UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Level

FURTHER MATHEMATICS   **9231/02**

Paper 2

October/November 2004

3 hours

Additional materials: Answer Booklet/Paper
                         Graph paper
                         List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10 \, \mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

1 A straight non-uniform beam AB rests horizontally in equilibrium on supports at its ends. The beam has mass 200 kg and its centre of mass is at the point G, where $AG = 1$ m and $BG = 1.5$ m. Find the vertical forces exerted on the supports by the beam. [4]

2 A snooker ball is moving with speed u towards a point P on one cushion of a snooker table. It rebounds from P and strikes a second cushion at a point Q, from which it rebounds with speed w. The cushions are smooth and perpendicular to each other, and the coefficient of restitution at each impact is e. Show that $w = eu$. [6]

3
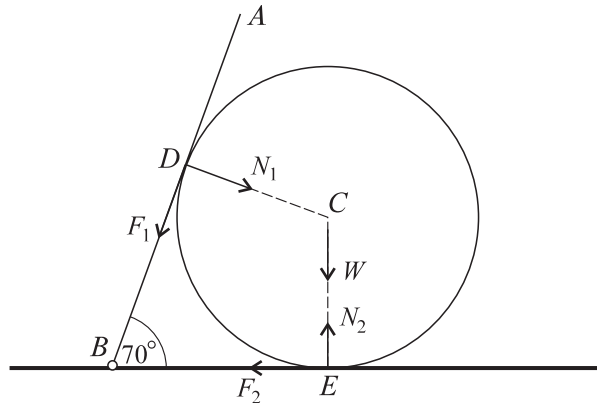
A uniform sphere of weight W rests at a point E on a rough horizontal floor. A straight heavy rod AB rests in equilibrium on the sphere, in a vertical plane through its centre C. The end B of the rod is hinged to the floor, and angle $ABE = 70°$. D is the point of contact of the rod and the sphere. F_1, N_1 and F_2, N_2 are the friction and normal forces acting at D and E respectively (see diagram). Show that $F_1 = F_2$ and that $N_1 + W = N_2$, and deduce that $\dfrac{F_1}{N_1} > \dfrac{F_2}{N_2}$. [5]

The coefficients of friction at D and E are each equal to μ, and the sphere is in limiting equilibrium. State whether the equilibrium is limiting at D or E, and find the value of μ. [5]

4 H is a point on horizontal ground and C is the point vertically above H such that $CH = r$. One end of a light inextensible string of length r is attached to C and a particle of mass m is attached to the other end. When the particle is hanging in equilibrium at H, it is given a horizontal speed U.

(i) Show that when the string makes an angle θ with the downward vertical the tension is

$$m\left[\frac{U^2}{r} - g(2 - 3\cos\theta)\right].$$ [4]

(ii) Deduce that, for the particle to describe a complete circle,

$$U^2 \geqslant 5gr.$$ [2]

It is given that $U^2 = 6gr$ and that when the particle reaches its highest point it becomes detached from the string. The particle then travels freely until it reaches the ground at the point K. Show that $HK = (2\sqrt{2})r$. [5]
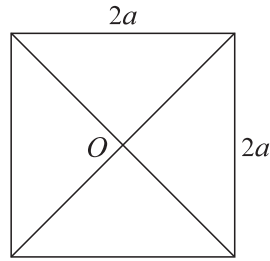
5

Fig. 1

Four isosceles right-angled triangular laminas, each uniform with mass m and base $2a$, are put together to form a square lamina of side $2a$ and centre O (see Fig. 1). Write down the moment of inertia of the square lamina about a perpendicular axis through O, and show that the moment of inertia of each triangular lamina about this axis is $\frac{2}{3}ma^2$. [3]
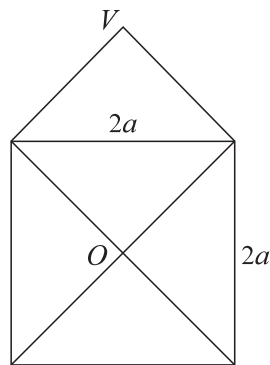

Fig. 2

Another such triangle, with vertex V, is now attached to one side of the square lamina (see Fig. 2). Find the moment of inertia of the combined lamina about an axis l through V, perpendicular to its plane. [4]

The axis l is horizontal, and the combined lamina makes small oscillations under gravity about l. At time t the angle between VO and the vertical is θ. Write down the equation of motion of the combined lamina, and find the approximate period of oscillation. [5]

6 Health and safety regulations have been introduced in 8 factories, and the working times (in hours) lost in randomly chosen weeks before and after their introduction are noted. The figures are given in the following table.

Factory	A	B	C	D	E	F	G	H
Hours lost before introduction	82	45	26	12	46	57	77	10
Hours lost after introduction	78	50	15	14	40	50	75	12

Carry out a suitable test, at the 5% level of significance, to determine whether working time lost has decreased. [7]

State the assumption necessary for your test. [1]

7 A police cadet on a firearms course shoots at a target in a series of shots numbered $1, 2, \ldots$. On any individual shot his probability of hitting the target is p. The result of each shot is independent of the results of previous shots.

