ADDITIONAL MATHEMATICS

Paper 0606/01
Paper 1

General comments

The examination produced a wide variation of marks which covered a wide range of marks. Many candidates were able to make a good attempt at the paper. However, **Questions 8**, **10** and **12 OR** did cause many candidates problems. There were also a significant number of candidates scoring marks in single figures.

Accuracy of working caused quite a few candidates to lose marks unnecessarily. Candidates are requested to give non-exact answers correct to 3 significant figures (or 1 decimal place in the case of angles in degrees), which means they should be working to at least 4 significant places during the solution of the question. Some candidates were working to 1 or 2 significant figures throughout which then affected not only final answers, but answers which required use of previously calculated answers.

Comments on specific questions

Question 1

Most candidates realised that rationalisation of surds was involved and were able to attempt a correct method. Most errors occurred when attempting to simplify the result; some candidates had problems dealing with the – 2 obtained in the resulting numerator.

Answer: $-25 + 18\sqrt{2}$.

Question 2

Many candidates were able to score full marks, showing a good understanding of the syllabus requirements. A few mistakenly attempted to use permutations rather than combinations and were thus unable to achieve any marks.

(i) Most candidates were able to achieve the solution immediately by use of ${}^{10}C_5$; others chose to consider every single possible case, but invariably still obtained the correct answer although a lot more work was required.

(ii) The great majority of candidates realised that they had two different cases to consider and went on to produce a correct solution. Common errors included the addition of an extra case and also the use of ${}^{5}C_1 + {}^{5}C_4$ instead of the correct product of combinations.

Answers: **(i)** 252; **(ii)** 66.

Question 3

(i) Nearly all candidates realised that a quadratic equation in one variable was needed and most were able to obtain the correct quadratic equation. Most realised that they needed to look at the discriminant of the equation although some mistakenly attempted to solve the equation. Many correct values for *k* were obtained although quite a few candidates forgot to include the negative value.

(ii) The majority of candidates who used the above method were able to obtain the correct values of x and went on to obtain the corresponding values of y, however the incorrect point (2, 22) was a common mistake.

Some candidates chose to use an alternative (but equally acceptable) method of equating differentials, which in general was successful provided they were able to differentiate correctly. All this meant was that they obtained their x and y values first and then had to calculate corresponding values of k. Again, some candidates forgot to consider the negative solution to their equation.

Answers: **(i)** 16, –16; **(ii)** (2, –10), (–2, 10).

Question 4

Most candidates attempted to obtain the equation of the straight line in the form of either $y = mx + c$ or $e^y = mx^2 + c$ as an initial approach to the solution of this question

(i) Most candidates were able to obtain a correct value of the gradient and then move on to obtain the correct value of the intercept. There were occasional arithmetic slips involved in this. Some candidates were content to leave their answer as $y = 0.6$, rather than the correct form of $e^y = 0.6$. Some mistakenly thought that the intercept was $e^{0.6}$.

(ii) A good many candidates were able to obtain $e^y = x^2 + 0.6$, but some were unable to use logarithms correctly, preferring to have 2 single logarithms on the right hand side of their equation.

Answers: **(i)** 0.6; **(ii)** $y = \ln(2x^2 + 0.6)$.

Question 5

Most candidates realised that they had to differentiate and a good many were able to do so correctly. Problems involved use of an incorrect formula for either a product or quotient, depending upon how they had decided to deal with equation and incorrect differentiation of tan x. Many candidates then attempted to try to simplify their often correct derivative with the result that unnecessary errors were introduced, even though they then went on to use a correct method. Most realised that they then had to substitute in for x and multiply their result by 2.

Answer: –1.14 units per second.

Question 6

The great majority of candidates were able to produce a correct solution to this question showing a good understanding of the syllabus requirements. There were few errors, usually with candidates leaving their solution in terms of factors, or sign errors from an incorrect factorisation.

Answer: 1, 1.5, –4.

Question 7

Most candidates were able to make a reasonable attempt at this question. This was an example where premature approximation caused problems, although most candidates were able to obtain final answers that were acceptable. Most candidates chose to work in radians, but those that chose to work in degrees usually produced correct and acceptable solutions.

(i) Most were able to obtain a correct answer in radians.

(ii) Very few candidates were unable to make an attempt to find the appropriate lengths and hence the required perimeter.

Answers: **(i)** 1.25 rad; **(ii)** 25.7 cm.

