Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the March 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Remove logarithm and obtain $1+2^x = e^2$	**B1**
	Use correct method to solve an equation of the form $2^x = a$, where $a > 0$	**M1**
	Obtain answer $x = 2.676$	**A1**
	Total:	**3**

Question	Answer	Marks
2	*EITHER*:	
	State or imply non-modular inequality $(x-4)^2 < (2(3x+1))^2$, or corresponding quadratic equation, or pair of linear equations $x-4 = \pm 2(3x+1)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -\frac{6}{5}$ and $x = \frac{2}{7}$	**A1**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**A1)**
	OR:	
	Obtain critical value $x = -\frac{6}{5}$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = \frac{2}{7}$ similarly	**B2**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**B1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Sketch a relevant graph, e.g. $y = e^{-\frac{1}{2}x}$	**B1**
	Sketch a second relevant graph, e.g. $y = 4 - x^2$, and justify the given statement	**B1**
	Total:	**2**
3(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	**M1**
	complete the argument correctly with correct calculated values	**A1**
	Total:	**2**

Question	Answer	Marks
3(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer – 1.41	**A1**
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval (– 1.415, – 1.405)	**A1**
	Total:	**3**

Question	Answer	Marks
4(i)	State $R = 17$	**B1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 61.93°$ with no errors seen	**A1**
	Total:	**3**
4(ii)	Evaluate $\cos^{-1}(4/17)$ to at least 1d.p. (76.39° to 2 d.p.)	**B1**$\sqrt{}$
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer, e.g. x = 7.2°	**A1**
	Obtain second answer, e.g. x = 110.8°and no others	**A1**
	[Ignore answers outside the given interval.]	
	[Treat answers in radians as a misread.]	
	Total:	**4**

Question	Answer	Marks
5	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	**M1**
	Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent	**A1**
	Use the condition for a quadratic to have only one root	**M1**
	Obtain answer $a = 2$	**A1**
	Obtain answer $x = \frac{1}{4}\pi$	**A1**
	Total:	**7**

Question	Answer	Marks
6(i)	Verify that the point with position vector $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ lies in the plane	**B1**
	EITHER:	
	Find a second point on *l* and substitute its coordinates in the equation of *p*	**(M1**
	Verify that the second point, e.g. (3, 1, – 2), lies in the plane	**A1)**
	OR:	
	Expand scalar product of a normal to *p* and the direction vector of *l*	**(M1**
	Verify scalar product is zero	**A1)**
	Total:	**3**

Question	Answer	Marks
6(ii)	*EITHER*:	
	Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a-b+c=0$	**(B1**
	Obtain a second relevant equation, e.g. $3a+b-5c=0$, and solve for one ratio e.g. $a:b$	**M1**
	Obtain $a:b:c=4:13:5$, or equivalent	**A1**
	Substitute (3, – 1, 2) and the values of a, b and c in the general equation and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*1:	
	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i}-\mathbf{j}+\mathbf{k})\times(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i}+13\mathbf{j}+5\mathbf{k}$	**A1**
	Substitute (3, – 1, 2) in $4x+13y+5z=d$, or equivalent, and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*2:	
	Using the relevant point and relevant vectors form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*3:	
	Using the relevant point and relevant vectors form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & -1 & 1 \\ 3 & 1 & -5 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**

Question	Answer	Marks
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	Total:	**5**

Question	Answer	Marks
7(i)	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t} = 2\frac{\mathrm{d}h}{\mathrm{d}t}$	**B1**
	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t} = 1 - 0.2\sqrt{h}$	**B1**
	Obtain the given answer correctly	**B1**
	Total:	**3**
7(ii)	State or imply $\mathrm{d}u = -\frac{1}{2\sqrt{h}}\ \mathrm{d}h$, or equivalent	**B1**
	Substitute for h and dh throughout	**M1**
	Obtain $T = \int_3^5 \frac{20(5-u)}{u}\ \mathrm{d}u$, or equivalent	**A1**
	Integrate and obtain terms $100\ln u - 20u$, or equivalent	**A1**
	Substitute limits $u = 3$ and $u = 5$ correctly	**M1**
	Obtain answer 11.1, with no errors seen	**A1**
	Total:	**6**

Question	Answer	Marks
8(i)	Substitute $z=-1+\mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**
	Use $\mathrm{i}^2=-1$ at least once	**M1**
	Complete the verification correctly	**A1**
	Total:	**3**
8(ii)	State second root $z=-1-\mathrm{i}$	**B1**
	Carry out a complete method for finding a quadratic factor with zeros $-1+\mathrm{i}$ and $-1-\mathrm{i}$	**M1**
	Obtain z^2+2z+2, or equivalent	**A1**
	Attempt division of p(z) by z^2+2z+2 and reach a partial quotient z^2+kz	**M1**
	Obtain quadratic factor z^2-2z+5	**A1**
	Solve 3-term quadratic and use $\mathrm{i}^2=-1$	**M1**
	Obtain roots 1 + 2i and 1 – 2i	**A1**
	Total:	**7**

Question	Answer	Marks
9(i)	State or imply the form $\frac{A}{2+x}+\frac{Bx+C}{4+x^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A=-2$, $B=1$, C = 4	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**

Question	Answer	Marks
9(ii)	Use correct method to obtain the first two terms of the expansion of $(1+\frac{1}{2}x)^{-1}$, $(2+x)^{-1}$, $(1+\frac{1}{4}x^2)^{-1}$ or $(4+x^2)^{-1}$	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	M1
	Obtain final answer $\frac{3}{4}x-\frac{1}{2}x^2$	A1
	[Symbolic binomial coefficients, e.g. ${}_{-1}C_2$, are not sufficient for the first M1. The f.t. is on A, B, C.]	
	[In the case of an attempt to expand $x(6-x)(2+x)^{-1}(4+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	
	Total:	5

Question	Answer	Marks
10(i)	State or imply derivative is $2\frac{\ln x}{x}$	B1
	State or imply gradient of the normal at x = e is $-\frac{1}{2}\mathrm{e}$, or equivalent	B1
	Carry out a complete method for finding the x-coordinate of Q	M1
	Obtain answer $x=\mathrm{e}+\frac{2}{\mathrm{e}}$, or exact equivalent	A1
	Total:	4
10(ii)	Justify the given statement by integration or by differentiation	B1
	Total:	1
10(iii)	Integrate by parts and reach $ax(\ln x)^2+b\int x.\frac{\ln x}{x}\,\mathrm{d}x$	M1*
	Complete the integration and obtain $x(\ln x)^2-2x\ln x+2x$, or equivalent	A1
	Use limits x = 1 and x = e correctly, having integrated twice	DM1
	Obtain exact value e – 2	A1
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e}-2+\frac{1}{\mathrm{e}}$	B1√
	Total:	5