   (i) Write down the expected number of shots up to and including the first shot that hits the target.

                                                                                    [1]

   (ii) Find the expected number of shots up to and including the second shot that hits the target.   [2]

   (iii) Find the probability that he hits the target for the second time on his sixth shot.   [3]

8 Three suppliers, P, Q and R, are under examination for the quality of their items, which are rated poor, fair or good. A random sample of items is taken from each supplier, and the numbers in each category are given in the table.

Supplier	Poor	Fair	Good
P	17	40	123
Q	12	17	31
R	11	43	66

Test, at the 5% significance level, whether quality rating is independent of supplier.   [8]

9 Shoppers enter a supermarket randomly and at a uniform average rate of 4 per minute. Show that the probability that at least one shopper enters in a period of t minutes is

$$1 - e^{-4t}.$$ [3]

The time that passes after one shopper has entered before the next enters is denoted by T minutes. Show that T has a negative exponential distribution.   [2]

Find the median value of T.   [3]

10 A fisherman believes that the mean length of the fish that he catches in the centre of a lake is greater than the mean length of those he catches in the shallows. He takes a random sample of 12 fish caught in the centre and measures their lengths, x m. He summarises his results in the form $\Sigma x = 3.40$, $\Sigma x^2 = 1.04$. Obtain a 99% confidence interval for the mean length of fish caught in the centre of the lake, assuming the lengths to be distributed normally.   [5]

He also measures a random sample of 20 fish caught in the shallows, and finds their lengths to be y m, where $\Sigma y = 5.20$ and $\Sigma y^2 = 1.66$. Assuming that this second sample also comes from a normal distribution, with the same variance as the first, obtain a pooled estimate of that variance.   [3]

Test, at the 5% significance level, whether the fisherman's catches provide evidence to support his belief.   [5]

11 Answer only **one** of the following two alternatives.

EITHER
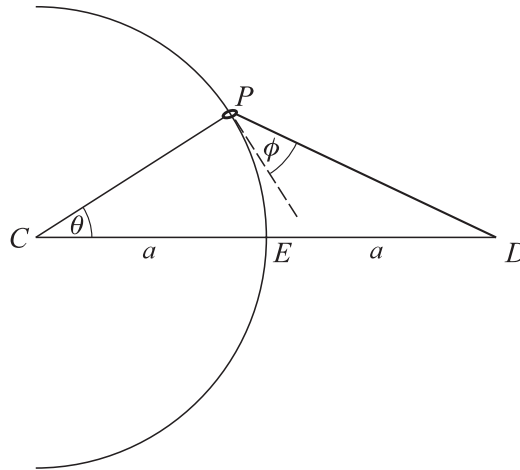
A ring P of mass m kg is threaded on a smooth semicircular wire with centre C and radius a m, fixed in a horizontal plane. A light elastic string connects P to D, a fixed point on the axis of the semicircle with $CD = 2a$ m (see diagram). The natural length of the string is less than a m. P is displaced through a small distance from its equilibrium position E, and is released from rest at time $t = 0$. In the subsequent motion, angle $PCD = \theta$, and ϕ is the angle which PD makes with the tangent at P. Given that θ is small enough for θ^2 and higher powers of θ to be neglected, show that

  (i) $PD = a$ m,   [2]

  (ii) the tension T N in the string is constant,   [1]

(iii) $\phi = \frac{1}{2}\pi - 2\theta$.   [3]

Write down an approximate equation of motion of P on the wire.   [2]

The period of the motion of P is approximately 2 s. Find the value of T in terms of m and a, and find the solution of the approximate equation of motion for which $\theta > 0$ when $t = 0$ and, 0.5 s after release, $\dot{\theta} = -0.03$ rad s^{-1}.   [6]

[The alternative for Q11 is printed overleaf.]

OR

A random sample of size n from a bivariate distribution is denoted by (x_r, y_r), $r = 1, 2, \ldots , n$. Show that, if the regression line of y on x passes through the origin of its scatter diagram, then

$$\bar{y} \sum_{r=1}^{n} x_r^2 = \bar{x} \sum_{r=1}^{n} x_r y_r, \qquad \text{(A)}$$

where $(\bar{x}, \bar{y})$ is the mean point of the sample. [4]

The table below gives a random sample of 5 pairs of values.

$x_1 = 0$	$y_1 = 3$
$x_2 = 4$	$y_2 = 12$
$x_3 = 2$	$y_3 = -1$
$x_4 = 1$	$y_4 = 2$
$x_5 = 3$	$y_5 = 14$

Verify that condition (A) is satisfied by this sample. [1]

Find the regression line, L, of y on x for the above sample. [3]

The equation $y = kx$ represents a line l through the origin of the scatter diagram of this sample, and $S = \sum_{r=1}^{5} (y_r - kx_r)^2$. Draw a diagram to show what S represents. [2]

By expressing S as a quadratic in k, show that, as k varies, S is a minimum when l is L. [4]

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