Question 8

This question caused many candidates problems. Parts **(i)** and **(ii)** were meant to help the candidates with part **(iii)**, but many chose to start afresh in part **(iii)** in spite of the use of the word 'Hence', designed to lead candidates in the correct direction.

(i) Many candidates were unable to change bases correctly and an answer of 2 was very common.

(ii) Many candidates were unable to change bases correctly and an answer of 3 was very common.

(iii) Few were able to obtain 2 equations in base 3 logarithms; those that did were usually unable to cope with the logarithms correctly. Completely correct solutions were few, with extra negative solutions introduced and not discarded as an additional error.

Answers: **(i)** $\frac{1}{2}$; **(ii)** $\frac{1}{3}$; **(iii)** $x = 27,\ y = \frac{1}{3}$.

Question 9

(i) There were many good solutions, however many candidates were unable to integrate correctly, answers of the form $2\sin(x^2 - \frac{\pi}{2}x)$ being too common. The constant of integration was very often ignored as well. Some candidates chose to propose a straight line equation for the curve using the given derivative as the gradient.

(ii) Fewer candidates were able to gain a correct solution although many attempted a correct method. Most were able to obtain a perpendicular gradient and try to obtain the equation of a straight line, however many mistakenly used an incorrect value for y and sometimes x as well, using the point given in part **(i)**

Answers: **(i)** $y = \sin\left(2x - \frac{\pi}{2}\right) + 2$; **(ii)** $y = \frac{1}{2}x - \frac{3\pi}{8} + 2$.

Question 10

Many candidates chose not to attempt this question and, if they did, if was very often as an afterthought at the end of their paper when all other questions had been attempted. From the great majority of attempts it is clear that candidates did not cope well with this vector question. Those candidates that were able to progress beyond the first part of the question usually managed to produce reasonable solutions and gain some marks, whereas most candidates failed to score any marks at all.

(i) Most candidates were unable to produce a velocity vector, failing to use the fact that each of the components had an equal magnitude due to the direction. This was usually as far as many got.

(ii) This part relied on part **(i)** and was intended as a check that candidates had the correct vector in part **(i)**. Most were unable to obtain this correctly.

(iii) Again, if candidates had been unable to achieve parts **(i)** and **(ii)** they were highly unlikely to obtain marks in this part. The first two parts were meant to lead the candidates to an appropriate method for part **(iii)**.

(iv) Very few candidates realised what was meant by relative velocity, in spite of it being mentioned specifically as a syllabus requirement.

(v) Those candidates that had managed to provide good solutions for the first three parts of the question were usually able to calculate the required position vector, usually using a method that did not involve relative velocity.

Answers: **(i)** 15**i** + 15**j**, **(iii)** (2 + 15*t*)**i** + (3 + 15*t*)**j**; **(iv)** 15**i** – 10**j**; **(v)** 47**i** + 48**j**.

Question 11

(a) Most candidates were able to obtain an equation in terms of $\tan x$ and go on to obtain correct solutions. There were however errors due to incorrect rounding.

(b) Most candidates were able to use either the correct trigonometric identity or $\sin x$ and $\cos x$ appropriately and obtain a quadratic equation in terms of either $\sec x$ or $\cos x$. Most were able to attempt to factorise and obtain solutions in terms of either $\sec x$ or $\cos x$. There were many correct solutions from $\cos x = 0.75$, but many candidates chose to discount $\sec x = -1$ or $\cos x = -1$ as providing a further solution.

(c) Completely correct solutions were few with many candidates choosing to write that $\sin(2z - 0.6) = \sin 2z - \sin 0.6$. Those that did attempt a correct solution sometimes lost marks due to premature approximation forgetting that the 1 decimal place of working applies to answers in degrees. Some managed to obtain a first solution but were unable to obtain a correct second solution. Those that chose to work in degrees invariable forgot that the 0.6 radians would have to be changed to degrees as well.

Answers; **(a)** 121.0°, 301.0°; **(b)** 41.4°, 318.6°, 180°; **(c)** 0.764 rad, 1.41 rad.

Question 12 EITHER

This alternative was attempted by the great majority of candidates.

(i) Most candidates were able to obtain at least one of the points of intersection, however many forgot that $x^2 = 3$ has two solutions. There were also spurious solutions from attempts to solve $e^{-x} = 0$.

