

Cambridge International Examinations

Cambridge International Advanced Level

	CANDIDATE NAME									
* 0 1	CENTRE NUMBER					CANDIDATE NUMBER				
2 3	FURTHER MAT	HEMAT	ICS						923	31/01
4 5	Paper 1					Fo	r Exami	inatior	າ from	2017
6 7	SPECIMEN PAR	PER							3	nours
6 8	Candidates ans	wer on t	he Quest	tion Pa	aper.					
*	Additional Mater	rials:	List of	Formu	lae (MF10))				

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page. Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of 21 printed pages and 1 blank page.



1 The curve *C* is defined parametrically by

 $x = 2\cos^3 t$ and $y = 2\sin^3 t$, for $0 < t < \frac{1}{2}\pi$.

Show that, at the point with parameter *t*,

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{1}{6}\sec^4 t \operatorname{cosec} t.$$
 [4]

2 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2.$$
 [6]

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3 Given that a is a constant, prove by mathematical induction that, for every positive integer n,

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x\mathrm{e}^{ax}) = na^{n-1}\mathrm{e}^{ax} + a^n x\mathrm{e}^{ax}.$$
[6]

		n+5	n + 6	
		$a_n = \frac{n+5}{\sqrt{n^2 - n + 1}}$	$-\frac{1}{\sqrt{n^2+n+1)}}$.	
	۸ <i>۲</i>			
The	sum $\sum_{n=1}^{N} a_n$ is denoted by	by S.		
The	$\sum_{n=1}^{\infty} a_n$ is denoted to	S_{N}		
(i)	Find the value of S_{30} of	correct to 3 decimal pla	aces.	[3]
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(11)	I find the least value of	N for which $S_N > 4.9$.	•	[+]
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The sequence a_1, a_2, a_3, \dots is such that, for all positive integers n,

5 The cubic equation $x^3 + px^2 + qx + r = 0$, where p, q and r are integers, has roots α , β and γ , such that

$$\alpha + \beta + \gamma = 15,$$

$$\alpha^2 + \beta^2 + \gamma^2 = 83.$$

(i) Write down the value of p and find the value of q. [3] (ii) Given that α , β and γ are all real and that $\alpha\beta + \alpha\gamma = 36$, find α and hence find the value of *r*. [5]

6 The matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 10 & -7 & 10 \\ 7 & -5 & 8 \end{pmatrix},$$

has eigenvalues 1 and 3.

(i)	Find corresponding eigenvectors.	[3]
It is	given that $\begin{pmatrix} 0\\2\\1 \end{pmatrix}$ is an eigenvector of A .	
(ii)	Find the corresponding eigenvalue.	[2]

(iii)	Find a diagonal matrix D and matrices P and \mathbf{P}^{-1} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$.	[5]

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7	The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix M , where	ere
7	The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix M , when	ere

$$\mathbf{M} = \begin{pmatrix} 1 & -2 & -3 & 1 \\ 3 & -5 & -7 & 7 \\ 5 & -9 & -13 & 9 \\ 7 & -13 & -19 & 11 \end{pmatrix}.$$

Find the rank of M and a basis for the null space of T .	
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(ii) The vector
$$\begin{pmatrix} 1\\ 2\\ 4 \end{pmatrix}$$
 is denoted by e. Show that there is a solution of the equation $\mathbf{Mx} = \mathbf{Me}$ of the form $\mathbf{x} = \begin{pmatrix} a\\ b\\ -1\\ -1 \end{pmatrix}$, where the constants a and b are to be found. [4]

8	The curve <i>C</i> has equation $y = \frac{2x^2 + kx}{x+1}$, where <i>k</i> is a constant.
	(i) Find the set of values of k for which C has no stationary points. [5]

(ii) For the case k = 4, find the equations of the asymptotes of *C* and sketch *C*, indicating the coordinates of the points where *C* intersects the coordinate axes. [6]

- 9 It is given that $I_n = \int_1^e (\ln x)^n dx$ for $n \ge 0$.
 - (i) Show that

$$I_n = (n-1)[I_{n-2} - I_{n-1}] \text{ for } n \ge 2.$$
 [6]

	Hence find, in an exact form, the mean value of $(\ln x)^3$ with respect to x over the interval	
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10 (i) Using de Moivre's theorem, show that

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$
[5]

(ii)	Hence show that the equation $x^2 - 10x + 5 = 0$ has roots $\tan^2(\frac{1}{5}\pi)$ and $\tan^2(\frac{2}{5}\pi)$. [4]

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11 Answer only **one** of the following two alternatives.

EITHER

The points *A*, *B* and *C* have position vectors **i**, 2**j** and 4**k** respectively, relative to an origin *O*. The point *N* is the foot of the perpendicular from *O* to the plane *ABC*. The point *P* on the line-segment *ON* is such that $OP = \frac{3}{4}ON$. The line *AP* meets the plane *OBC* at *Q*.

(i) Find a vector perpendicular to the plane ABC and show that the length of ON is $\frac{4}{\sqrt{21}}$. [4] (ii) Find the position vector of the point Q. [5]

(iii)	Show that the acute angle between the planes <i>ABC</i> and <i>ABQ</i> is $\cos^{-1}(\frac{2}{3})$. [5]

OR

The curve *C* has polar equation $r = a(1 - \cos \theta)$ for $0 \le \theta < 2\pi$.

(i) Sketch C.

[2]

(ii)	Find the area of the region enclosed by the arc of <i>C</i> for which $\frac{1}{2}\pi \le \theta \le \frac{3}{2}\pi$, the half-line $\theta = \frac{1}{2}\pi$ and the half-line $\theta = \frac{3}{2}\pi$. [5]

(iii) Show that

$$\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^2 = 4a^2\sin^2(\frac{1}{2}\theta),$$

where <i>s</i> denotes arc length, and find the length of the arc of <i>C</i> for which $\frac{1}{2}\pi \le \theta \le \frac{3}{2}\pi$. [7]		

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