CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2012 series

9231 FURTHER MATHEMATICS

9231/22 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question Number	Mark Scheme Details			Part Mark	Total
1	Find MI of disc A about O:	$I_A = ½\, ma^2 + m(4a)^2$ [$= (33/2)ma^2$]	B1		
	Find MI of disc B about O:	$I_B = ½\, ma^2 + m(6a)^2$ [$= (73/2)ma^2$]	B1		
	Find MI of rod AB about O:	$I_{rod} = ⅓\, 3m(5a)^2 + 3ma^2$ [$= 28ma^2$]	B1		
	Find MI of body about O:	$I_{body} = I_A + I_B + I_{rod} = 81ma^2$ **A.G.**	M1 A1	5	**[5]**
2 (i)	Find eqn of motion for disc:	$T \times 0{\cdot}4 = 0{\cdot}2\ d^2\theta/dt^2$	M1		
	Find eqn of motion for particle:	$1{\cdot}5g - T = 1{\cdot}5 \times 0{\cdot}4\ d^2\theta/dt^2$	M1		
	Eliminate T to find angular accel.:	$1{\cdot}5g = (0{\cdot}6 + 0{\cdot}5)\ d^2\theta/dt^2$	M1		
		$d^2\theta/dt^2 = 15g/11$ *or* $13{\cdot}6$ [rad s^{-2}]	A1		
	S.R.: M1 only for $1{\cdot}5g \times 0{\cdot}4 = 0{\cdot}2\ d^2\theta/dt^2$				
	[$d^2\theta/dt^2 = 30$, $(d\theta/dt)^2 = 10\pi$, $v = 2{\cdot}24$]			4	
(ii)	EITHER				
	Integrate to find $(d\theta/dt)^2$:	$½\,(d\theta/dt)^2 = (15g/11)\theta$ [$+ c$]	M1		
	Apply initial conds. and $\theta = \pi/6$:	$(d\theta/dt)^2 = 5\pi g/11$ *or* $14{\cdot}3$	A1		
	OR				
	Use energy to find $(d\theta/dt)^2$:	$½\ 0{\cdot}2\ (d\theta/dt)^2 + ½\ 1{\cdot}5\ (0{\cdot}4\ d\theta/dt)^2$			
		$= 1{\cdot}5g \times 0{\cdot}4 \times \pi/6$	(M1)		
	Simplify:	$(d\theta/dt)^2 = 5\pi g/11$ *or* $14{\cdot}3$	(A1)		
	Find speed of particle:	$v = 0{\cdot}4\ d\theta/dt = 51$ [m s^{-1}]	B1	3	**[7]**
3	Use energy to find speed v when AP vertical:	$½mv^2 = mga$ [$v^2 = 2ga$]	B1		
	Use energy to find speed w when AP at angle θ:	$½mw^2 = ½mv^2$			
		$- mg(a - x)(1 - \cos\theta)$	M1 A1		
	(note that v need not be found)	[$mw^2 = 2mg\{x + (a - x)\cos\theta\}$]			
	Use $F = ma$ radially to find tension T:	$T - mg\cos\theta = mw^2/(a - x)$	M1 A1		
	Substitute for w^2:	$T = mg\{3\cos\theta + 2x/(a-x)\}$ **A.G.**	M1 A1	7	
	Find x/a if $T = 0$ when $\theta = \pi$:	$2x = 3(a-x)$, $x/a = 3/5$	M1 A1	2	**[9]**
4	Resolve speeds parallel to barrier:	$v\cos\theta = u\cos 60°$ [$= u/2$]	B1		
	Resolve speeds perpendicular to barrier:	$v\sin\theta = ⅓\, u \sin 60°$ [$= u/2\sqrt{3}$]	M1		
	Find v^2	$v^2 = u^2\,(1/12 + 1/4) = ⅓\, u^2$	A1		
	Relate loss of K.E. to that before collision:	$½\ 2m(u^2 - v^2) = ⅔ \times ½ 2mu^2$ **A.G.**	M1 B1	5	