(ii) Most candidates realised that they had to differentiate a product and this was very often done correctly with the occasional slips caused by sign errors when differentiating e^{-x}. Most were able to obtain a quadratic equation in terms of x only and go on to obtain correct x-values for the coordinates of the stationary points. Problems involving accuracy arose again when candidates attempted to find the corresponding y-values and did not leave their answers in terms of e (decimal answers correct to 3 significant figures were acceptable).

(iii) Most candidates made attempts to find the second derivative, but many got muddled as there was usually more than one product involved. Those candidates that had correctly simplified their answer to part **(ii)** usually fared better.

Answers: **(i)** $(\sqrt{3}, 0)$, $(-\sqrt{3}, 0)$; **(ii)** (3, 0.299), (–1, 5.44); **(iii)** maximum, minimum.

Question 12 OR

Few candidates attempted this question and those that did were not usually very successful as most candidates appeared to be unable to differentiate logarithmic functions.

(i) Those candidates that were able to differentiate $\ln(t + 1)$ were usually able to obtain the correct solution.

(ii) Those candidates that were able to differentiate the correct logarithmic function were usually able to obtain the correct solution. However there were those who were under the misapprehension that the differential of $\ln 16$ was $\frac{1}{16}$.

(iii) Those candidates that realised that they had to differentiate again to obtain the acceleration were usually able to obtain the correct solution. Those that had made the error mentioned above involving $\ln 16$, were usually able to recover and obtain full credit for this part.

(iv) Few candidates realised that they had to use their answer to part **(ii)**; those that did were usually successful provided their answer to part **(ii)** was correct.

(v) Very few correct solutions appeared for this part. Most candidates appeared not to realise that they had to find a difference of displacements and the most common response was 1.51.

Answers: **(i)** 1 ms^{-1}; **(ii)** 0.05 ms^{-1}; **(iii)** -0.085 ms^{-2}; **(iv)** 5; **(v)** 0.123 m.

ADDITIONAL MATHEMATICS

Paper 0606/02
Paper 2

General comments

The paper was generally found accessible by candidates. Many candidates scored high marks and there were few poor scripts.

The layout of work was, with some exceptions, good, with questions answered in the correct order (apart from **Question 7** which candidates struggled with and often left to the end). However there were a few candidates who divided the page into two columns which often led to cluttered and confusing working.

There was some sloppy notation, particularly where differentiation and integration were concerned. It was very common to see expressions such as $\int x(x-1)^2 = \int x^3 - 2x^2 + x = \int \frac{x^4}{4} - 2\frac{x^3}{3} + \frac{x^2}{2}$. Constants were frequently omitted and dx and integral signs were used inconsistently.

While **Question 7** was the only question which proved particularly difficult, **Questions 2, 4** and **10** proved testing for some candidates.

Questions 3 and **9** were found particularly straightforward. **Question 11** was also a question on which the weakest candidates were able to obtain some reward. On this question however, almost always the more elegant ways of solving parts **(iv)** and **(v)** were missed and part **(v)** often involved large quantities of complex algebra.

There was often a large variability in the length of responses to questions. For example, whilst some might begin with a concise correct solution to **Question 1**, others might offer a page or more of indeterminate work. **Question 6** was also a question where a brief correct solution might be given, or a response could go on at length with little progress being made.

Comments on specific questions

Question 1

Good candidates found this to be very straightforward. Weaker ones could usually make some progress by finding *a* but often had difficulty with the signs. The simplest way to answer this question was to expand $(x+4)^2$ and then compare coefficients. Some candidates did not appear to understand the method of equating coefficients and often 'solutions' consisted of expressions containing *a*, *b* and *x*.

Those who chose to complete the square of the first expression made more slips and took longer. Since part **(ii)** had only a single mark allocated to it, this was an indication that very little work was required. The connection between parts **(i)** and **(ii)** was often not appreciated with candidates differentiating rather than using the completed square form.

Answers: **(i)** 8, –13; **(ii)** (–4, –13).

Question 2

As with the first question this proved to be a good discriminator. Part **(a)(i)** was done well by many candidates though several thought that they could show *A* and *B* overlapping and then write $\varnothing$ in the intersection part, suggesting they knew the meaning of $\cap$ but perhaps not $\varnothing$. There were many correct solutions in part **(a)(ii)** although some candidates just showed any three sets representing *C*, *D* and *E*.

Part **(b)** was rather poorly done with $X' \cap Y'$ or $(X \cap Y)'$ being the most common answers. Some candidates also included conventional arithmetic symbols such as $(X \cup Y) - (X \cap Y)$ or $(X \cap Y') + (X' \cap Y)$.

