UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/22 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Apply $v^2 = \omega^2 (A^2 - x^2)$ at both points:	$16^2 = \omega^2(A^2 - 3^2)$, $12^2 = \omega^2(A^2 - 4^2)$	M1 A1		
	Combine to find A and ω (in either order):	$A = 5$ [m], $\omega = 4$	B1 B1		
	Find period T using ω (√ on ω):	$T = 2\pi/\omega = \pi/2$ *or* 1·57 [s]	B1√	5	**[5]**
2	Find T by Hooke's Law for whole [or half] string:	$T = \lambda\{2\sqrt{(a^2 + x^2)} - 2l\} / 2l$	M1 A1		
	Neglect x^2: [M0 here for motion along string]	$= (\lambda/l)(a - l)$ **A.G.**	A1	3	
	Find eqn of motion at general point:	$m\mathrm{d}^2x/\mathrm{d}t^2 = -2T x/\sqrt{(a^2 + x^2)}$	M1		
	Neglect x^2 to give SHM eqn for $\mathrm{d}^2x/\mathrm{d}t^2$:	$-(2T/ma)\,x$ *or* $-\{2\lambda(a-l)/alm\}x$	M1 A1		
	State period of motion using $2\pi/\omega$ (A.E.F.):	$2\pi\sqrt{\{alm / 2\lambda(a - l)\}}$	B1	4	**[7]**
	M.R. Vertical motion loses one A/B mark				
	S.R. Motion along string earns max 1 mark:	$\mathrm{d}^2x/\mathrm{d}t^2 = -2\lambda x/lm$	(B1)		
3 (i)	*EITHER:* Resolve horizontally:	$F\cos\alpha = R\sin\alpha$	B2		
	OR: Resolve along plane to find friction F:	$F = (M + m)g\sin\alpha$	(B1)		
	Resolve normal to plane for reaction R:	$R = (M + m)g\cos\alpha$	(B1)		
	Use $F/R \le ½$:	$\tan\alpha \le ½$ **A.G.**	M1 A1	4	
(ii)	*EITHER:* Take moments about pt of contact:	$mg(a - a\sin\alpha) = Mga\sin\alpha$			
	OR: Take moments about centre:	$mg = F = (M + m)g\sin\alpha$	M1 A1		
	Find inequality for $\sin\alpha$:	$\sin\alpha \le 1/\sqrt{5}$	B1		
	Combine:	$m = M/(1/\sin\alpha - 1) \le M/(\sqrt{5} - 1)$			
		$m \le M(1 + \sqrt{5}) / 4$ **A.G.**	M1 A1	5	**[9]**
4	Use conservation of momentum for 1st collision:	$3mu_A + mu_B = 3mu$	M1		
	Use Newton's law of restitution (A1 if both correct):	$u_A - u_B = -0{\cdot}6\,u$	M1 A1		
	Solve for u_A and u_B :	$u_A = 0{\cdot}6\,u$ *and* $u_B = 1{\cdot}2u$	M1 A1	5	
	Find speed u'_B of B after striking wall:	$u'_B = eu_B$ [$= 1{\cdot}2eu$]	M1		
	Use conservation of momentum for 2nd collision:	$mv_B = 3mu_A - mu'_B$	B1		
	Use Newton's law of restitution:	$v_B = 0{\cdot}6\,(u_A + u'_B)$	B1		
	Combine to find e:	$1{\cdot}6e \times 1{\cdot}2u = 2{\cdot}4 \times 0{\cdot}6u$, $e = ¾$	M1 A1	5	**[10]**

Question Number	Mark Scheme Details			Part Mark	Total
5	Use conservation of energy:	$\frac{1}{2}mv^2 = mga(\cos\alpha - \cos\theta)$	M1 A1		
	Equate radial forces [may imply $R = 0$]:	$mv^2/a + R = mg\cos\theta$	M1		
