FURTHER MATHEMATICS **9231/02**

Paper 2 **October/November 2009**

3 hours

Additional Materials: Answer Booklet/Paper
                             Graph Paper
                             List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10 \, \text{m s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

[Turn over

1   A particle of mass m is attached to one end A of a light inextensible string of length a. The other end of the string is attached to a fixed point O and the particle hangs in equilibrium under gravity. The particle is projected horizontally so that it starts to move in a vertical circle. The string slackens after turning through an angle of $120°$. Show that the speed of the particle is then $\sqrt{(\frac{1}{2}ga)}$ and find the initial speed of projection. [5]

2   A circular wheel is modelled as a uniform disc of mass $6\,\text{kg}$ and radius $0.25\,\text{m}$. It is rotating with angular speed $2\,\text{rad s}^{-1}$ about a fixed smooth axis perpendicular to its plane and passing through its centre. A braking force of constant magnitude is applied tangentially to the rim of the wheel. The wheel comes to rest $5\,\text{s}$ after the braking force is applied. Find the magnitude of the braking force and the angle turned through by the wheel while the braking force acts. [7]

3   Two small smooth spheres A and B of equal radius have masses m and $3m$ respectively. They lie at rest on a smooth horizontal plane with their line of centres perpendicular to a smooth fixed vertical barrier with B between A and the barrier. The coefficient of restitution between A and B, and between B and the barrier, is e, where $e > \frac{1}{3}$. Sphere A is projected directly towards B with speed u. Show that after colliding with B the direction of motion of A is reversed. [5]

   After the impact, B hits the barrier and rebounds. Show that B will subsequently collide with A again unless $e = 1$. [3]

4   A uniform rod AB, of length $2a$ and mass $2m$, can rotate freely in a vertical plane about a smooth horizontal axis through A. A small rough ring of mass m is threaded on the rod. The rod is held in a horizontal position with the ring at rest at the mid-point of the rod. The rod is released from rest. Using energy considerations, show that, until the ring slides,
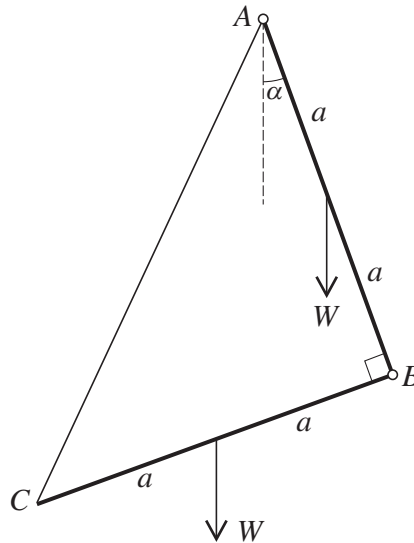
$$a\dot{\theta}^2 = \tfrac{18}{11}g\sin\theta,$$

   where θ is the angle turned through by the rod. [3]

   Show that, until the ring slides, the magnitudes of the friction force and normal contact force acting on the ring are $\frac{29}{11}mg\sin\theta$ and $\frac{2}{11}mg\cos\theta$ respectively. [6]

   The coefficient of friction between the ring and the rod is μ. Find, in terms of μ, the value of θ when the ring starts to slide. [2]

5

Two uniform rods, AB and BC, each have length $2a$ and weight W. They are smoothly jointed at B, and A is attached to a smooth fixed pivot. A light inextensible string of length $(2\sqrt{2})a$ joins A to C so that angle $ABC = 90°$. The system hangs in equilibrium, with AB making an angle α with the vertical (see diagram). By taking moments about A for the system, or otherwise, show that $\alpha = 18.4°$, correct to the nearest $0.1°$. [3]

Find the tension in the string in the form kW, giving the value of k correct to 3 significant figures. [3]

Find, in terms of W, the magnitude of the force acting on the rod BC at B. [6]

6 A machine produces metal discs whose diameters have a normal distribution. The mean of this distribution is intended to be $10\,\text{cm}$. Accuracy is checked by measuring the diameters of a random sample of six discs. The diameters, in cm, are as follows.

$$10.03 \quad 10.02 \quad 9.98 \quad 10.06 \quad 10.08 \quad 10.01$$

Calculate a 99% confidence interval for the mean diameter of all discs produced by the machine. [5]

Deduce a 99% confidence interval for the mean circumference of all discs produced by the machine. [1]

7 A continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(x^3 + 1) & -1 \leqslant x \leqslant 1, \\ 1 & x > 1. \end{cases}$$

Find $P\left(X \geqslant \frac{3}{4}\right)$, and state what can be deduced about the upper quartile of X. [3]

Obtain the cumulative distribution function of Y, where $Y = X^2$. [5]

8 150 sheep, chosen from a large flock of sheep, were divided into two groups of 75. Over a fixed period, one group had their grazing controlled and the other group grazed freely. The gains in weight, in kg, were recorded for each animal and the table below shows the sample means and the unbiased estimates of the population variances for the two samples.