(i)	Find (reversed) speed of P using impulse:	$2mw_P = ⅔mu(1 + \sqrt{3}) - 2mv$			
		$w_P = ⅓\, u$ **A.G.**	M1 A1	2	
(ii)	Find (reversed) speed of Q using impulse:	$mw_Q = ⅔mu(1 + \sqrt{3}) - mu$			
	OR by conservation of momentum:	$2mu/3 - mw_Q = -2mu/\sqrt{3} + mu$			
		$w_Q = (2/\sqrt{3} - 1/3)\, u$ (A.E.F.)	M1 A1		
	Find coefficient of restitution:	$(w_P + w_Q) / (v + u)$			
		$= 2/(1 + \sqrt{3})$ *or* $\sqrt{3} - 1$	M1 A1	4	**[11]**

Question Number	Mark Scheme Details			Part Mark	Total
5	Find (or verify) *AP* by equating equilibrium tensions:				
		$8mg\,(AP - 2a)/2a$	M1 A1		
		$= 16mg\,(6a - AP)/4a$ $AP = 32a/8 = 4a$	A1 **A.G** A1	3	
(i)	Apply Newton's law at general point, e.g.: (lose A1 for each incorrect term)	$m\,d^2x/dt^2 = 8mg\,(2a - x)/2a$ $- 16mg\,(2a + x)/4a$			
	Or	$m\,d^2y/dt^2 = - 8mg\,(2a + y)/2a$ $+ 16mg\,(2a - y)/4a$	M1 A2		
	Simplify to give standard SHM eqn, e.g.: **S.R.**: B1 if no derivation (max 3/6) Find period *T* using SHM with $\omega = \sqrt{(8g/a)}$:	$d^2x/dt^2 = - 8gx/a$ $T = 2\pi/\sqrt{(8g/a)}] = \pi\sqrt{(a/2g)}$	A1 **A.G** M1 A1	6	
(ii)	Find max speed using ωA with $A = a$:	$v_{max} = \sqrt{(8g/a)} \times a$ $= \sqrt{(8ag)}$ *or* $2\sqrt{(2ag)}$	M1 A1	2	**[11]**
6 (i)	Find prob. that first snow falls on 20th:	$(1 - 0{\cdot}2)^{19} \times 0{\cdot}2 = 0{\cdot}00288$	M1 A1	2	
(ii)	Find prob. that first snow falls before 5th:	$1 - (1 - 0{\cdot}2)^4 = 0{\cdot}59[0]$	M1 A1	2	
(iii)	Formulate condition for day *n* of month: Take logs (any base) to give bound for *n*: Find n_{min}:	$1 - (1 - 0{\cdot}2)^n \ge 0{\cdot}95,\ 0{\cdot}8^n \le 0{\cdot}05$ $n > \log 0{\cdot}05/\log 0{\cdot}8$ $n > 13{\cdot}4,\ n_{min} = 14$	M1 M1 A1	3	**[7]**
7	Integrate f(*x*) to find F(*x*) for $1 \le x \le 4$: Relate dist. fn. G(*y*) of *Y* to *X* for $1 \le x \le 4$:	$F(x) = x^2/15 + c = (x^2 - 1)/15$ $G(y) = P(Y < y) = P(X^3 < y)$ $= P(X < y^{1/3}) = F(y^{1/3})$ $= (y^{2/3} - 1)/15$	M1 A1 **A.G** M1 A1	4	
(i)	Find relation for median *m* of *Y*: Evaluate *m*:	$G(m) = ½,\ m^{2/3} = 17/2$ $m = 24{\cdot}8$	M1 A1 A1	3	
(ii)	EITHER Find g(*y*) and formulate E(*Y*):	$g(y) = 2y^{-1/3}/45$ $E(Y) = \int y g(y) dy = \int 2y^{2/3}/45\, dy$	M1 A1		
	OR Formulate E(*Y*) in terms of *X*:	$E(Y) = E(X^3) = \int 2x^4/15\, dx$	(M1 A1)		
