$. This confusion between linear and circular motion was also frequently evident in attempts at the second part. Solving it correctly essentially requires the angle turned by the wheel to be found, and then divided by 2π to give the number of revolutions, but a very common fault was to base the working on an un-stated assumption of constant angular speed, usually that calculated earlier for 30 s after starting from rest.

Answers: 6 rad s^{-1}; 14.3.

Question 3

In each of the two collisions the resulting velocities are readily found by formulating and then solving the conservation of momentum and the restitution equations. This shows that after the second collision, *A* is moving in the opposite direction to *B* and *C* with speed $3.5u$, with *C*'s speed of $21u$ faster than the u of *B*, showing that there can be no further impacts. Where errors occurred, they were usually due to either faulty arithmetic or confusion over signs. Some candidates wrongly deduced that after the first collision, *B* continues to move in the same direction, but faster than *C*. Although clearly absurd, these candidates then considered a second collision between *B* and *C*. In order to find the total loss of kinetic energy it is necessary to either add the losses in the first and second collisions, or more easily to consider the difference between the initial kinetic energy of *B* and the sum of the final kinetic energies of all three balls; some candidates considered only one or two balls in the latter case.

Answer: 30%.

Question 4

If moments are taken about either *A* or *B*, so that only one of the reactions at these points is introduced, then the given expression for *T* can be produced by using a single horizontal or vertical resolution to eliminate the reaction. Indeed a single moment equation about the point 2 m vertically above *B* needs no other equation, but this approach was rarely seen. Instead most candidates started with the two resolutions and then took moments about a variety of points such as *A*, *B*, *C* or *O*. The given inequality for $\tan\theta$ follows from noting that the denominator of the expression for *T* is positive, but invalid arguments based on the angle *ABO* being greater than θ were often seen. The final part was, however, usually answered more successfully, by substituting the appropriate values $\frac{3}{5}$ and $\frac{4}{5}$ for $\sin\theta$ and $\cos\theta$ in the expression for *T*.

Answer: $\frac{15}{14}W$.

Question 5

The required equation of motion should relate the product of m and the tangential acceleration $l\,\frac{d^2\theta}{dt^2}$ of the bob to the force in the tangential direction, but many candidates produced instead rather suspect equations involving some, usually undefined, linear variable. Although the approximation $\sin\theta \approx \theta$ was often stated correctly, very few attempts were made at the general solution, to the extent that the concept of a general solution seemed largely unknown. The usual expression $\frac{2\pi}{\omega}$ for the period in simple harmonic motion was widely used, even if an incorrect SHM equation did not yield the correct ω. The final part was often tackled correctly, essentially based on the fact that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Answer: $2\pi\sqrt{\frac{l}{g}}$.

Question 6

The probability of the bulb still burning after having been switched on 100 times was usually found correctly from $(1 - p)^{100}$ with $p = \frac{1}{200}$, though some candidates calculated instead some other probability, such as it failing on the one hundredth occasion. Almost all recalled that $E(N)$ is $\frac{1}{p}$, but the final part was not so well done. It requires both the formulation of the correct expression for $P(N \leqslant n)$, namely $1 - (1 - p)^n$, and the handling of the resulting inequality.

Answers: 0.606; 200; 21.

Question 7

The principal potential stumbling block is obtaining an unbiased estimate of the population variance, which should be found from $\frac{10}{9}\left(\frac{299.82}{10} - m^2\right)$, where m is the sample mean, here 5.44. The confidence interval may then be found in the usual way, using the tabular t-value 2.821. Although most candidates realised that normality was involved in the assumption, only a minority stated explicitly and unambiguously that the parent population of the masses of Indian elephants should have a normal distribution.

Answer: [4.85, 6.03].

Question 8

Since the question requires that the mean of X be shown to be θ, it is insufficient to simply state that the mean is known to be θ for such an exponential distribution. Instead $x\mathrm{f}(x)$ must be integrated by parts over the interval $x \geqslant 0$. Integration of $\mathrm{f}(x)$ over $[0, \theta]$ produces a value $1 - \frac{1}{e}$ which is greater than $\frac{1}{2}$, showing that $P(X < \theta)$ is the larger. The next part requires replacement of the given conditional probability by $P(X > a + b) \mid P(X > a)$, and then substitution of the appropriate exponential expressions to give the simplified result $e^{-b/\theta}$ and hence $P(X > b)$, but many candidates failed to do this correctly. Even fewer were able to give a satisfactory explanation of this equation, either in terms of understanding precisely what the two probabilities represent, or the significance of their equality.

Question 9

This question presented few difficulties to most candidates, who began by estimating the two-sample common population variance as 11.35, and then comparing their calculated *t*-value of magnitude 1.76 with the tabular value 1.734, concluding that stockbrokers claim more than bankers. Some candidates who employed a broadly correct method overlooked the fact that the given estimates of the population variances are said to be unbiased rather than biased. The correct approach was usually used to determine the confidence interval, though not all candidates chose the correct tabular *t*-value of 2.101, and incorrect estimates of the common population variance were of course carried forward from earlier.

Answer: **(ii)** [–0.53, 5.93].

Question 10

Although the distribution function may be found by calculating the triangular areas under the probability density function directly, most candidates used integration instead. While this was usually performed correctly for the range $-1 \leqslant x \leqslant 0$, a very common fault for $0 < x \leqslant 1$ was to overlook the addition of F(0) even though the given result was not produced. Even more candidates ignored the fact that their invalid working, usually finding $\mathrm{F}(\sqrt{y})$ rather than $\mathrm{F}(\sqrt{y}) - \mathrm{F}(-\sqrt{y})$, did not yield the given expression for $\mathrm{P}(Y \leqslant y)$. $\mathrm{E}(Y)$ may be found by differentiating this expression to give the probability density function $\mathrm{g}(y)$ of Y and then integrating $y\mathrm{g}(y)$, or alternatively by integrating the appropriate expressions for $x^2\mathrm{f}(x)$ over $-1 \leqslant x \leqslant 0$ and $0 < x \leqslant 1$, but a good many candidates did not appreciate this.

Answer: $\frac{1}{6}$.

Question 11 EITHER

Although less popular than the Statistics option, this question usually elicited reasonable attempts. The first result follows from considering the horizontal and vertical components of motion between *A* and *B*, or equivalently from the trajectory equation, though care must be taken over the angle of projection in the latter case since it is here $\frac{\pi}{2} - \alpha$. Conservation of energy yields the given result for U^2, while the greatest and least values of *R* follow from a resolution of radial forces at *C* and *A* respectively.

Answer: $mg\,(\operatorname{cosec}\alpha - \sin\alpha)$.

Question 11 OR

Finding the least squares regression line and the product moment correlation coefficient presented no difficulty to most candidates, though rounding errors during the calculation often gave an insufficiently accurate value of the constant in the equation of the regression line or else candidates retained only two significant figures rather than three. The predicted price of the article was also usually found correctly by taking $x = 30$, and most candidates produced at least one comment from among the various possibilities, namely extrapolation is usually inadvisable but $x = 30$ is here only just outside the range of data, and the product moment correlation is very close to unity. The final part should have been answered by adapting the formula given in the List of Formula to the case of *y* and *z*, replacing *z* by $1.62x$ in numerator and denominator, and noting that the factor 1.62 cancels out to give the formula for *x* and *y*. However many candidates either failed to follow this argument through correctly, or used a variety of unsatisfactory approaches such as simply stating that the result is obvious or using a table of observations of *z* and *y* to calculate r_2, even though the question specified no further calculation.

Answers: $y = 1.44x - 0.284$; 0.985; 42.9.