Answer: **(b)** $(X \cap Y') \cup (X' \cap Y)$.

Question 3

The vast majority of candidates knew to eliminate one variable with almost all of them sensibly opting to replace *y* by $2x - 3$, although a few wasted time and occasionally made mistakes by first rearranging the equation of the curve. The few candidates who replaced *x* by $\frac{y+3}{2}$ usually made errors when substituting.

Answer: (3, 3), (–1, –5).

Question 4

The offerings for the sketch of the modulus function varied, with sometimes one or other of the branches missing. Some excellent sketches were seen and some candidates drew careful diagrams on graph paper. In part **(ii)** the number of roots offered was, at times, inconsistent with the number shown on the sketch. Sometimes the solutions appeared to have been taken from the sketch, but exact integer values were almost always given.

Answer: **(ii)** 4, 6.

Question 5

(i) Most candidates at least attempted to include the binomial coefficients though many candidates used $3x^3$ in their expansion rather than $(3x)^3$ leading to 168 rather than 1512. A handful of candidates left out the binomial coefficients. Some candidates also wasted time by finding the full expansion whilst a handful appeared not to be aware of the binomial expansion and needed to expand the bracket 8 times though a number of candidates expanded $(1+3x)^2$ and then squared their answer.

(ii) Some candidates did not include both contributions to the required term. A few also did far too much calculation involving other terms, even in a very few cases finding the complete expansion.

Answers: **(i)** 1512; **(ii)** 504.

Question 6

For those candidates who knew the necessary trigonometric definitions this was a straightforward question, although many solutions for part **(b)** included working with both sides of the identity simultaneously. For weaker candidates several sides of working were produced, often going round in circles and not reaching a conclusion. Some answers to part **(a)** were in terms of both *p* and *x*.

(a) Most of the better candidates had little difficulty with this part of this question while the weaker candidates were able to find a suitable expression for $\sec^2 x$ but were often unable to follow it up accurately to a correct expression in terms of *p*. Those who took an approach via $\sec^2 x = 1 + \tan^2 x$ often ended up with an expression containing *p* and $\cos^2 x$.

(b) Correct solutions were usually obtained if candidates converted to sine and cosine but there was sometimes a reluctance to convert the tan *A* term, presumably since tan *A* appeared on the right-hand side.

Answer: **(a)** $\dfrac{1}{1-p^2}$.

Question 7

This was the single most problematic question for the candidates as a whole, and it was not unusual to find it attempted last. It seems that the slightly unusual nature of the question left many candidates not knowing how to start.

(i) Although there were a few completely correct solutions, the commonest answer to this part was simply $x^2 + y^2$ and many candidates failed to give their answers in terms of *x*.

(ii) The greatest success was achieved by those who differentiated $x^2 + \dfrac{32}{x^4}$, although a few did manage to obtain the correct solution from the quotient rule.

(iii) Most candidates who answered the first two parts correctly usually then managed to complete the solution although some stopped at the *y*-value or found $(OP)^2$.

Answers: **(i)** $x^2 + \dfrac{32}{x^4}$; **(ii)** $2x - \dfrac{128}{x^5}$; **(iii)** 2, $\sqrt{6}$.

Question 8

This question was generally answered well, although there were some cases of premature approximation in part **(i)**.

(i) Many candidates were able to solve correctly but the solutions were often rather laborious. Manipulation and substitution of $y = 2^x$ leading to $y = \sqrt{10}$ before taking logarithms was not an uncommon approach. This simply made the solution longer and not simpler. Weaker candidates found that the manipulation of the logarithms to create a correct solution was beyond them even though quite a number realised the need to introduce logarithms.

(ii) The commonest mistake was to write $\dfrac{5^{4y-1}}{5^{2y}} = \dfrac{5^{3y+9}}{5^{4-2y}}$ followed by $\dfrac{4y-1}{2y} = \dfrac{3y+9}{4-2y}$. There were also a number of sign slips and inaccuracies in the answers seen.

Answers: **(i)** 1.66; **(ii)** –2.

Question 9

This question was very well answered and for weaker candidates gained them more marks than any other question. Parts **(i)** and **(ii)** caused very few problems with the occasional difficulty in multiplying by zero and not getting zero. In part **(iii)** candidates had been particularly well trained in finding the inverse of a matrix. The most common mistake was to think that $\mathbf{AX} = \mathbf{B} \Rightarrow \mathbf{X} = \mathbf{BA}^{-1}$. A few candidates worked out the inverse by solving simultaneous equations and were usually successful; only a few candidates forgot to put their final answer as a matrix.

