UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2 **May/June 2008**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE International Examinations

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1 A bullet of mass 20 grams is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed $240\,\mathrm{m\,s^{-1}}$ and emerges 0.004 s later with speed $10\,\mathrm{m\,s^{-1}}$. Assuming that the motion occurs in a horizontal straight line, find

(i) the magnitude of the impulse that acts on the bullet, [2]

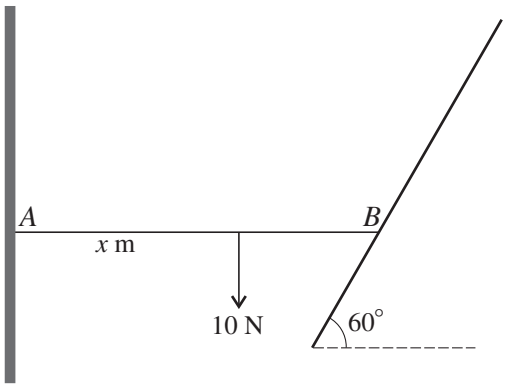
(ii) the magnitude of the resisting force, assuming that it is constant. [2]

2 A particle P moves on a straight line AOA' in simple harmonic motion with period T. The centre of the motion is O and P is instantaneously at rest at A and A'. The mid-points of OA and OA' are B and B' respectively.

(i) Find, in terms of T, the time for P to move directly from B to B'. [5]

(ii) Find the ratio of the speed of P at B to its speed at O. [3]

3



A light rod AB of length 0.8 m has a particle of weight 10 N attached to a point at a distance x m from A. The rod rests in equilibrium in a horizontal position with A against a rough vertical wall and B on a smooth plane inclined at 60° to the horizontal (see diagram). Find, in terms of x, the magnitude of the force that the inclined plane exerts on the rod. [2]

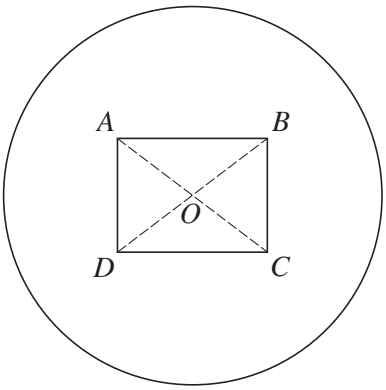
The rod can remain in equilibrium when the particle is attached to any point on the half of the rod nearer to B. Find the least possible value of the coefficient of friction between the rod and the wall. [6]

4

Two identical smooth balls, P and Q, are projected simultaneously towards each other from two points on horizontal ground. P is projected with speed u at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal and Q is projected with speed ku at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal (see diagram). The balls collide when they are moving horizontally. It may be assumed that there is no air resistance. Find the value of k. [5]

The coefficient of restitution between the balls is e. Find, in terms of e, u and g, the distance between the points where the balls first hit the ground. [5]

5



A uniform circular disc has radius 0.5 m, centre O and mass per unit area 6.20 kg m^{-2}. A rectangle $ABCD$, where AC and BD intersect at O, is removed (see diagram). The lengths of AB and BC are 0.4 m and 0.3 m respectively. Show that the moment of inertia of the resulting lamina, about an axis through A perpendicular to its plane, is 0.851 kg m^2, correct to 3 significant figures. [8]

The lamina is free to rotate in a vertical plane about a smooth horizontal axis through A. It is held with AB horizontal and with DC below AB. It is then given an initial angular speed ω rad s^{-1}, so that B moves upwards. Find the least possible value of ω for the lamina to make complete revolutions. [5]

6 Cylindrical metal tubes have internal diameters which are normally distributed with mean 1.275 cm and standard deviation 0.015 cm. Rods, which are intended to be inserted in the tubes, are made from a different metal and have diameters which are normally distributed with mean 1.245 cm and standard deviation 0.028 cm. Both tubes and rods are kept in a temperature-controlled store. On a particular day there is a system failure and the diameters of all tubes increase by 2% and the diameters of all rods increase by 4%. Find the probability that a randomly chosen rod cannot be inserted into a randomly chosen tube. [6]

7 The time for a certain kind of plaster to dry has a normal distribution with mean μ minutes. In order to find a confidence interval for μ, the times for a random sample of 10 specimens to dry were measured. The results, x minutes, are summarised by

$$\Sigma x = 285.4, \quad \Sigma(x - \bar{x})^2 = 448.65.$$

Calculate a 95% confidence interval for μ. [5]

Estimate the width of a 95% confidence interval for μ based on a random sample of 20 specimens. [3]

8 The number of serious faults on a randomly chosen 1 km stretch of motorway has a Poisson distribution with mean 2.1. The random variable X km is the distance between two successive serious faults on the motorway.

(i) Explain, using a Poisson distribution, why, for $x \geqslant 0$,

$$\mathrm{P}(X > x) = \mathrm{e}^{-2.1x}.$$ [3]

(ii) Obtain the distribution function and the probability density function of X. [3]

(iii) Obtain the mean distance between successive serious faults. [1]

(iv) Find the median distance between successive serious faults. [2]

9 A study was carried out to investigate the effect of moderate exercise on blood cholesterol levels. The levels were measured before and after a training programme administered to a random sample of 8 people. The results, in suitable units, are given in the following table.

Person	1	2	3	4	5	6	7	8
Before	195	209	184	215	295	198	310	191
After	197	201	192	204	266	183	272	168

Stating any assumption you need to make, test at the 10% significance level whether, after the training programme,

(i) there is a reduction in the population mean cholesterol level, [7]

(ii) there is a reduction of more than 5 units in the population mean cholesterol level. [3]

10 The time in the air, x seconds, and the score, y, for 12 randomly chosen divers competing at a diving competition for a reverse half-somersault event are summarised as follows.

$$\Sigma x = 5.22 \quad \Sigma x^2 = 2.2774 \quad \Sigma y = 85.9 \quad \Sigma y^2 = 618.11 \quad \Sigma xy = 37.338$$

Obtain an estimate for the score of a diver who takes 0.40 seconds. [4]

Find the product moment correlation coefficient for the sample. [2]

Stating your hypotheses, test at the 5% significance level whether there is a non-zero correlation between the two variables. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m moves on an arc of a circle with centre O and radius a. At time $t = 0$ the particle is at the point A and at time t the angle $POA = \theta$, where $\theta = \sin^2 kt$ and k is a positive constant. Find constants b and c such that $\ddot{\theta} = b + c\theta$, and deduce that P moves in simple harmonic motion along the arc. State the centre and amplitude of the motion. [9]

Show that the magnitude of the resultant force acting on P at time t is $mak^2(1 + \cos^2 2kt)$. [5]

OR

Each of 100 coins is tossed repeatedly until a head is obtained, and the number, x, of tosses needed is recorded. The results are summarised in the following table.

x	1	2	3	4	5	$\geqslant 6$
Frequency	56	20	11	10	3	0

Assume that, for each coin, the probability of obtaining a head has the same value p. State a distribution which could be used to fit the above data, and use the sample mean to obtain 0.5435, correct to 4 decimal places, as an estimate of p. [4]

Carry out a goodness of fit test at the $2\frac{1}{2}\%$ level. [8]

Without further calculation, state two changes which would be necessary if a similar method is to be used on the data above to test whether all the coins are fair. [2]

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