CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2012 series

9231 FURTHER MATHEMATICS

9231/23 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Find radial acceleration when $t = 3$:	$(k - 3^2)^2 / 1{\cdot}5$ [m s^{-2}]	B1		
	Find transverse accel. (ignoring sign) when $t = 3$:	$2t = 6$ [m s^{-2}]	B1		
	Equate magnitudes to find k:	$(k - 9)^2 = 9$, $k = 6$ or 12	M1 A1	4	**[4]**
2	Use conservation of energy:	$\frac{1}{2}mv^2 = \frac{1}{2}mkga - mga(1 - \cos\theta)$	B1		
	Use $F = ma$ radially:	$R + 4mg - mg\cos\theta = mv^2/a$	M1 A1		
	Eliminate v to find R:	$R = mg(3\cos\theta + k - 6)$ **A.G.**	M1 A1	5	
	Find k from $v \geq 0$ (*or* > 0) when $\theta = \pi$:	$k \geq 4$ (*or* $k > 4$)	M1 A1	2	**[7]**
3 (i)	Find R_C by moments for BC about B:	$R_C\, 2a \sin\beta = mg\, a\cos\beta$			
		$R_C = \frac{1}{2}\, mg \cot\beta$ **A.G.**	M1 A1	2	
(ii)	*EITHER:* Moments for system about A:	$R_C\,(2a\sin\alpha + 2a\sin\beta)$			
		$= mg\,(3a\cos\alpha + a\cos\beta)$	M1 A1		
	Substitute for R_C from (i):	$\frac{1}{2}\cos\beta\,(2\sin\alpha + 2\sin\beta)$			
		$= \sin\beta\,(3\cos\alpha + \cos\beta)$	M1 A1		
		$\tan\alpha = 3\tan\beta$ **A.G.**	A1		
	OR: Moments for AB about B:	$R_A\, 2a\cos\alpha = F_A\, 2a\sin\alpha$			
		$+\ mg\, a\cos\alpha$	(M1 A1)		
	Substitute $R_A = 2mg$, $F_A = R_C$:	$4\cos\alpha = (\frac{1}{2}\cot\beta)\sin\alpha + \cos\alpha$	(M1 A1)		
		$\tan\alpha = 3\tan\beta$ **A.G.**	(A1)	5	
(iii)	Find μ_{min} using $F_A \leq \mu R_A$:	$\mu_{min} = \frac{1}{4}\cot\beta = \frac{3}{4}\cot\alpha = \frac{1}{4}\sqrt{3}$	M1 A1	2	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
4 (i)	Use cons. of momentum for 1st collision:	$mu_A + 2mu_B = 2mu$	B1		
	Use Newton's law of restitution:	$u_A - u_B = -e\,2u$	B1		
	Eliminate u_A to find u_B:	$u_B = 2u(1+e)/3$ **A.G.**	M1 A1	4	
(ii)	Use cons. of momentum for 2nd collision:	$2mv_B + mv_C = 2mu_B - mu$	M1		
	Use Newton's law of restitution:	$v_B - v_C = -e\,(u_B + u)$	M1		
	Substitute and solve for v_B :	$v_B = u(1+e)(1-2e)/9$ (A.E.F.)	A1	3	
(iii)	Find u_A:	$u_A = \frac{2}{3}u(1-2e)$	B1		
	State or imply dirns. in which A, B move:	$e > \frac{1}{2}$ so A/B change direction			
	(needs u_A, v_B correct)	in 1st/2nd collision (A.E.F.)	B1		
	Show $\lvert u_A\rvert > \lvert v_B\rvert$: (needs u_A, v_B correct):	$\lvert u_A\rvert / \lvert v_B\rvert = \frac{2}{3}/(1+e)/9$			
		$= 6/(1+e) > 1$ (A.E.F.)	M1 A1	4	**[11]**
5	State or find MI of rod AB (or AD) about A:	$I_{AB} = \frac{1}{3}ma^2 + ma^2 = (4/3)ma^2$	B1		