	Integrate and apply limits:	$E(Y) = \left[\frac{2y^{5/3}}{75}\right]_1^{64}$ *or* $\left[\frac{2x^5}{75}\right]_1^4$ $= 2(1024 - 1)/75$ $= 682/25$ *or* $27{\cdot}3$	 M1 A1	4	**[11]**

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8	**(i)**	Calculate gradient b in $y - \bar{y} = b(x - \bar{x})$:	$b = (761{\cdot}3 - 72{\cdot}4 \times 78/8)/(769{\cdot}9 - 72{\cdot}4^2/8)$ M1 $= 55{\cdot}4/114{\cdot}68$ $[or\ 6{\cdot}925/14{\cdot}335]$ $= 1385/2867\ or\ 0{\cdot}483[1]$ A1		
		Find regression line:	$y - 9{\cdot}75 = 0{\cdot}483\,(x - 9{\cdot}05)$ *Or* $y = 5{\cdot}38 + 0{\cdot}483x$ M1 A1	4	
	(ii)	Find correlation coefficient r: $r = (761{\cdot}3 - 72{\cdot}4 \times 78/8) / \surd\{(769{\cdot}9 - 72{\cdot}4^2/8)\,(820 - 78^2/8)\}$ M1	$= 55{\cdot}4 / \surd(114{\cdot}68 \times 59{\cdot}5)$ $[or\ 6{\cdot}925 / \surd(14{\cdot}335 \times 7{\cdot}4375)]$ A1 $= 0{\cdot}671$ *A1	3	
	(iii)	State both hypotheses: State or use correct tabular one-tail r value: Valid method for reaching conclusion: Correct conclusion (AEF, dep *A1, *B1):	$H_0: \rho = 0,\ H_1: \rho > 0$ B1 $r_{8,\ 5\%} = 0{\cdot}621$ *B1 Reject H_0 if $\lvert r \rvert >$ tabular value M1 There is positive correlation A1	4	**[11]**
9		Estimate population variance using A's sample: (allow use of biased here: $1{\cdot}489$ *or* $1{\cdot}22^2$) Find confidence interval: State or use correct tabular value of t: Evaluate C.I. correct to 3 s.f.:	$s_A^2 = (481{\cdot}1 - 57{\cdot}4^2/7) / 6$ $= 521/300\ or\ 1{\cdot}737\ or\ 1{\cdot}318^2$ M1 A1 $57{\cdot}4/7 \pm t\,\surd(s_A^2/7)$ M1 $t_{6,0.975} = 2{\cdot}447\ [or\ 2{\cdot}45]$ A1 $8{\cdot}2 \pm 1{\cdot}22\ or\ [6{\cdot}98, 9{\cdot}42]$ A1	5	
		State suitable assumptions (A.E.F.): State hypotheses: Estimate population variance using B's sample: (allow use of biased here: $0{\cdot}988$ *or* $0{\cdot}994^2$) Estimate population variance for combined sample: Calculate value of t (to 2 d.p.): State or use correct tabular value Correct conclusion (AEF, dep *A1, *B1): **S.R.**: Deduct only A1 if intermediate result to 3 s.f. **S.R.**: Invalid method for calculating t (max 6/9):	Population of B is Normal *and* has same variance as for A B1 $H_0: \mu_A = \mu_B,\ H_1: \mu_A > \mu_B$ B1 $s_B^2 = (278{\cdot}74 - 37^2/5) / 4$ $= 1{\cdot}235\ or\ 1{\cdot}111^2$ B1 $s^2 = (6s_A^2 + 4s_B^2) / 10$ $= 192/125\ or\ 1{\cdot}536\ or\ 1{\cdot}239^2$ M1 A1 $t = (57{\cdot}4/7 - 37/5)/\ s\surd(1/7+1/5)$ M1 $= 0{\cdot}8/0{\cdot}726 = 1{\cdot}10[2]$ *A1 $t_{10,0.95} = 1{\cdot}812\ [or\ 1{\cdot}81]$ *B1 μ_A is not greater than μ_B B1 $t = 0{\cdot}8/\surd(s_A^2/7 + s_B^2/5)$ (M1) $= 0{\cdot}8/0{\cdot}704 = 1{\cdot}14$ (A1)	9	**[14]**

