UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/23 Paper 23, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Find eqn of motion for P, denoting P's accel. by A:	$2mg - T = 2mA$	M1		
	Find eqn of motion for disc:	$Ta = I_{disc}A/a$	M1		
	($2mga = I_{disc}A/a$ can earn this M1; usually 1/5)				
	Substitute $\frac{1}{2}ma^2$ for MI I_{disc} of disc:	$T = \frac{1}{2}mA$	A1		
	Eliminate T to find accel. A:	$2mg = (5/2)mA$	M1		
		$A = 4g/5$ *or* 8	A1	5	**[5]**
2	*EITHER:* Resolve vertically:	$T \cos 30° + R \cos 30° = mg$ $[T + R = 2mg/\sqrt{3}]$	M1 A1		
	Resolve horizontally:	$T \sin 30° - R \sin 30° = mu^2/a \sin 60°$ $[T - R = 4mu^2/a\sqrt{3}]$	M1 A1		
	OR: Any 2 other independent resolutions, e.g.				
	Along BV (gives T):	$T \cos 30° - mg \cos 60° = (mu^2/a \sin 60°) \sin 60°$			
	Normal to BV:	$R + T \cos 60° - mg \cos 30° = -(mu^2/a \sin 60°) \cos 60°$			
	Along BA:	$T + R \cos 60° - mg \cos 30° = (mu^2/a \sin 60°) \cos 60°$			
	Normal to BA:	$R \cos 30° - mg \cos 60° = -(mu^2/a \sin 60°) \sin 60°$			
	Solve for T:	$T = (m/\sqrt{3}) (g + 2u^2/a)$ **A.G.**	A1		
	Solve for R:	$R = (m/\sqrt{3}) (g - 2u^2/a)$	A1*	6	
	Valid reason for deducing $u^2 \leqslant \frac{1}{2}ga$ (dep A1*):	$R \geqslant 0$ (A.E.F.)	B1	1	**[7]**
3	Resolve vertically at equilibrium:	$0{\cdot}1g = \lambda\, 0{\cdot}01/0{\cdot}25$	M1		
	Evaluate λ:	$\lambda = 25$ **A.G.**	A1	2	
	Use Newton's Law at general point:	$0{\cdot}1\, d^2x/dt^2 = 0{\cdot}1g - \lambda(0{\cdot}01 + x)/0{\cdot}25$	M1		
	Simplify:	$d^2x/dt^2 = -1000x$ **A.G.**	A1	2	
	Use SHM formula for x:	$x = 0{\cdot}02 \cos (t\sqrt{1000})$ [*or* sin]	M1 A1		
	Find reqd. time t:	$t = (1/\sqrt{1000}) \cos^{-1}(-0{\cdot}01/0{\cdot}02)$			
	or	$2\pi/4\sqrt{1000} + (1/\sqrt{1000}) \sin^{-1}(1/2)$	M1 A1		
	Evaluate:	$t = (1/\sqrt{1000})\, 2\pi/3 = 0{\cdot}0662$ [s]	A1	5	**[9]**
4	Find speed u_Q of Q when striking plane:	$u_Q = \sqrt{(10\,g)}$ *or* 10 [m s^{-1}]	M1		
	Find speed v_Q of Q when rebounding from plane:	$v_Q = \frac{1}{2}\, u_Q = 5$ [m s^{-1}]	B1		
	Find magnitude of impulse:	$0{\cdot}04\, (u_Q + v_Q) = 0{\cdot}6$ [N s]	M1 A1	4	
	Find height risen by Q to collision in time t:	$h_Q = v_Q\, t - \frac{1}{2}g\, t^2 = 5t - 5t^2$	M1		
	EITHER: Find time for Q to fall to plane:	$u_Q /g = 1$	M1		
	Find height fallen by P in time $(1 + t)$:	$h_P = \frac{1}{2}g(1 + t)^2 = 5 + 10t + 5t^2$	M1		
	OR: State or imply P is at B when Q is at plane:	[P's speed at B is 10]	(M1)		
	Find height fallen by P in time t:	$h'_P = 10t + \frac{1}{2}gt^2 = 10t + 5t^2$	(M1)		
	Use $h_P + h_Q = 10$ *or* $h'_P + h_Q = 5$ to find t:	$15t = 5,\ t = 1/3$	M1 A1		
	[*or* relative motion can earn previous M1 M1 A1]				
	Evaluate height above plane of collision:	$h_Q = 10/9$ *or* $1{\cdot}11$ [m]	A1	6	**[10]**

Question Number	Mark Scheme Details			Part Mark	Total
5	Consider speed components at Y normal to KN:	$v \sin \varphi = ¾\, u \sin \theta$	M1		
	Consider speed components at Y parallel to KN:	$v \cos \varphi = u \cos \theta$	M1		
	Combine to eliminate u, v:	$\tan \varphi = ¾ \tan \theta$ **A.G.**	A1	3	
	Find XM in terms of a, θ:	$XM = 2a - a \cot \theta - a \cot \varphi$			