	Take $R = 0$ when contact lost:	$mv^2/a = mg\cos\theta$	A1		
	Eliminate v^2 to find show $\cos\theta = 3/5$ when $R = 0$:	$2(\cos\alpha - \cos\theta) = \cos\theta$			
	[or can show $R = 0$ when $\cos\theta = 3/5$]	$\cos\theta = (2/3)\cos\alpha = 3/5$ **A.G.**	B1		
	Find v:	$v = \sqrt{(3ag/5)}$ *or* $\sqrt{(6a)}$	B1	6	
	Find horizontal distance moved:	$x = 7a/5 - 4a/5$ [$= 3a/5$]	B1		
	EITHER: Find time of subsequent motion:	$t = x / v\cos\theta$			
		[$= a/v = \sqrt{(5a/3g)} = \sqrt{(a/6)}$]	M1		
	Find distance h fallen in time t:	$h = (v\sin\theta)\,t + \frac{1}{2}gt^2$	M1		
	OR: Use trajectory eqn to find h:	$h = x\tan\theta + \frac{1}{2}g(x / v\cos\theta)^2$	(M2)		
	Substitute and simplify h:	$h = 4a/5 + 5a/6 = 49a/30$	M1 A1		
	Find distance below horizontal through O:	$h - 3a/5 = 31a/30$ **A.G.**	B1	6	**[12]**
6	Find expected values (to 1 dp):	66·72 44·48 27·80			
	(lose A1 if rounded to integers)	53·28 35·52 22·20	M1 A1		
	State (at least) null hypothesis (A.E.F.):	H_0: No association between them	B1		
	Calculate value of χ^2:	$\chi^2 = 5{\cdot}36 \pm 0{\cdot}03$	M1 A1		
	Compare with consistent tabular value (to 2 dp):	$\chi_{2,\,0.95}{}^2 = 5{\cdot}991$	B1		
	Conclusion consistent with values (A.E.F):	No association	A1 √	7	**[7]**
7	State valid assumption (A.E.F.):	Popln. of masses has normal distn.	B1		
	State both hypotheses:	H_0: $\mu = 1{\cdot}2$, H_1: $\mu > 1{\cdot}2$	B1		
	Calculate sample mean:	$\bar{x} = 12{\cdot}11 / 10$ [$= 1{\cdot}211$]	B1		
	Estimate population variance:	$s^2 = (14{\cdot}6745 - 12{\cdot}11^2/10) / 9$			
	(allow biased: $0{\cdot}000929$ *or* $0{\cdot}0305^2$)	[$= 0{\cdot}00103$ *or* $0{\cdot}0321^2$]	M1		
	Calculate value of t (to 2 dp):	$t = (\bar{x} - 1{\cdot}2) / (s/\sqrt{10}) = 1{\cdot}08$	M1 A1		
	Compare with correct tabular t value:	$t_{9,\,0.90} = 1{\cdot}38[3]$	B1		
	Conclusion consistent with values (A.E.F):	Claim is not correct	A1 √	8	**[8]**
8	Calculate sample mean:	$\bar{d} = 35{\cdot}15$	B1		
	Estimate population variance (allow biased):	$s^2 = 30{\cdot}24$ [$5{\cdot}5^2$] (or $25{\cdot}2$ [$5{\cdot}02^2$])	B1		
	Find confidence interval (allow z in place of t) e.g.:	$35{\cdot}15 \pm t\sqrt{(30{\cdot}24/6)}$	M1		
	(inconsistent use of 5 or 6 loses M1)				
	Use of correct tabular value:	$t_{5,\,0.975} = 2{\cdot}571$ (2 d.p.)	A1		
	Evaluate C.I. correct to 3 s.f. (needs correct s, t):	$35{\cdot}15 \pm 5{\cdot}77$ *or* [29·4, 40·9]	A1	5	
	State inequality involving sample size n:	$1{\cdot}96 \times 5{\cdot}6 / \sqrt{n} \le$ (*or* $<$) $2{\cdot}5$	M1 A1		
