UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/13 Paper 13, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Relevant working from

$\mathbf{A} - 5\mathbf{I} = \begin{pmatrix} 0 & -3 & 0 \\ 1 & -3 & 1 \\ -1 & 3 & -1 \end{pmatrix}$ or the equivalent in equations M1

to obtain an eigenvector of the form $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ A1

[2]

An eigenvalue of $\mathbf{A} + \mathbf{A}^2$ is 5 + 25 = 30 B1
Corresponding eigenvector, as above B1
[2]

No penalty for not hence methods.

Accept λ = 6 with $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

and λ = 20 with $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

Accept linear scaling of eigenvectors.

2 $2\sin\alpha\sum_{n=1}^{N}\sin(2n\alpha) = \cos\alpha - \cos[(2N+1)\alpha]$ M1A1

$\Rightarrow$ displayed result (AG) M1A1
[4]

$\cos(2N+1)\pi/3$ oscillates finitely as $n \to \infty \Rightarrow \sum_{n=1}^{\infty}\sin(2n\pi/3)$ does not converge (CWO) B1

Require $\alpha = \dfrac{\pi}{3}$, 'oscillate' or values of $\cos(2N+1)\dfrac{\pi}{3}$ given as $\dfrac{1}{2}$ or -1 [1]

3 $H_k : x_k > 2$ for some k B1

$x_{k+1} - 2 = (2x_k^2 - 8)/(2x_k + 3)$ M1A1

$H_k \Rightarrow 2x_k^2 - 8 > 0 \Rightarrow x_{k+1} > 2 \Rightarrow H_{k+1}$ A1

$x_1 = 3 > 2 \Rightarrow H_1$ is true B1 CWO

Completion of the induction argument A1
[6]

Alternatively for lines 2 and 3:

$x_{k+1} = x_k + \frac{1}{2} - \frac{3\frac{1}{2}}{(2x_k + 3)}$ M1A1

$H_k \Rightarrow 2x_k + 3 > 7 \Rightarrow H_{k+1}$ A1

OR $x_{k+1} = x_k + \frac{x_{k-2}}{(2x_k + 3)}$ M1A1

$x_k > 2 \Rightarrow x_{k+1} > 2$ A1

OR $x_{k+1} - x_k = \frac{x_{k-2}}{(2x_k + 3)}$ M1A1

$x_k > 2 \Rightarrow x_{k+1} > x_k > 2$ A1

Minimum conclusion is 'Hence true for $n \geqslant 1$'.

4 $dx/dt = t\cos t,\ dy/dt = t\sin t$ (both) B1

$\sqrt{(dx/dt)^2 + (dy/dt)^2} = t$ B1

$S = 2\pi \int_0^{\pi/2} \left(t\sin t - t^2 \cos t\right) dt$ (AEF) M1A1

$\int t\sin t\, dt = -t\cos t + \sin t$ B1

$\int t^2 \cos t\, dt = t^2 \sin t + 2t\cos t - 2\sin t$ B1

$S = (\pi/2)\left[12 - \pi^2\right]$

Accept forms such as $6\pi - \pi^3/2$, etc. A1

[7]

OR for lines 4 and 5

$2\pi\left[(-t\cos t) + \int \cos t\, dt - \left\{(t^2 \sin t) - \int 2t\sin t\, dt\right\}\right]_0^{\pi/2}$ (LNR) M1

$= 2\pi\left[-t\cos t + \sin t - t^2 \sin t - 2t\cos t + 2\sin t\right]_0^{\pi/2}$ (LNR) A1

5 $(c+is)^5 = c^5 + 5c^4(is) + 10c^3(-s^2) + 10c^2(is)^3 + 5c(is)^4 + (is)^5$ M1

$\theta = 5c^4s - 10c^2s^3 + s^5$ M1

$= 5s(1-s^2)^2 - 10s^3(1-s^2) + s^5$ M1

$= 16s^5 - 20s^3 + 5s$ (AG) A1 OEW
[4]