	State or find MI of rod BC (or CD) about A:	$I_{BC} = \frac{1}{3}ma^2 + m5a^2$ [$=(16/3)ma^2$]	M1		
	Find MI of frame about A:	$I = 2(I_{AB} + I_{BC}) = 40ma^2/3$ **A.G.**	M1 A1	4	
	Use energy to find ang. vel. ω at angle θ:	$\frac{1}{2}I\omega^2 = \frac{1}{2}I\,(6g/5a)$			
	(lose A1 for one incorrect term)	$-\ 4mg\,a\sqrt{2}\,(1 - \cos\theta)$	M1 A2		
	Substitute for I and simplify (A.E.F.):	$\omega = \sqrt{\{(3g/5a)(2 - \sqrt{2}(1 - \cos\theta))\}}$	M1 A1	5	
	Equate $AC\,\omega$ to $k\sqrt{(ga)}$ to find k when $\theta = 90°$:	$k\sqrt{(ga)} = 2\sqrt{2}a\,\sqrt{\{(3g/5a)(2 - \sqrt{2})\}}$	M1 A1		
		$k = 2\sqrt{\{6(2 - \sqrt{2})/5\}} = 1{\cdot}68$	A1	3	**[12]**
6 (i)	State or find by integration F(x):	$F(x) = 1 - e^{-x/6}$ ($x \ge 0$), 0 otherwise	M1 A1	2	
(ii)	State or find mean μ:	$\mu = 1/(1/6) = 6$	B1		
	Find $\pm P(m \le X \le \mu)$ [$m = 4{\cdot}16$ not reqd]:	$F(\mu) - \frac{1}{2} = 1 - e^{-1} - \frac{1}{2}$	M1 A1		
		Reqd. prob. = 0·132	A1	4	**[6]**

Question Number	Mark Scheme Details			Part Mark	Total
7 (i)	State suitable assumption (A.E.F.):	Population is Normal	B1		
	Find confidence interval:	$1110{\cdot}8/10 \pm t\sqrt{(333{\cdot}9/90)}$	M1 A1		
		$= 111{\cdot}1 \pm t\sqrt{3{\cdot}71}$	A1		
	State or use correct tabular value of t:	$t_{9,0.995} = 3{\cdot}25$	A1		
	Evaluate C.I.:	111 ± 6 *or* [105, 117]	A1	6	
(ii)	Compare t, est. variance s and n:	t and s smaller, n larger	M1		
	Deduce effect on width of C.I. (A.E.F.):	Width is less than in **(i)**	A1	2	**[8]**
	S.R. B1 if valid apart from considering n				
8	Find value of p for binomial dist.:	mean = 150/50 = 3, $p = ¾$	M1 A1		
	Find expected binomial values (to 2 d.p.):	0·20 2·34 10·55 21·09 15·82	M1 A1		
	Combine adjacent cells since exp. value < 5:	O: 14 17 19			
		E: 13·09 21·09 15·82	*M1		
	Calculate value of χ^2 (to 2 d.p. ; A1 dep *M1):	$\chi^2 = 1{\cdot}50$	M1 *A1		
	State or use consistent tabular value (to 2 d.p.):	$\chi_{1,\,0.9}{}^2 = 2{\cdot}706$ (cells combined)	*B1		
		[$\chi_{2,\,0.9}{}^2 = 4{\cdot}605$, $\chi_{3,\,0.9}{}^2 = 6{\cdot}251$]			
	Correct conclusion (A.E.F., dep *A1, *B1):	$1{\cdot}50 < 2{\cdot}71$ so distn. does fit	A1	9	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
9	State hypotheses:	H_0: $\mu_P = \mu_Q$, H_1: $\mu_P \neq \mu_Q$	B1		
	Estimate population variance using P's sample: s_P^2 = $(2120 – 321{\cdot}2^2/50) / 49$				
	(allow use of biased: $\sigma_{P,50}^2$ = $1{\cdot}132$ *or* $1{\cdot}064^2$)	[= $1{\cdot}155$ *or* $1{\cdot}075^2$]	M1		
	Estimate population variance using Q's sample: s_Q^2 = $(3310 – 475{\cdot}3^2/70) / 69$				