	(Equality or wrong critical value loses A1)				
	Solve for limiting value of n:	$4{\cdot}39^2 = 19{\cdot}3$	A1		
	State smallest sample size:	$n_{\min} = 20$	A1	4	**[9]**

Question Number	Mark Scheme Details	Part Mark	Total
9	State hypotheses: H_0: $\mu_b = \mu_g$, H_1: $\mu_b > \mu_g$ B1 Estimate population variance using boys' sample: $s_b^2 = (216{\cdot}5 - 92{\cdot}0^2/40) / 39$ (allow use of biased: $\sigma_{b,40}^2 = 0{\cdot}1225$ *or* $0{\cdot}35^2$) [$= 0{\cdot}126$ *or* $0{\cdot}354^2$] M1 Estimate population variance using girls' sample: $s_g^2 = (288{\cdot}8 - 129{\cdot}8^2/60) / 59$ (allow use of biased: $\sigma_{g,60}^2 = 0{\cdot}133$ *or* $0{\cdot}365^2$) [$= 0{\cdot}136$ *or* $0{\cdot}368^2$] M1 Estimate population variance for combined sample: $s^2 = s_b^2/40 + s_g^2/60$ $= 0{\cdot}00541$ *or* $0{\cdot}0736^2$ (allow use of $\sigma_{b,40}^2$, $\sigma_{g,60}^2$) (*or* $0{\cdot}00528$ or $0{\cdot}0727^2$) M1 A1 Calculate value of z (to 2 dp): $z = (2{\cdot}3 - 2{\cdot}163)/ s$ $= 0{\cdot}1367/0{\cdot}0736 = 1{\cdot}86$ (*or* $1{\cdot}88$) M1 A1 **S.R.** Allow (implicit) assumption of equal variances: Find pooled estimate of common variance s^2: $(40\sigma_{b,40}^2 + 60\sigma_{g,60}^2)/98 = 0{\cdot}132$ (M1A1) Calculate value of z (to 2 dp): $z = (2{\cdot}3 - 2{\cdot}163)/s\sqrt{(1/40+1/60)}$ $= 1{\cdot}85$ (M1 A1) Compare with correct tabular t value: $z_{0.975} = 1{\cdot}96$ B1 Conclusion consistent with values (A.E.F): Claim is not correct A1 √	9	**[9]**
10 (i)	Find correlation coefficient r: $r = (92{\cdot}01 - 18{\cdot}7 \times 34{\cdot}7/12) / \sqrt{\{(106{\cdot}43 - 18{\cdot}7^2/12)(133{\cdot}43 - 34{\cdot}7^2/12)\}}$ M1 $= 37{\cdot}94 / \sqrt{(77{\cdot}29 \times 33{\cdot}09)}$ $= 37{\cdot}94 / (8{\cdot}791 \times 5{\cdot}752)$ *or* $3{\cdot}161 / \sqrt{(6{\cdot}441 \times 2{\cdot}757)}$ $= 3{\cdot}161 / (2{\cdot}538 \times 1{\cdot}661)$ A1 $= 0{\cdot}750$ [allow $0{\cdot}75$] *A1	3	
(ii)	State both hypotheses: H_0: $\rho = 0$, H_1: $\rho \neq 0$ B1 Use correct tabular 2-tail r value: $r_{12,\,1\%} = 0{\cdot}708$ (to 2 dp) *B1 Valid method for reaching conclusion: Reject H_0 if $\lvert r \rvert >$ tabular value M1 Correct conclusion (AEF, dep *A1, *B1): There is non-zero correlation A1	4	