$x = \sin\theta \Rightarrow \sin 5\theta = -1/2$ M1

Roots are $\sin q\pi$ where q = 7/30, 11/30, 31/30, 35/30, 43/30 A3

A1 for 1 root: + A1 for 2 further roots: + A1 for completion [4]
CWO CWO

Alternative answers

$q = \frac{11}{30}, \frac{23}{30}, \frac{35}{30}, \frac{47}{30}, \frac{59}{30}$
or
$q = \frac{7}{30}, \frac{19}{30}, \frac{31}{30}, \frac{43}{30}, \frac{55}{30}$

6 **(i)** One asymptote is $x = -1$ B1
$y = x - 4 - 3/(x+1)$ M1
Require $y = x$ + non-zero constant.
Other asymptote is $y = x - 4$ A1
[3]

Alternatively for last two marks:
OR $x + k \approx (x^2 - 3x - 7)/(x+1)$ for large $x \Rightarrow x^2 + (k+1)x + k \approx x^2 - 3x - 7$ for large x M1
$\Rightarrow k + 1 = -3 \Rightarrow k = -4 \Rightarrow$ other asymptote is $y = x - 4$ A1

OR $(x^2 - 3x - 7)/(x+1) = mx + c \Rightarrow (m-1)x^2 + (m+c+3)x + c + 7 = 0$ M1
Put $m - 1 = 0, m + c + 3 = 0$ to obtain other asymptote is $y = x - 4$ A1

(ii) Obtains any correct result for dy/dx M1

$\Rightarrow \ldots \Rightarrow dy/dx = \left[(x+1)^2 + 3\right]/(x+1)^2 = 1 + 3/(x+1)^2$ A1
No comment required. [2]

(iii) Axes and asymptotes correctly placed B1
Right-hand branch B1
Left-hand branch B1

Deduct 1 overall for bad form(s) at infinity [3]

7 **(i)** $dx/dt = (2t - 2t^3)e^{-t^2}$, $dy/dt = (1 - 2t^2)e^{-t^2}$ (both, AEF) B1

Obtains displayed result (AG) M1A1

[3]

(ii) Any correct result for $d(dy/dx)/dt$ in terms of t (*) M1A1A1

M1 – Quotient Rule A1A1 for terms in numerator

$d^2y/dx^2 = (*) \times dt/dx$ expressed in terms of t M1

Simplify to, e.g., $\dfrac{(-1 + t^2 - 2t^4)e^{t^2}}{4t^3(1-t^2)^3}$ A1

Two terms in numerator must be combined. [5]

8 Complementary function is $Ae^{-x} + Be^{-4x}$ M1A1

Particular integral of form $P\sin 3x + Q\cos 3x$, so that

$-5P + 15Q = 10$ and $15P - 5Q = -20$ M1

$\Rightarrow P = -7/5,\ Q = -1/5$ A1

General solution is $y = Ae^{-x} + Be^{-4x} - 1.4\sin 3x - 0.2\cos 3x$ A1

[5]

$Ae^{-x} + Be^{-4x} \to 0$ as $x \to +\infty$, M1

$\tan\phi = \dfrac{1}{7}$ M1

$R = \sqrt{2} = 1.41,\ \phi = \pi + \arctan(1/7) = 3.28$ B1A1

Accept $R = \sqrt{2}$, but must be positive. [4]

9 $D\left[s^{n+1}c\right] = -s^{n+2} + (n+1)s^n c^2$, where $D \equiv d/dx$, $s = \sin x$, $c = \cos x$ M1

$= \ldots = -(n+2)s^{n+2} + (n+1)s^n$ A1

Integrates w.r.t. x over the range $\left[0, \pi/2\right]$ to obtain $0 = -(n+2)I_{n+2} + (n+1)I_n$ M1

Result (AG) A1
[4]

OR $I_n = -\left[cs^{n-1}\right]_0^{\pi/2} + (n-1)\int_0^{\pi/2} c^2 s^{n-2} dx$ M1