10	**(a)**	For A, let contact pts with plane, sphere C be P, S Stating or implying reactions R_P, R_S same as for B: B1 Stating or implying $F_P = F_S$ by moments about O_A: B1 Stating or implying 3 indep. eqns for F, R_P, R_S e.g.: 3 × M1 A1 Up to 2 resolutions of forces, e.g. ↑ for system: $2R_P = 3W$ ↑ for A: $R_P = W + R_S \cos\theta + F_S \sin\theta$ ↑ for C: $2R_S \cos\theta + 2F_S \sin\theta = W$ $O_A \to O_C$ for A: $R_P \cos\theta + F_P \sin\theta = R_S + W\cos\theta$ Moments about S for A: $F_P(r + r\cos\theta) + Wr\sin\theta = R_P r\sin\theta$ Find R_P: $R_P = 3W/2$ A1 Find R_S: $R_S = W/2$ A1 Find F at P and/or S: $F = (W\sin\theta) / 2(1 + \cos\theta)$ A1 Use $F \le \mu R_P$ to find bound for μ: $\mu \ge \sin\theta / 3(1 + \cos\theta)$ **A.G.** M1 A1 Use $F \le \mu' R_S$ to find bound for μ': $\mu' \ge \sin\theta / (1 + \cos\theta)$ **A.G.** M1	14	**[14]**
	(b)	Find E(X) using $\int x\mathrm{f}(x)\mathrm{d}x$: $\mathrm{E}(X) = \int_2^4 3(5x^2 - x^3 - 4x)/10\,\mathrm{d}x$ M1 A1 $= \tfrac{1}{2}(4^3 - 2^3) - 3(4^4 - 2^4)/40 - 3(4^2 - 2^2)/5$ $= 28 - 18 - 7{\cdot}2 = 2{\cdot}8$ *A1 Verify E(X) within 10% of 2·69 (A1 dep *A1): $(\mathrm{E}(X) - 2{\cdot}69)/2{\cdot}69 = 0{\cdot}041 < 0{\cdot}1$ *or* $1{\cdot}1 \times 2{\cdot}69 = 2{\cdot}96 > \mathrm{E}(X)$ M1 A1	5	
		Show derivation of tabular entry: $60\int_{3.2}^{3.6} 3(5x - x^2 - 4)/10\,\mathrm{d}x$ M1 $= 60[3(5x^2/2 - x^3/3 - 4x)/10]_{3.2}^{3.6}$ *or* $[45x^2 - 6x^3 - 72x]_{3.2}^{3.6}$ $= 122{\cdot}4 - 83{\cdot}328 - 28{\cdot}8$ *or* $60 \times 0{\cdot}1712$ $= 10{\cdot}272$ **A.G** A1	2	
		State (at least) null hypothesis: H_0: f(x) fits data (A.E.F.) B1 Combine last 2 cells since exp. value < 5: O: . . . 8 E: . . . 14·208 B1 Calculate χ^2 (to 2 d.p.): $\chi^2 = 0{\cdot}8126 + 0{\cdot}0584 + 0{\cdot}2011 + 2{\cdot}7135 = 3{\cdot}78[47]$ M1 *A1 State or use consistent tabular value (to 2 d.p.): $\chi_{3\,0.9}^2 = 6{\cdot}25[1]$ [or if no cells combined: $\chi_{4,\,0.9}^2 = 7{\cdot}78$] *B1 Valid method for reaching conclusion: Accept H_0 if χ^2 < tabular value M1 Conclusion (A.E.F., dep *A1, *B1): $3{\cdot}78 < 6{\cdot}25$ so f(x) does fit A1	7	**[14]**