(iii)	*EITHER*: Calculate gradient b in $y - \bar{y} = b(x - \bar{x})$: $b = (92{\cdot}01 - 18{\cdot}7 \times 34{\cdot}7/12) / (106{\cdot}43 - 18{\cdot}7^2/12)$ $= 37{\cdot}94 / 77{\cdot}29 = 0{\cdot}491$ B1 Use regression line for y at $x = 2$: $y = 34{\cdot}7/12 + 0{\cdot}491(x - 18{\cdot}7/12)$ M1 [$y = 0{\cdot}491x + 2{\cdot}13$] $= 2{\cdot}13 + 0{\cdot}491x = 3{\cdot}11 \pm 0{\cdot}01$ A1 *OR*: Calculate gradient b' in $x - \bar{x} = b'(y - \bar{y})$: $b' = (92{\cdot}01 - 18{\cdot}7 \times 34{\cdot}7/12) / (133{\cdot}43 - 34{\cdot}7^2/12)$ $= 37{\cdot}94 / 33{\cdot}09 = 1{\cdot}15$ (B1) Use regression line for y at $x = 2$: $y = 34{\cdot}7/12 + (x - 18{\cdot}7/12)/1{\cdot}15$ (M1) [$x = 1{\cdot}15y - 1{\cdot}76$] $= 1{\cdot}53 + x/1{\cdot}15 = 3{\cdot}28 \pm 0{\cdot}01$ (A1)	3	**[10]**

Question Number	Mark Scheme Details			Part Mark	Total
11 (a)	Find MI of one side of square about midpoint:	$\frac{1}{3}(M/4)\,a^2/2 = Ma^2/24$	M1 A1		
	Find MI of side of square (or of square) about O:	$Ma^2/24 + (M/4)\,a^2/2 = Ma^2/6$	M1 A1		
	Find MI of ring about O:	$2Ma^2$	B1		
	Sum to find MI of system about O: **A.G.**	$I_O = 4 \times Ma^2/6 + 2Ma^2 = 8\,Ma^2/3$	A1	6	
	Find MI of system about axis through A:	$I_A = \frac{1}{2}\,I_O + 3Ma^2 = 13\,Ma^2/3$	M1 A1	2	
	State or imply that speed is max when AC vertical		M1		
	Use energy when AC vertical (or at general point):	$\frac{1}{2}(I_A + 4Ma^2)\,\omega^2$ [$=25\,Ma^2/3$] *or* $\frac{1}{2}I_A\,\omega^2 + \frac{1}{2}Mv^2$			
	(A1 for each side of eqn, A.E.F.)	$= 5Mga\,(1 + \cos 60°)$ *or* $15Mga/2$	M1 A1 A1		
	Substitute for I_A and find max speed $v = 2a\omega$:	$\omega^2 = (15Mga/2) / (25Ma^2/6) = 9g/5a$ $v = 6\sqrt{(ga/5)}$ *or* $6\sqrt{(2a)}$ *or* $8{\cdot}49\sqrt{a}$	M1 A1	6	**[14]**
(b)	Find k by equating area under graph to 1:	$\frac{1}{2}k + k = 1,\ k = \frac{2}{3}$	M1 A1		
	Find f(x) for $0 < x \le 1$ and $1 < x \le 3$:	kx [$= \frac{2}{3}x$], $\frac{1}{2}k(3 - x)$ [$= 1 - \frac{1}{3}x$]	M1 A1		
	Integrate to find F(x) for $0 < x \le 1$ and $1 < x \le 3$:	$\frac{1}{3}x^2,\ x - \frac{1}{2} - x^2/6$ **A.G.**	M1 A1	6	
	(**i**) Relate dist. fn. G(y) of Y to X: (working may be omitted)	$G(y) = P(Y < y) = P(X^2 < y)$ $= P(X < y^{1/2}) = F(y^{1/2})$ $= \frac{1}{3}y,\ y^{1/2} - \frac{1}{2} - y/6$	M1 A1		
	Differentiate to find g(y):	$g(y) = \frac{1}{3}\quad (0 < y \le 1)$, $\frac{1}{2}\,y^{-1/2} - 1/6\ \ (1 < y \le 9)$, [0 otherwise]	M1 A1	4	
	(**ii**) Use $G(m) = \frac{1}{2}$ to find eqn for median m:	$m^{1/2} - \frac{1}{2} - m/6 = \frac{1}{2}$	M1		
	Solve quadratic for $\sqrt{m}$:	$(\sqrt{m})^2 - 6\sqrt{m} + 6 = 0, \sqrt{m} = 3 \pm \sqrt{3}$	M1 A1		
	Select value of m in interval:	$m = (3 - \sqrt{3})^2 = 12 - 6\sqrt{3}$ *or* $1{\cdot}61$	B1	4	**[14]**