$\Rightarrow (n-1)I_{n-2} - (n-1)I_n \quad (n \geqslant 2)$ M1A1

$\Rightarrow I_n = \left[(n-1)/n\right]I_{n-2} \Rightarrow I_{n+2} = \left[(n+1)/(n+2)\right]I_n$ for $n \geqslant 0$ A1

OR Starts with I_{n+2} and relates to I_n directly: mark as above

OR $I_n = \int_0^{\pi/2} \sin^{n+2}\theta \operatorname{cosec}^2\theta \, d\theta$

$= \left[-\sin^{n+2}\theta \cot\theta\right]_0^{\pi/2} + \int_0^{\pi/2} (n+2)\sin^{n+1}\theta \cos\theta \cot\theta d\theta$ (LNR) M1

$= 0 + (n+2)\int_0^{\pi/2} \sin^n\theta\left(1 - \sin^2\theta\right)d\theta$ (LR) M1

$= (n+2)I_n - (n+2)I_{n+2}$ A1

$\Rightarrow I_{n+2} = \dfrac{(n+1)}{(n+2)}I_n$ A1

$$\bar{y} = \frac{\frac{1}{2}\int_0^{\pi/2m} \sin^8 mx\,dx}{\int_0^{\pi/2m} \sin^4 mx\,dx}$$ M1 (LR)

let $u = mx$ M1

$$\bar{y} = \frac{\frac{1}{2}\int_0^{\pi/2} \sin^8 u\,du}{\int_0^{\pi/2} \sin^4 u\,du}$$ A1

$I_0 = \frac{\pi}{2} \quad I_2 = \frac{1}{2}\times\frac{\pi}{4} \quad I_4 = \frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\left(=\frac{3\pi}{16}\right)$ B1

$I_6 = \frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2} \quad I_8 = \frac{7}{8}\times\frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\left(=\frac{105\pi}{768}\right)$ B1

$\bar{y} = \frac{1}{2}\frac{I_8}{I_4} = \frac{35}{96}$ (or 0.365) B1

[6]

OR for last 3 marks

$\frac{I_8}{I_4} = \frac{35}{48}$ (oe) M1A1

$\therefore \bar{y} = \frac{35}{96}$ A1

10 **(i)** $x = 1/y \Rightarrow 2y^4 - 4y^3 - cy^2 - y - 1 = 0$ M1A1

[2]

(ii) $\sum \alpha^2 = 1 - 2c$ M1A1

$\sum \alpha^{-2} = 4 + c$ A1

(M1 is for use of $\sum \alpha^2 = \left(\sum \alpha\right)^2 - 2\sum \alpha\beta$ in either part.) [3]

(iii) $S = \sum\left(\alpha - \alpha^{-1}\right)^2 = \sum \alpha^2 + \sum \alpha^{-2} - 8 = -c - 3$ M1A1√

A1ft is for adding answers to **(ii)** correctly and subtracting 8. [2]

(iv) $c = -3 \Rightarrow S = 0$ so that if all roots are real then $\alpha = \pm 1$

and similarly for β, γ, δ M1A1 CWO

This is impossible since e.g., $\alpha\beta\gamma\delta = -2,$ or any other contradiction A1 CWO

[3]