	Sample mean	Unbiased estimate of population variance
Controlled grazing	19.14	20.54
Free grazing	15.36	9.84

It is required to test whether the population mean for sheep having their grazing controlled exceeds the population mean for sheep grazing freely by less than 5 kg. State, giving a reason, if it is necessary for the validity of the test to assume that the two population variances are equal. [1]

Stating any other assumption, carry out the test at the 5% significance level. [8]

9 It has been found that 60% of the computer chips produced in a factory are faulty. As part of quality control, 100 samples of 4 chips are selected at random, and each chip is tested. The number of faulty chips in each sample is recorded, with the results given in the following table.

Number of faulty chips	0	1	2	3	4
Number of samples	2	12	27	49	10

The expected values for a binomial distribution with parameters $n = 4$ and $p = 0.6$ are given in the following table.

Number of faulty chips	0	1	2	3	4
Expected value	2.56	15.36	34.56	34.56	12.96

Show how the expected value 34.56 corresponding to 2 faulty chips is obtained. [2]

Carry out a goodness of fit test at the 5% significance level, and state what can be deduced from the outcome of the test. [8]

10 An archer shoots at a target. It may be assumed that each shot is independent of all other shots and that, on average, she hits the bull's-eye with 3 shots in 20. Find the probability that she requires at least 6 shots to hit the bull's-eye. [3]

When she hits the bull's-eye for the third time her total number of shots is Y. Show that

$$P(Y = r) = \tfrac{1}{2}(r - 1)(r - 2)\left(\tfrac{3}{20}\right)^3\left(\tfrac{17}{20}\right)^{r-3}.$$ [3]

Simplify $\dfrac{P(Y = r + 1)}{P(Y = r)}$, and hence find the set of values of r for which $P(Y = r + 1) < P(Y = r)$. Deduce the most probable value of Y. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A light elastic string, of natural length l and modulus of elasticity $4mg$, is attached at one end to a fixed point and has a particle P of mass m attached to the other end. When P is hanging in equilibrium under gravity it is given a velocity $\sqrt{(gl)}$ vertically downwards. At time t the downward displacement of P from its equilibrium position is x. Show that, while the string is taut,
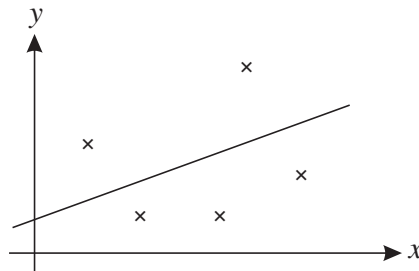
$$\ddot{x} = -\frac{4g}{l}x.$$ [4]

Find the speed of P when the length of the string is l. [4]

Show that the time taken for P to move from the lowest point to the highest point of its motion is

$$\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)\sqrt{\left(\frac{l}{g}\right)}.$$ [6]

OR

The scatter diagram shows a sample of size 5 of bivariate data, together with the regression line of y on x. State what is minimised in obtaining this regression line, illustrating your answer on a copy of this diagram. [2]

State, giving a reason, whether, for the data shown, the regression line of y on x is the same as the regression line of x on y. [1]

A car is travelling along a stretch of road with speed $v\,\text{km}\,\text{h}^{-1}$ when the brakes are applied. The car comes to rest after travelling a further distance of z m. The values of z (and $\sqrt{z}$) for 8 different values of v are given in the table, correct to 2 decimal places.

v	25	30	35	40	45	50	55	60
z	2.83	4.63	4.84	5.29	9.73	10.30	14.82	15.21
$\sqrt{z}$	1.68	2.15	2.20	2.30	3.12	3.21	3.85	3.90

[$\Sigma v = 340$, $\Sigma v^2 = 15\,500$, $\Sigma \sqrt{z} = 22.41$, $\Sigma z = 67.65$, $\Sigma v\sqrt{z} = 1022.15$.]

(i) Calculate the product moment correlation coefficient between v and $\sqrt{z}$. What does this indicate about the scatter diagram of the points $(v, \sqrt{z})$? [4]

(ii) Given that the product moment correlation coefficient between v and z is 0.965, correct to 3 decimal places, state why the regression line of $\sqrt{z}$ on v is more suitable than the regression line of z on v, and find the equation of the regression line of $\sqrt{z}$ on v. [5]

(iii) Comment, in the context of the question, on the value of the constant term in the equation of the regression line of $\sqrt{z}$ on v. [2]

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