	(allow use of biased: $\sigma_{Q,70}^2$ = $1{\cdot}182$ *or* $1{\cdot}087^2$)	[= $1{\cdot}199$ *or* $1{\cdot}095^2$]	M1		
	Estimate population variance for combined sample:	$s^2 = s_P^2/50 + s_Q^2/70$			
		$= 0{\cdot}04023$ *or* $0{\cdot}2006^2$			
	(allow use of $\sigma_{P,50}^2$, $\sigma_{Q,70}^2$)	(*or* $0{\cdot}03949$ *or* $0{\cdot}1987^2$)	M1 A1		
	Calculate value of z (to 2 d.p., either sign):	$z = (6{\cdot}424 – 6{\cdot}79) / s$	M1 A1		
		$= –0{\cdot}366/0{\cdot}2006 = –1{\cdot}82[5]$			
		(*or* $–1{\cdot}84$)	A1		
	S.R. Allow (implicit) assumption of equal variances,				
	but deduct A1 if not explicit:				
	Find pooled estimate of common variance s^2:	$(50\sigma_{P,50}^2 + 70\sigma_{Q,70}^2)/118$			
		$= 1{\cdot}180$ *or* $1{\cdot}086^2$	(M1A1)		
	Calculate value of z (to 2 d.p.):	$z = (6{\cdot}424 – 6{\cdot}79)/s\sqrt{(1/50+1/70)}$	(M1 A1)		
		$= –1{\cdot}82$	(A1)		
	State or use correct tabular z value:	$z_{0.95} = 1{\cdot}645$ (to 2 d.p.)	B1		
	Conclusion consistent with values (A.E.F):	Breaking strengths not the same	A1√	10	**[10]**

Question Number	Mark Scheme Details		Part Mark	Total
10	Calculate gradient b in $y - \bar{y} = b(x - \bar{x})$:			
	$b = (47136 - 610 \times 578/8) / (49682 - 610^2/8)$			
	$= 3063{\cdot}5 / 3169{\cdot}5 = 0{\cdot}966[6]$	B1		
	Find regression line of y on x (A.E.F.): $y = 578/8 + 0{\cdot}967\ (x - 610/8)$	M1		
	$= 72{\cdot}2[5] + 0{\cdot}967\ (x - 76{\cdot}2[5])$			
	or $-1{\cdot}45 + 0{\cdot}967x$	A1		
	Calculate gradient b' in $x - \bar{x} = b'\ (y - \bar{y})$:			
	$b' = (47136 - 610 \times 578/8) / (45212 - 578^2/8)$			
	$= 3063{\cdot}5 / 3451{\cdot}5 = 0{\cdot}887[6]$	B1		
	Find regression line of x on y (A.E.F.): $x = 610/8 + 0{\cdot}888\ (y - 578/8)$	M1		
	$= 76{\cdot}2[5] + 0{\cdot}888\ (y - 72{\cdot}2[5])$			
	or $12{\cdot}1 + 0{\cdot}888y$	A1	6	
	Use regression line for x on y at $y = 100$: $x = 101$ [mins]	M1 A1	2	
	S.R. Using regression line for y on x at $y = 100$: $x = 105$ [mins]	(B1)		
	Find correlation coefficient r:			
	EITHER: $r^2 = bb' = 0{\cdot}8580,\ r = 0{\cdot}926$	M1 A1		
	OR: $r = (47136 - 610 \times 578/8) /$			
	$\sqrt{\{(49682 - 610^2/8)(45212 - 578^2/8)\}}$			
	$= 3063{\cdot}5 / \sqrt{(3169{\cdot}5 \times 3451{\cdot}5)}$			
	$= 0{\cdot}926$	(M1 A1)	2	**[10]**

Question Number	Mark Scheme Details			Part Mark	Total
11 (a)	Resolve vertically at equilibrium with extn. e:	$8mge / a = mg \quad [e = a/8]$	B1		
	EITHER: Use Newton's Law at general point:	$m\,d^2x/dt^2 = mg - 8mg(e+x)/a$	M1 A1		
		$[$ *or* $-mg + 8mg(e–x)/a\ ]$			