11	**(i)**	$dr/d\theta = -a/(1+\theta)^2$	M1
		Since $a > 0$, then $dr/d\theta < 0$, $\forall$ points of C	A1 [2]
		OR $a(>0)$ constant and as $\theta(>0)$ increases, $1+\theta$ increases	M1
		$\therefore \dfrac{a}{1+\theta}$ decreases	A1
	(ii)	$y = a\sin\theta/(1+\theta)$	M1
		$dy/d\theta = 0 \Rightarrow \ldots \Rightarrow (1+\theta)\cos\theta - \sin\theta = 0$	A1
		$\Rightarrow \tan\theta = 1+\theta$ (AG)	A1
		$\tan\theta - 1 - \theta = -0.135$ when $\theta = 1.1 : \tan\theta - 1 - \theta = +0.372$ when $\theta = 1.2$	B1 [4]
		OR equivalent argument for B1 such as:	
		$\tan(1.1) \approx 1.96 < 2.1,\ \tan(1.2) \approx 2.37 > 2.2$	B1
		or $f(\theta) = \tan\theta - (1+\theta) \Rightarrow f(1.1) = -0.14 \quad f(1.2) = 0.37$	
	(iii)	Sketch:	
		Approximately correct shape and placement for $0 \leqslant \theta \leqslant \pi/2$ and passing through $(a, 0)$ and $(0.4a,\ \pi/2)$, approximately, indicated in some way	B1B1
		Maximum in interval $(\pi/4,\ \pi/2)$	B1 [3]
	(iv)	$A = (a^2/2)\int_0^{\pi/2}(1+\theta)^{-2}\,d\theta$	M1
		$= (a^2/2)\left[-(1+\theta)^{-1}\right]_0^{\pi/2}$	A1
		$= \ldots = \pi a^2/2(\pi+2)$	A1
		Do not accept double minus signs or fractions in the denominator for the final mark.	[3]

12 EITHER

(i) $(\mathbf{i} + \mathbf{j}) \times (\mathbf{j} + 2\mathbf{k}) = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ M1A1

$PQ = (4\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})/3 = 9/3 = 3$ M1A1
[4]

(ii) $(4 + \lambda)/2 = (1 + \lambda - \mu)/(-2) = (3 - 2\mu)/1$ (AEF)

OR $(4 + \lambda) + (1 + \lambda - \mu) = 0$, $(1 + \lambda - \mu + 2(3 - 2\mu) = 0$, both M1A1

$\Rightarrow \ldots \Rightarrow \mu = 1$ M1A1

Position vector of Q is $-\mathbf{i} + \mathbf{j} + \mathbf{k}$ A1
[5]

Parts **(i)** and **(ii)** together:

$\overrightarrow{PQ} = \begin{pmatrix} 4+\lambda \\ 1+\lambda-\mu \\ 3-2\mu \end{pmatrix}$ B1

$\overrightarrow{PQ}.\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \Rightarrow 5 + 2\lambda - \mu = 0$ M1A1

$\overrightarrow{PQ}.\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = 0 \Rightarrow 7 + \lambda - 5\mu = 0$ A1

$\lambda = -2, \ \mu = 1$ M1A1

$\overrightarrow{PQ}.\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \Rightarrow \left|\overrightarrow{PQ}\right| = 3$ M1A1

$\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ A1

[9]

(iii) $(\mathbf{i} + \mathbf{j}) \times (4\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$ M1A1

$p_2 = \left| [(\mathbf{i} - \mathbf{j} - \mathbf{k}).(4\mathbf{i} + \mathbf{k})] / \sqrt{3} \right|$ M1

$= \ldots = \sqrt{3}$ A1

[4]

for final 2 marks
$\pi : x - y - z = 0$

perpendicular distance $= \left| \left(\dfrac{-1-1-1}{\sqrt{1^2+1^2+1^2}} \right) \right| = \sqrt{3}$

or any other method.
M1 is for a complete strategy and A1 for $\sqrt{3}$.

12 OR

(i) (a) $\begin{pmatrix} 1 & 1 & 5 & 7 \\ 3 & 9 & 17 & 25 \\ 1 & 7 & 7 & 11 \\ 3 & 6 & 16 & 23 \end{pmatrix} \to \ldots \to \begin{pmatrix} 1 & 1 & 5 & 7 \\ 0 & 6 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

OR Establishes equivalent result for $\mathbf{M}^{\mathrm{T}}$

OR Establishes 2 linearly independent relations for rows or columns M1A1

$\Rightarrow r(\mathbf{M}) = 2 \Rightarrow \dim(R) = 2$ A1

(b) Basis for R_{T} is $\left\{ \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 9 \\ 7 \\ 6 \end{pmatrix} \right\}$ M1A1