		$= 2a - (7/3)\, a \cot \theta$ **A.G.**	M1 A1		
	Find alternative form of XM in terms of a, θ:	$XM = a \cot MXN = a\,(4/3)^2 \cot \theta$	M1 A1		
	Equate two forms of XM:	$(37/9) \cot \theta = 2$ [$\tan \theta = 37/18$]	M1		
	Evaluate θ:	$\theta = 1{\cdot}12$ rad *or* $64{\cdot}1°$	A1	6	
	EITHER: Combine speed components at N:	$w = u \sqrt{(\cos^2 \theta + (9/16)^2 \sin^2 \theta)}$	M1 A1		
	OR: Consider speed components along KN:	$w \cos MXN = u \cos \theta$ *and*			
		$MXN = \cot^{-1}((16/9) \cot \theta)$	(M1)		
	√ on θ	$= 0{\cdot}858$ rad *or* $49{\cdot}1°$	(A1√)		
	Evaluate speed w at N:	$w = 0{\cdot}669\, u$ (allow $0{\cdot}668\, u$)	A1	3	**[12]**
6	Find confidence interval (allow z in place of t): (using 24 in place of 25 loses A1)	$110{\cdot}4 \pm t \sqrt{(50{\cdot}42/25)}$	M1 *A1		
	Use of correct tabular value:	$t_{24,\, 0.95} = 1{\cdot}71[1]$	*B1		
	Evaluate C.I. correct to 3 sf (dep *A1, *B1):	$110{\cdot}4 \pm 2{\cdot}4$ *or* $[108{\cdot}0, 112{\cdot}8]$	A1	4	**[4]**
7	State (at least) null hypothesis (A.E.F.):	H_0: Data conforms to Law	B1		
	Calculate expected values (to 1 dp):	30·10 17·61 12·49 9·69 7·92 22·18	M1 A1		
	Calculate value of χ^2:	$\chi^2 = 4{\cdot}23 \pm 0{\cdot}02$	M1 A1		
	Compare with consistent tabular value (to 2 dp):	$\chi_{5,\, 0.9}{}^2 = 9{\cdot}236$	B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (A.E.F., requires correct values):	Data conforms to Benford's Law	A1	8	**[8]**
8 **(a)** **(i)**	Find prob. of 5 operating after 3 months:	$\{\exp(-3/2{\cdot}5)\}^5 = \{\exp(-3 \times 0{\cdot}4)\}^5$			
		$= 0{\cdot}3011^5 = 0{\cdot}00248$ (to 2 sf)	M1 A1	2	
(ii)	Find prob. of 1 failing within one month:	$p_1 = 1 - \exp(-1/2{\cdot}5)$ [$= 0{\cdot}3297$]	B1		
	Find prob. of 2 failing within one month:	${}^5C_2\, p_1{}^2 (1 - p_1)^3 = 0{\cdot}327$	M1 A1	3	
	Using 2·5 as parameter, not mean, can earn M1s only in **(a)**				
(b)	Find prob. of n operating after c months:	$\{\exp(-c\lambda)\}^n$ [$\lambda = 0{\cdot}4$]	M1 A1		
	Show same as 1 operating after nc months:	$= \exp(-nc\lambda)$	M1 A1	4	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
9 (a) (i)	State valid comment on each diagram (A.E.F.):	Str. line with negative gradient	B1		
(ii)		(Almost) no linear relation	B1		
(iii)		Close to str. line with +ve gradient	B1	3	
(b) (i)	Calculate correlation coefficient: $r = (13040 - 700 \times 275/7) / \sqrt{\{(149000 - 700^2/7)(17351 - 275^2/7)\}}$ $= -14460 / \sqrt{(79000 \times 6547{\cdot}4285} = -0{\cdot}636$		M1 A1 A1	3	
(ii)	State hypotheses:	H_0: $\rho = 0$, H_1: $\rho < 0$	B1		
	Compare with consistent tabular value:	$r_{7,\,5\%} = 0{\cdot}669$	B1		
	Valid method for reaching conclusion:	Reject H_0 if $r < -$ tabular value	M1		
	Correct conclusion (needs correct values):	No negative correlation (A.E.F.)	A1	4	[10]
10	State valid assumption (A.E.F.):	Popln. of diffs. has Normal distn.	B1		
	State hypotheses:	H_0: $\mu_2 - \mu_1 = 0$, H_1: $\mu_2 - \mu_1 > 0$	B1		
	Consider differences eg:	4·0 5·1 2·3 3·1 0 1·1 0·9 1·2	M1		
	Calculate sample mean:	$\bar{x} = 17{\cdot}7/8$ [$= 2{\cdot}2125$]	M1		
