Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

9022343494

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 **(i)** Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

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(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

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2 The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

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(ii) Find the coordinates of B. [3]

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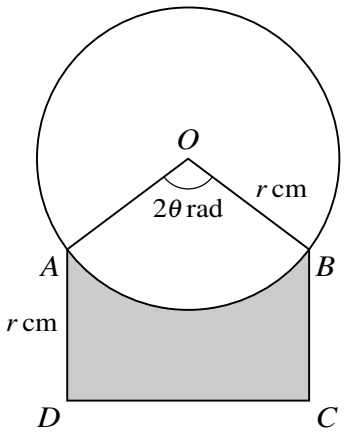
3 **(i)** Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1-\sin\theta}{1+\sin\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

4



The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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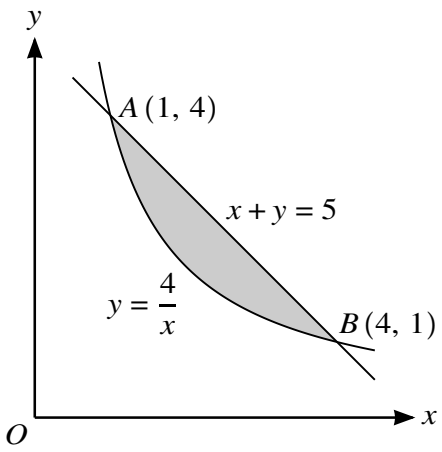
(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

5 A curve has equation $y = 3 + \frac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

6



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \frac{4}{x}$ at the points $A\,(1,\ 4)$ and $B\,(4,\ 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

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7 **(a)** The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

8 Relative to an origin O, the position vectors of three points A, B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) In the case where $p = 2$, use a scalar product to find angle AOB. [4]

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(ii) In the case where $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, find the values of p and q. [4]

9 The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

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(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

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10 The function f is defined by $f(x) = 3\tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

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(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1}. [5]

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(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

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