Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

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| CANDIDATE NAME | | | | | | | | | |
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*2032562688*

**FURTHER MATHEMATICS** **9231/23**

Paper 2 **May/June 2017**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

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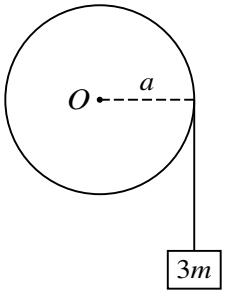
**1**




A uniform disc with centre $O$, mass $m$ and radius $a$ is free to rotate without resistance in a vertical plane about a horizontal axis through $O$. One end of a light inextensible string is attached to the rim of the disc and wrapped around the rim. The other end of the string is attached to a block of mass $3m$ (see diagram). The system is released from rest with the block hanging vertically. While the block is in motion, it experiences a constant vertical resisting force of magnitude $0.9mg$. Find the tension in the string in terms of $m$ and $g$. [5]

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**2** A particle $P$ moves on a straight line in simple harmonic motion. The centre of the motion is $O$, and the amplitude of the motion is 2.5 m. The points $L$ and $M$ are on the line, on opposite sides of $O$, with $OL = 1.5$ m. The magnitudes of the accelerations of $P$ at $L$ and at $M$ are in the ratio 3 : 4.

**(i)** Find the distance $OM$. [2]

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The time taken by $P$ to travel directly from $L$ to $M$ is 2 s.

**(ii)** Find the period of the motion. [5]

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**(iii)** Find the speed of $P$ when it passes through $L$. [2]

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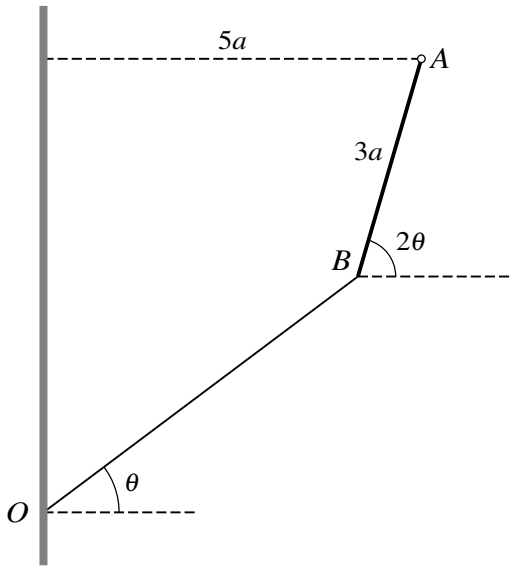
**3** Two uniform small smooth spheres $A$ and $B$ have equal radii and each has mass $m$. Sphere $A$ is moving with speed $u$ on a smooth horizontal surface when it collides directly with sphere $B$ which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$. Sphere $B$ is initially at a distance $d$ from a fixed smooth vertical wall which is perpendicular to the direction of motion of $A$. The coefficient of restitution between $B$ and the wall is $\frac{1}{3}$.

**(i)** Show that the speed of $B$ after its collision with the wall is $\frac{5}{18}u$. [4]

**(ii)** Find the distance of $B$ from the wall when it collides with $A$ for the second time. [6]

**4**




A uniform rod $AB$ of length $3a$ and weight $W$ is freely hinged to a fixed point at the end $A$. The end $B$ is below the level of $A$ and is attached to one end of a light elastic string of natural length $4a$. The other end of the string is attached to a point $O$ on a vertical wall. The horizontal distance between $A$ and the wall is $5a$. The string and the rod make angles $\theta$ and $2\theta$ respectively with the horizontal (see diagram). The system is in equilibrium with the rod and the string in the same vertical plane. It is given that $\sin\theta = \frac{3}{5}$ and you may use the fact that $\cos 2\theta = \frac{7}{25}$.

**(i)** Find the tension in the string in terms of $W$. [3]

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**(ii)** Find the modulus of elasticity of the string in terms of $W$. [4]

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**(iii)** Find the angle that the force acting on the rod at $A$ makes with the horizontal. [3]

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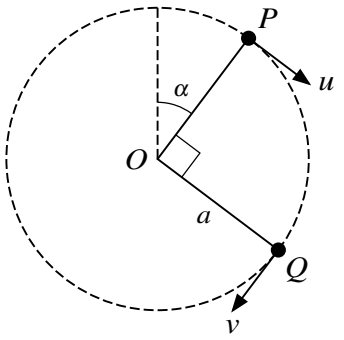
**5**




A particle of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle is moving in complete vertical circles with the string taut. When the particle is at the point $P$, where $OP$ makes an angle $\alpha$ with the upward vertical through $O$, its speed is $u$. When the particle is at the point $Q$, where angle $QOP = 90°$, its speed is $v$ (see diagram). It is given that $\cos\alpha = \frac{4}{5}$.

**(i)** Show that $v^2 = u^2 + \frac{14}{5}ag$. [2]

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The tension in the string when the particle is at $Q$ is twice the tension in the string when the particle is at $P$.

