Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
	Use Pythagoras	**M1**
	Obtain the given answer	**A1**
	Total:	**3**
2	*EITHER:* State a correct unsimplified version of the x or x^2 term in the expansion of $\left(1+\frac{2}{3}x\right)^{-3}$ or $\left(3+2x\right)^{-3}$ [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$, are not sufficient for **M1**.]	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x)=k(3+2x)^{-4}$	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	Total:	**4**
3	Rearrange as $3u^2+4u-4=0$, or $3\mathrm{e}^{2x}+4\mathrm{e}^{x}-4=0$, or equivalent	**B1**
	Solve a 3-term quadratic for e^x or for u	**M1**
	Obtain $\mathrm{e}^x=\frac{2}{3}$ or $u=\frac{2}{3}$	**A1**
	Obtain answer $x = -0.405$ and no other	**A1**
	Total:	**4**

Question	Answer	Marks
4	Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ \mathrm{d}\theta$	*M1
	Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
	Total:	**4**
5(i)	Use the chain rule	M1
	Obtain correct derivative in any form	A1
	Use correct trigonometry to express derivative in terms of $\tan x$	M1
	Obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4\tan x}{4+\tan^2 x}$, or equivalent	A1
	Total:	**4**
5(ii)	Equate derivative to –1 and solve a 3–term quadratic for tan x	M1
	Obtain answer x=1.11 and no other in the given interval	A1
	Total:	**2**
6(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	**2**
6(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}\left(1/\left(1-x\right)\right)$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	B1
	Total:	**2**
6(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	**3**

Question	Answer	Marks
7(i)	Use correct quotient rule or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = 2$	**A1**
	Total:	**4**
7(ii)	State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
	Obtain answer 2.93 only	**A1**
	Total:	**3**
7(iii)	Explain why the estimate would be less than E	**B1**
	Total:	**1**
8(i)	Justify the given differential equation	**B1**
	Total:	**1**
8(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 0$, $t = 0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x) = kt - \ln 50$, or equivalent	**A1**
	Use $x = 25$, $t = 10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50 - \ln(50-x) = \frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x = 50(1 - \exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

Question	Answer	Marks
9(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A=3$, $B=-2$, $C=-6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2}+\frac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**
9(ii)	Integrate and obtain terms $3\ln x=\frac{2}{x}-2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x+\frac{2}{x}-3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x+\frac{b}{x}+c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**
10(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$, or equivalent	**A1**
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda=\frac{5}{7}$ or $\mu=\frac{3}{7}$	**A1**
	Obtain $m=3$	**A1**
	Total:	**5**

Question	Answer	Marks
10(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $a-2b-4c=0$	**(B1**
	Form a second relevant equation, e.g. $2a+3b-c=0$ and solve for one ratio, e.g. a : b	**M1**
	Obtain final answer $a : b : c = 14 : -7 : 7$	**A1**
	Use coordinates of a relevant point and values of a, b and c and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}-4\mathbf{k})\times(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $14\mathbf{i}-7\mathbf{j}+7\mathbf{k}$	**A1**
	Substitute coordinates of a relevant point in $14x-7y+7z=d$, or equivalent, and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 2: Using a relevant point and relevant vectors, form a 2–parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+s(\mathbf{i}-2\mathbf{j}-4\mathbf{k})+t(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**A1**
	State 3 correct equations in x, y, z, s and t	**A1**
	Eliminate s and t	**M1**
	Obtain answer $2x-y+z=6$, or equivalent	**A1)**
	OR 3: Using a relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & -2 & -4 \\ 2 & 3 & -1 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	Total:	**5**

Question	Answer	Marks
11(a)	Solve for z or for w	M1
	Use $\mathrm{i}^2 = -1$	M1
	Obtain $w = \dfrac{\mathrm{i}}{2-\mathrm{i}}$ or $z = \dfrac{2+\mathrm{i}}{2-\mathrm{i}}$	A1
	Multiply numerator and denominator by the conjugate of the denominator	M1
	Obtain $w = -\frac{1}{5} + \frac{2}{5}\mathrm{i}$	A1
	Obtain $z = \frac{3}{5} + \frac{4}{5}\mathrm{i}$	A1
	Total:	**6**
11(b)	*EITHER:* Find $\pm\left[2 + \left(2 - 2\sqrt{3}\right)\mathrm{i}\right]$	(B1
	Multiply by 2i (or –2i)	M1*
	Add result to v	DM1
	Obtain answer $4\sqrt{3} - 1 + 6\mathrm{i}$	A1)
	OR: State $\dfrac{z-v}{v-u} = k\mathrm{i}$, or equivalent	(M1
	State $k = 2$	A1
	Substitute and solve for z even if i omitted	M1
	Obtain answer $4\sqrt{3} - 1 + 6\mathrm{i}$	A1)
	Total:	**4**