Cambridge International A Level

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MATHEMATICS **9709/31**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	**EITHER:** State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
	Obtain critical values $x = 1$ and $x = 7$	**A1**
	State final answer $x < 1$ and $x > 7$	**A1)**
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = 1$ similarly	**B2**
	State final answer $x < 1$ and $x > 7$	**B1)**
	Total:	**4**
2	**EITHER:** State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	**(M1**
	State correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	**A1)**
	OR: Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = k(1+6x)^{-\frac{4}{3}}$	**(M1**
	Obtain correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ in final answer	**A1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	**B1**
	Total:	**2**
3(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1+e^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	**A1**
	Total:	**4**
4(i)	Use chain rule to differentiate x $\left(\frac{dx}{d\theta} = -\frac{\sin\theta}{\cos\theta}\right)$	**M1**
	State $\frac{dy}{d\theta} = 3 - \sec^2\theta$	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$	**A1**
	Obtain $\frac{dy}{dx} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	**A1**
	Total:	**5**
4(ii)	Equate gradient to −1 and obtain an equation in $\tan\theta$	**M1**
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	**M1**
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
5(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
5(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**
6(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
6(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x + 2y - z = k$	**B1**
	Obtain answer $2x + 2y - z = 2$	**B1**
	Total:	**3**

Question	Answer	Marks
7(i)	State that $u - 2w = -7 - \mathrm{i}$	**B1**
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by 3 – 4i, or equivalent	**(M1**
	Simplify the numerator to 25 + 25i or denominator to 25	**A1**
	Obtain final answer 1 + i	**A1)**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = 1$ or $y = 1$	**A1**
	Obtain final answer 1 + i	**A1)**
	Total:	**4**
7(ii)	Find the argument of $\frac{u}{w}$	**M1**
	Obtain the given answer	**A1**
	Total:	**2**
7(iii)	State that OB and CA are parallel	**B1**
	State that $CA = 2OB$, or equivalent	**B1**
	Total:	**2**
8(i)	Use $\sin(A - B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**
	Collect terms and reach $\sqrt{3}\sin x - 2\cos x$, or equivalent	**A1**
	Obtain $R = \sqrt{7}$	**A1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 49.11°$ with no errors seen	**A1**
	Total:	**5**

Question	Answer	Marks
8(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	**B1 FT**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer 71.3°	**A1**
	[ignore answers outside given range.]	
	Total:	**3**
9(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	**M1**
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	**A1**
	Total:	**2**
9(ii)	Separate variables and integrate one side	**B1**
	Obtain term $\ln y$	**B1**
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	**B2 FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	**A1**
	Obtain answer $y = 1.29$ (3s.f. only)	**A1**
	Total:	**7**
10(i)	State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**
	Using correct double angle formula, express the integral in terms of u and $\mathrm{d}u$	**M1**
	Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2\,\mathrm{d}u$ with no errors seen	**A1**
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

Question	Answer	Marks
10(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**