	Simplify to give ω^2 in $d^2x/dt^2 = -\omega^2 x$:	$d^2x/dt^2 = -(8g/a)\,x$ *or* $\omega^2 = 8g/a$	A1		
	(allow stating result without derivation)				
	OR: Assume SHM and find ω^2 from speed v when first slack, found from energy as below:	$v^2 = \omega^2\{(\tfrac{1}{4}a)^2 - e^2\}$	(M1)		
		$3ga/8 = \omega^2(a^2/16 - a^2/64)$	(A1)		
		$\omega^2 = 8g/a$	(A1)		
	Use $x = \tfrac{1}{4}a\cos\omega t$ *or* $\tfrac{1}{4}a\sin\omega t$ to find ωt:	$\omega t = \cos^{-1}(-\tfrac{1}{2})$ *or* $\tfrac{1}{2}\pi + \sin^{-1}(\tfrac{1}{2})$	M1 A1		
		$= 2\pi/3$	A1		
	Substitute $\omega = \sqrt{(8g/a)}$:	$t = (2\pi/3)\sqrt{(a/8g)}$ **A.G.**	A1	8	
	EITHER: Find v^2 when first slack from an SHM eqn:	$v^2 = \omega^2(a^2/16 - e^2) = 3ga/8$			
		or $\tfrac{1}{4}a\omega\sin 2\pi/3 = 3ga/8$	M1 A1		
	OR: Find v^2 when first slack using energy:	$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\,8mg(e + \tfrac{1}{4}a)^2 / a$			
		$- mg(e + \tfrac{1}{4}a)$			
	(this result may be used above)	$v^2 = 9ga/8 - 3ga/4 = 3ga/8$	(M1 A1)		
	Find further distance s_2 to rest:	$2gs_2 = v^2,\ s_2 = 3a/16$	M1 A1		
	Find total distance:	$\tfrac{1}{4}a + e + s_2 = 9a/16$ *or* $0{\cdot}562[5]a$	M1 A1	6	**[14]**

Question Number	Mark Scheme Details			Part Mark	Total
(b)	Find k by equating area under graph to 1:	$k + 3k = 1,\ k = ¼$	M1 A1		
	Find f(x) for $0 < x \le 2$ and $2 < x \le 5$:	$½kx = x/8$ and $k = ¼$ **A.G.**	B1	3	
(i)	Integrate to find F(x):	$F(x) = x^2/16 \quad (0 \le x \le 2)$			
		$¼x - ¼ \quad (2 < x \le 5)$	M1 A1		
	Relate dist. fn. G(y) of Y to X:	$G(y) = P(Y < y) = P(X^2 < y)$			
	(working may be omitted)	$= P(X < y^{1/2}) = F(y^{1/2})$			
		$= y/16$ and $¼y^{1/2} - ¼$	M1 A1		
	Differentiate to find g(y):	$g(y) = 1/16$ *or* $0{\cdot}0625 \quad (0 \le y \le 4)$			
	(both results reqd. for M1)	$1/8\sqrt{y} \quad (4 < y \le 25)$	M1 A1		
		[0 otherwise]		6	
(ii)	*EITHER:* Find E(Y) using $\int y\, g(y)\, dy$:	$E(Y) = (1/16)\int y\, dy + (1/8)\int y^{1/2}\, dy$	M1		
	Integrate and insert limits:	$= [y^2/32]_0^4 + [y^{3/2}/12]_4^{25}$	A1		
		$= ½ + 117/12 = 10{\cdot}25$ **A.G.**	A1		
	OR: Find E(Y) using $\int x^2 f(x)\, dx$:	$E(Y) = (1/8)\int x^3\, dx + ¼\int x^2\, dx$	(M1)		
	Integrate and insert limits:	$= [x^4/32]_0^2 + [x^3/12]_2^5$	(A1)		
		$= ½ + 117/12 = 10{\cdot}25$ **A.G.**	(A1)	3	
(iii)	*EITHER:* Find median m_x of X and	$F(m_x) = ¼\, m_x - ¼ = ½,\quad m_x = 3$			
	median m_y of Y (or $\sqrt{m_y}$):	$F(m_y) = ¼\, m_y^{1/2} - ¼ = ½,\quad m_y = 9$	M1 A1		
	OR: Show $m_y = m_x^2$:	$P(Y < m_x^2) = P(X^2 < m_x^2)$			
		$= P(X < m_x)$	(M1 A1)	2	[14]