Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

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FURTHER MATHEMATICS **9231/13**

Paper 1 **October/November 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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1 Given that

$$u_k = \frac{1}{\sqrt{(2k-1)}} - \frac{1}{\sqrt{(2k+1)}},$$

express $\sum_{k=13}^{n} u_k$ in terms of n. [4]

Deduce the value of $\sum_{k=13}^{\infty} u_k$. [1]

2 A curve C has parametric equations

$$x = e^t \cos t, \quad y = e^t \sin t, \quad \text{for } 0 \leqslant t \leqslant \tfrac{1}{2}\pi.$$

Find the arc length of C. [6]

3 It is given that $u_r = r \times r!$ for $r = 1, 2, 3, \ldots$. Let $S_n = u_1 + u_2 + u_3 + \ldots + u_n$. Write down the values of

$$2! - S_1, \quad 3! - S_2, \quad 4! - S_3, \quad 5! - S_4.$$ [2]

Conjecture a formula for S_n. [1]

Prove, by mathematical induction, a formula for S_n, for all positive integers n. [4]

4 A curve C has equation $y = \dfrac{2x^2 + x - 1}{x - 1}$. Find the equations of the asymptotes of C. [3]

Show that there is no point on C for which $1 < y < 9$. [4]

5 Find the value of a for which the system of equations

$$\begin{aligned} x - y + 2z &= 4, \\ x + ay - 3z &= b, \\ x - y + 7z &= 13, \end{aligned}$$

where a and b are constants, has no unique solution. [3]

Taking a as the value just found,

(i) find the general solution in the case $b = -5$, [4]

(ii) interpret the situation geometrically in the case $b \neq -5$. [1]

6 Use de Moivre's theorem to show that

$$\cos 5\theta \equiv \cos\theta(16\sin^4\theta - 12\sin^2\theta + 1).$$ [5]

By considering the equation $\cos 5\theta = 0$, show that the exact value of $\sin^2\left(\frac{1}{10}\pi\right)$ is $\dfrac{3 - \sqrt{5}}{8}$. [4]

7 Let $I_n = \int_0^1 (1-x)^n e^x \, dx$. Show that, for all positive integers n,

$$I_n = nI_{n-1} - 1.$$ [3]

Find the exact value of I_4. [4]

By considering the area of the region enclosed by the x-axis, the y-axis and the curve with equation $y = (1-x)^4 e^x$ in the interval $0 \leqslant x \leqslant 1$, show that

$$\tfrac{65}{24} < e < \tfrac{11}{4}.$$ [3]

8 A circle has polar equation $r = a$, for $0 \leqslant \theta < 2\pi$, and a cardioid has polar equation $r = a(1 - \cos\theta)$, for $0 \leqslant \theta < 2\pi$, where a is a positive constant. Draw sketches of the circle and the cardioid on the same diagram. [3]

Write down the polar coordinates of the points of intersection of the circle and the cardioid. [2]

Show that the area of the region that is both inside the circle and inside the cardioid is

$$\left(\tfrac{5}{4}\pi - 2\right)a^2.$$ [6]

9 Given that

$$x\frac{d^2y}{dx^2} + (2x+2)\frac{dy}{dx} + (2-3x)y = 10e^{2x}$$

and that $v = xy$, show that

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} - 3v = 10e^{2x}.$$ [4]

Find the general solution for y in terms of x. [7]

10 The line l_1 is parallel to the vector $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and passes through the point A, whose position vector is $3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. The line l_2 is parallel to the vector $-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and passes through the point B, whose position vector is $-3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find

(i) the length PQ, [5]

(ii) the cartesian equation of the plane Π containing PQ and l_2, [4]

(iii) the perpendicular distance of A from Π. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The roots of the quartic equation $x^4 + 4x^3 + 2x^2 - 4x + 1 = 0$ are α, β, γ and δ. Find the values of

(i) $\alpha + \beta + \gamma + \delta$, [1]

(ii) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$, [2]

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$, [2]

(iv) $\dfrac{\alpha}{\beta\gamma\delta} + \dfrac{\beta}{\alpha\gamma\delta} + \dfrac{\gamma}{\alpha\beta\delta} + \dfrac{\delta}{\alpha\beta\gamma}$. [2]

Using the substitution $y = x + 1$, find a quartic equation in y. Solve this quartic equation and hence find the roots of the equation $x^4 + 4x^3 + 2x^2 - 4x + 1 = 0$. [7]

OR

The square matrix **A** has λ as an eigenvalue with **e** as a corresponding eigenvector. Show that if **A** is non-singular then

(i) $\lambda \neq 0$, [2]

(ii) the matrix $\mathbf{A}^{-1}$ has λ^{-1} as an eigenvalue with **e** as a corresponding eigenvector. [2]

The 3×3 matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} -2 & 2 & -4 \\ 0 & -1 & 5 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = (\mathbf{A} + 3\mathbf{I})^{-1},$$

where **I** is the 3×3 identity matrix. Find a non-singular matrix **P**, and a diagonal matrix **D**, such that $\mathbf{B} = \mathbf{PDP}^{-1}$. [10]