**(ii)** Obtain another equation relating $u^2$, $v^2$, $a$ and $g$, and hence find $u$ in terms of $a$ and $g$. [5]

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**(iii)** Find the least tension in the string during the motion. [3]

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**6** The independent variables $X$ and $Y$ have distributions with the same variance $\sigma^2$. Random samples of $N$ observations of $X$ and $2N$ observations of $Y$ are taken, and the results are summarised by

$$\Sigma x = 4, \quad \Sigma x^2 = 10, \quad \Sigma y = 8, \quad \Sigma y^2 = 102.$$

These data give a pooled estimate of 10 for $\sigma^2$. Find $N$. [5]

**7** A random sample of twelve pairs of values of $x$ and $y$ is taken from a bivariate distribution. The equations of the regression lines of $y$ on $x$ and of $x$ on $y$ are respectively

$$y = 0.46x + 1.62 \qquad \text{and} \qquad x = 0.93y + 8.24.$$

**(i)** Find the value of the product moment correlation coefficient for this sample. [2]

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**(ii)** Using a 5% significance level, test whether there is non-zero correlation between the variables. [4]

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**8** The number, $x$, of beech trees was counted in each of 50 randomly chosen regions of equal size in beech forests in country $A$. The number, $y$, of beech trees was counted in each of 40 randomly chosen regions of the same equal size in beech forests in country $B$. The results are summarised as follows.

$$\Sigma x = 1416 \qquad \Sigma x^2 = 41\,100 \qquad \Sigma y = 888 \qquad \Sigma y^2 = 20\,140$$

Find a 95% confidence interval for the difference between the mean number of beech trees in regions of this size in country $A$ and in country $B$. [9]

**9** The continuous random variable $X$ has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} 0 & x < 0, \\ a\mathrm{e}^{-x\ln 2} & x \geqslant 0, \end{cases}$$

where $a$ is a positive constant.

**(i)** Find the value of $a$. [2]

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**(ii)** State the value of $\mathrm{E}(X)$. [1]

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**(iii)** Find the interquartile range of $X$. [4]

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The variable $Y$ is related to $X$ by $Y = 2^X$.

**(iv)** Find the probability density function of $Y$. [5]

**10** Roberto owns a small hotel and offers accommodation to guests. Over a period of 100 nights, the numbers of rooms, $x$, that are occupied each night at Roberto's hotel and the corresponding frequencies are shown in the following table.

| Number of rooms occupied ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $\geqslant 7$ |
|---|---|---|---|---|---|---|---|---|
| Number of nights | 4 | 9 | 18 | 26 | 20 | 16 | 7 | 0 |

**(i)** Show that the mean number of rooms that are occupied each night is 3.25. [1]

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The following table shows most of the corresponding expected frequencies, correct to 2 decimal places, using a Poisson distribution with mean 3.25.

| Number of rooms occupied ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $\geqslant 7$ |
|---|---|---|---|---|---|---|---|---|
| Observed frequency | 4 | 9 | 18 | 26 | 20 | 16 | 7 | 0 |
| Expected frequency | 3.88 | 12.60 | 20.48 | 22.18 | 18.02 | 11.72 | | |

**(ii)** Show how the expected value of 22.18, for $x = 3$, is obtained and find the expected values for $x = 6$ and for $x \geqslant 7$. [4]

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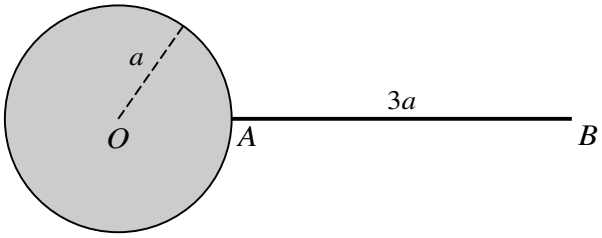
**(iii)** Use a goodness-of-fit test at the 5% significance level to determine whether the Poisson distribution is a suitable model for the number of rooms occupied each night at Roberto's hotel. [7]

**11** Answer only **one** of the following two alternatives.

**EITHER**




The diagram shows a uniform thin rod $AB$ of length $3a$ and mass $8m$. The end $A$ is rigidly attached to the surface of a sphere with centre $O$ and radius $a$. The rod is perpendicular to the surface of the sphere. The sphere consists of two parts: an inner uniform solid sphere of mass $\frac{3}{2}m$ and radius $a$ surrounded by a thin uniform spherical shell of mass $m$ and also of radius $a$. The horizontal axis $l$ is perpendicular to the rod and passes through the point $C$ on the rod where $AC = a$.

**(i)** Show that the moment of inertia of the object, consisting of rod, shell and inner sphere, about the axis $l$ is $\frac{289}{15}ma^2$. [6]

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The object is free to rotate about the axis $l$. The object is held so that $CA$ makes an angle $\alpha$ with the downward vertical and is released from rest.

**(ii)** Given that $\cos\alpha = \frac{1}{6}$, find the greatest speed achieved by the centre of the sphere in the subsequent motion. [6]

**OR**

The times taken to run 200 metres at the beginning of the year and at the end of the year are recorded for each member of a large athletics club. The time taken, in seconds, at the beginning of the year is denoted by $x$ and the time taken, in seconds, at the end of the year is denoted by $y$. For a random sample of 8 members, the results are shown in the following table.

| Member | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
|---|---|---|---|---|---|---|---|---|
| $x$ | 24.2 | 23.8 | 22.8 | 25.1 | 24.5 | 24.0 | 23.8 | 22.8 |
| $y$ | 23.9 | 23.6 | 22.8 | 24.5 | 24.2 | 23.5 | 23.6 | 22.7 |

$[\Sigma x = 191, \quad \Sigma x^2 = 4564.46, \quad \Sigma y = 188.8, \quad \Sigma y^2 = 4458.4, \quad \Sigma xy = 4510.99.]$

**(i)** Find, showing all necessary working, the equation of the regression line of $y$ on $x$. [4]

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The athletics coach believes that, on average, the time taken by an athlete to run 200 metres decreases between the beginning and the end of the year by more than 0.2 seconds.

**(ii)** Stating suitable hypotheses and assuming a normal distribution, test the coach's belief at the 10% significance level. [8]

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