Answers: **(i)** $\begin{pmatrix} 12 & -18 \\ 6 & -4 \end{pmatrix}$; **(ii)** $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$; **(iii)** $\begin{pmatrix} 0.9 & -1.7 \\ -0.6 & 1.8 \end{pmatrix}$.

Question 10

(a) (i) Although a number of candidates wrote $(2x-1)^{-3}$, many of them failed to get the correct multiplier of 2. Some ignored the fact that $(2x-1)^4$ was the denominator and integrated it as though it was a numerator.

(ii) This part was often quite well done with a few candidates even using parts or substitution first. However a number squared out wrongly and hence lost marks. Many candidates thought that they could integrate each term separately with $\frac{x^2}{2}\times\frac{(x-1)^3}{3}$ being common.

(b) (i) Practically all of the candidates knew that they had to use the product rule and the vast majority of them did this correctly obtaining $2(x+4)^{\frac{1}{2}}+2(x-5)\frac{1}{2}(x+4)^{-\frac{1}{2}}$. Although many candidates then completed the solution adequately, a number 'fudged' the answer, being unable to cope with the negative power and/or common denominator concept.

(ii) Although a proportion of candidates managed to get the multiplier $\frac{2}{3}$ there were many that did not. Many candidates failed to see the link with part **(i)** and produced lengthy working with meaningless algebraic manipulation.

Answers: **(a)(i)** $\frac{-2}{(2x-1)^3}+c$, **(ii)** $\frac{x^4}{4}-\frac{2x^3}{3}+\frac{x^2}{2}+c$; **(b)(ii)** $\frac{2(x-5)\sqrt{x+4}}{3}+c$.

Question 11

This was generally a high scoring question. Parts **(i)** and **(ii)** were found most difficult by many candidates and in parts **(iii)** and **(v)** many candidates used both the negative and positive square root, ignoring the restriction on the domain.

(i) There was some confusion between the domain and the range of f, so that answers such as -1 ⌀ f(x) ⌀ 2 were sometimes seen.

(ii) The commonest mistake was to think that $\mathrm{f}^2(1)=[\mathrm{f}(1)]^2$, thus obtaining the value 36.

(iii) This part was generally answered well, provided that candidates did not start answer their solution by expanding the expression for f (x). There were some scripts where a $\pm$ sign was incorrectly left.

(iv) This part was usually completed correctly, although a few candidates stopped after finding an expression for $\mathrm{g}^{-1}(x)$. A minority solved $\frac{20}{x+1}=2$, which was a lot quicker.

(v) Candidates frequently used several sides of paper when the expression $\left(\frac{20}{x+1}+1\right)^2$ was expanded. To their credit, many were still able to obtain the final solution after this mammoth effort.

Answers: **(i)** $\mathrm{f}(x)>2$; **(ii)** 51; **(iii)** $\sqrt{x-2}-1$; **(iv)** 9; **(v)** 3.

Question 12 EITHER

(i) This part was frequently completed correctly. Only weak candidates failed to differentiate in order to find the equation of the tangent. Those who failed to answer this part correctly usually failed to differentiate and attempted to find the gradient using $\frac{y_2 - y_1}{x_2 - x_1}$ or used $m = 4$.

(ii) This part was generally not answered well and for many candidates the only credit achieved was for integrating correctly, with the many candidates finding $\int_0^4 \left(4x - x^2\right)\mathrm{d}x$ instead of $\int_1^4 \left(4x - x^2\right)\mathrm{d}x$. A few candidates were then able to remedy the situation by finding $\int_0^1 \left(4x - x^2\right)\mathrm{d}x$, but others found $\int_{-\frac{1}{2}}^{1} \left(2x + 1 - \left[4x - x^2\right]\right)\mathrm{d}x$ or used other incorrect integrals.

Answers: **(i)** (4, 0), (–0.5, 0); **(ii)** 11.25.

Question 12 OR

Some candidates attempted this question without drawing a sketch, which might have proved helpful. Of the candidates who gained high marks the array method was used most commonly for finding the area, though where wrong working had led to a change in the position of *D*, the quadrilateral was still always treated as *ABCD*. Many candidates wasted time by finding the equations of the lines *AB* and *BC* rather than just using their gradients. A significant number treated *ABCD* as a trapezium.

Answer: 77.