	Estimate population variance: (allow biased: $2{\cdot}6511$ *or* $1{\cdot}628^2$)	$s^2 = (60{\cdot}37 - 17{\cdot}7^2/8) / 7$ [$= 3{\cdot}0298$ *or* $1{\cdot}741^2$]	M1		
	Calculate value of t (to 2 dp):	$t = \bar{x}/(s/\sqrt{8}) = 3{\cdot}60$ [*or* $3{\cdot}59$]	M1 *A1		
	Compare with correct tabular t value:	$t_{7,\,0.99} = 2{\cdot}998$ (to 2 dp)	*B1		
	Correct conclusion (AEF, dep *A1, *B1):	There is an increase	B1	9	
	Formulate inequality, with any tabular t value:	$(\bar{x} - w)/(s/\sqrt{8}) > t_{7,\,0.9}$	M1		
	Use correct tabular value (to 2 dp):	$t_{7,\,0.9} = 1{\cdot}415$	A1		
	Evaluate inequality for w:	$w < 1{\cdot}34$ [*or* $\leq$]	A1	3	
	S.R. Allow M1 A1 A0 if = or < used in inequality				[12]
11 EITHER	*EITHER:* Find 2 indep. eqns for R_B, F_B only:		M1		
	Moments for BA about A:	$F_B 2a \sin\beta - R_B 2a \cos\beta = Wa \sin\beta$	M1 A1		
	Moments for system about C:	$F_B 6a \sin\beta + R_B 2a \cos\beta = 9Wa \sin\beta$	M1 A1		
	Add eqns to find F_B:	$F_B = 5W/4$ **A.G.**	M1 A1		
	OR: If R_C, F_C introduced, resolve vertically:	$F_B + F_C = 3W$	(B1)		
	Any 2 moment eqns indep. of above resln. e.g.:				
	For BA about A:	$F_B 2a \sin\beta - R_B 2a \cos\beta = Wa \sin\beta$			
	For CA about A:	$F_C 4a \sin\beta - R_C 4a \cos\beta = 4Wa \sin\beta$			
	For system about B:	$F_C 6a \sin\beta - R_C 2a \cos\beta = 9Wa \sin\beta$			
	(2 system eqns are equiv. to vert. resolution)		(2 × M1 A1)		
	Solve eqns using $R_B = R_C$ to find F_B:	$F_B = 5W/4$ **A.G.**	(M1 A1)	7	
	Find F_C by eg vertical resolution for rods:	$F_C = 7W/4$	B1		
	Find R_B [or R_C] from a moment eqn:	R_B [$= R_C$] $= \frac{3}{4} W \tan\beta$	B1		
	Find one of F_B/R_B, F_C/R_C eg:	$F_B/R_B = 5 / (3 \tan\beta)$	M1 A1		
	Find other eg:	$F_C/R_C = 7 / (3 \tan\beta)$	A1		
	Find set of possible values of μ:	$\mu > 7 / (3 \tan\beta)$ (allow $\geqslant$)	M1 A1	7	[14]

Question Number	Mark Scheme Details			Part Mark	Total
11 OR	For time to pack 50 items, find $E(T_{50})$ *and* $Var(T_{50})$:	$E(T_{50}) = 50 \times 1{\cdot}5$ [= 75] *and* $Var(T_{50}) = 50 \times 0{\cdot}4^2$ [= 8]	B1		
	State valid justification (A.E.F.) e.g.:	By central limit theorem *or* Since n [or 50] is large	B1		
	Find Normal approximation to $P(T_{50} < 70)$:	$\Phi((70 - 75)/\sqrt{8})$	M1		
		$= 1 - \Phi(1{\cdot}768) = 0{\cdot}0385 \pm 0{\cdot}0001$	A1	4	
	For time T_n to pack n items, find $E(T_n)$ *and* $Var(T_n)$:	$E(T_n) = 1{\cdot}5n$ *and* $Var(T_n) = 0{\cdot}4^2\, n$	B1		
	Use Normal approximation for $P(T_n < 70) > 0{\cdot}9$:	$\Phi((70 - 1{\cdot}5n) / (0{\cdot}4\sqrt{n})) \geqslant 0{\cdot}9$	M1		
	Invert function:	$(70 - 1{\cdot}5n)/ 0{\cdot}4\sqrt{n} \geqslant 1{\cdot}282$	A1		
	Rearrange as quadratic expression in $\sqrt{n}$:	$(\sqrt{n})^2 + 0{\cdot}3419\sqrt{n} - 140/3$ [$\leqslant 0$]	M1		
	Find positive root when expression is zero:	44·4	A1		
	Find greatest integer value of n:	$n_{max} = 44$	A1	6	
	For difference in times, find $E(A–H)$ *and* $Var(A–H)$:	$E(A–H) = 75 - 65 = 10$ *and* $Var(A–H) = 8 + 12{\cdot}5 = 20{\cdot}5$	M1 A1		
	Find Normal approximation to $P(A–H > 0)$:	$\Phi((10)/\sqrt{20{\cdot}5}) = \Phi(2{\cdot}209) = 0{\cdot}986$	M1 A1	4	**[14]**