[5]

(ii) Let $\begin{pmatrix} 1 \\ -15 \\ -17 \\ -6 \end{pmatrix} = p \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \end{pmatrix} + q \begin{pmatrix} 1 \\ 9 \\ 7 \\ 6 \end{pmatrix}$

Solves any 2 of:

$p + q = 1$, $3p + 9q = -15$, $p + 7q = -17$, $3p + 6q = -6$ to obtain $p = 4$, $q = -3$ M1A1

Checks consistency with other 2 equations A1

[3]

(iii) $\mathbf{M}\begin{pmatrix}4\\-3\\0\\0\end{pmatrix}=\begin{pmatrix}1\\-15\\-17\\-6\end{pmatrix}$ **OR** Works with equations (**M** written out fully.) (AG) B1 OEW

$\mathbf{e}_1$ and $\mathbf{e}_2$ basis vectors for $\mathbf{T}\Rightarrow \mathbf{Me}_1=0,\ \mathbf{Me}_2=0$ M1

$$\mathbf{M}\begin{pmatrix}4\\-3\\0\\0\end{pmatrix}+\lambda\mathbf{Me}_1+\mu\mathbf{Me}_2=\begin{pmatrix}1\\-15\\-17\\-6\end{pmatrix}+\lambda 0+\mu 0=\begin{pmatrix}1\\-15\\-17\\-6\end{pmatrix},\ \forall\lambda,\ \mu$$ A1

[3]

(iv) Need $-3+\lambda+2\mu=0$ M1

$\lambda=1\Rightarrow\mu=1\Rightarrow \mathrm{x}=\begin{pmatrix}37\\0\\-3\\-3\end{pmatrix}$ A1

[2]

Accept a parametric answer

e.g. $\begin{pmatrix}46\\0\\-9\\0\end{pmatrix}+t\begin{pmatrix}-3\\0\\2\\-1\end{pmatrix}$

Augmented Matrix Method

(i) **(a)** $\left(\begin{array}{cccc:c}1&1&5&7&1\\3&9&17&25&-15\\1&7&7&11&-17\\3&6&16&23&-6\end{array}\right)\sim\left(\begin{array}{cccc:c}1&1&5&7&1\\0&6&2&4&-18\\0&6&2&4&-18\\0&3&1&2&-9\end{array}\right)\sim\left(\begin{array}{cccc:c}1&1&5&7&1\\0&3&1&2&-9\\0&0&0&0&0\\0&0&0&0&0\end{array}\right)$ M1A1

$\Rightarrow \mathrm{r}(\mathbf{M})=\dim(R)=2$ A1

(b) Basis for R is $\left\{\begin{pmatrix}1\\3\\1\\3\end{pmatrix},\begin{pmatrix}1\\9\\7\\6\end{pmatrix}\right\}$ M1A1

[5]

(ii) and **(iii)**

$x + y + 5z + 7t = 1$

$3y + z + 2t = -9$ M1A1

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 46 \\ 0 \\ -9 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 14 \\ 1 \\ -3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$ A1

$$\lambda = -3,\ \mu = 0 \Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 0 \\ 0 \end{pmatrix}$$ B1

$$\therefore \mathbf{M} \begin{pmatrix} 4 \\ -3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -15 \\ -17 \\ -6 \end{pmatrix}$$

$\mathbf{Me}_1 = \mathbf{Me}_2 = 0$ M1

$$\therefore \mathbf{M} \left(\begin{pmatrix} 4 \\ -3 \\ 0 \\ 0 \end{pmatrix} + \lambda \mathbf{e}_1 + \mu \mathbf{e}_2 \right) = \begin{pmatrix} 1 \\ -15 \\ -17 \\ -6 \end{pmatrix}$$ A1

(iv) $\lambda = 0$ $$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 46 \\ 0 \\ -9 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$ M1A1

[8]