UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

0606 ADDITIONAL MATHEMATICS

0606/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 **(i)** $2a^3 - 7a^2 + 7a^2 + 16 = 0$ leading to $a^3 = -8$, $a = -2$	M1 A1 [2]	M1 for use of $x = a$ and equated to zero, maybe implied
(ii) $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ $= 21$	M1 A1 [2]	M1 for substitution of $x = -\frac{1}{2}$ into their expression or f(x)
2 **(i)** **(ii)** $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$	B1, B1 [2] B2, 1, 0 [2]	B1 for each matrix, must be in correct order –1 for each error
3 $4(2k+1)^2 = 4(k+2)$ $4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}, -1$	M1 A1 M1 A1 [4]	M1 for use of '$b^2 - 4ac$' Correct quadratic expression M1 for correct attempt at solution A1 for both values
4 $(13 - 3y)^2 + 3y^2 = 43$ (or $x^2 + \frac{(13-x)^2}{3} = 43$) $6(2y^2 - 13y + 21) = 0$ (or $2(2x^2 - 13x + 20) = 0$) $(2y - 7)(y - 3) = 0$ (or $(2x - 5)(x - 4) = 0$ $y = 3$ or $\frac{7}{2}$ $\left(x = \frac{5}{2} \text{ or } 4\right)$ (or $x = 4$ or $\frac{5}{2}$ $\left(y = \frac{7}{2} \text{ or } 3\right)$)	M1 A1 DM1 A1,A1 [5]	M1 for eliminating one variable A1 for correct quadratic DM1 for correct attempt at solving quadratic A1 for each correct pair
5 **(i)** $\left(3+\sqrt{2}\right)^2 + \left(3-\sqrt{2}\right)^2 = 22$ AC = $\sqrt{22}$	M1 A1 [2]	M1 for use of Pythagoras Use of decimals M1, A0
(ii) $\tan A = \frac{3-\sqrt{2}}{3+\sqrt{2}}$ $\frac{\left(3-\sqrt{2}\right)\left(3-\sqrt{2}\right)}{\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)} = \frac{11-6\sqrt{2}}{7}$	M1 M1, A1 [3]	M1 for correct ratio M1 for rationalising 2 term denominator

6 **(i)**	$3x^2 - 10x - 8 = 0$ $(3x + 2)(x - 4) = 0$ critical values $-\frac{2}{3}, 4$ $A = \{x : -\frac{2}{3} \leqslant x \leqslant 4\}$	M1 A1 √A1 [3]	M1 for attempt to solve quadratic A1 for critical values Follow through on their critical values.
(ii)	$B = \{x : x \geqslant 3\}$ $A \cap B = \{x : 3 \leqslant x \leqslant 4\}$	B1 B1 [2]	B1 for values of x that define B. B1 (beware of fortuitous answers)
7 **(i)**	${}^{13}C_8 = 1287$	M1, A1 [2]	M1 for correct C notation
(ii)	7 teachers, 1 student : 6 6 teachers, 2 students ${}^7C_6 \times {}^6C_2$: 105 5 teachers, 3 students ${}^7C_5 \times {}^6C_3$: 420 531	B1 B1 B1 B1 [4]	
8 **(i)**	When $t = 0$, $N = 1000$	B1 [1]	
(ii)	$\frac{dN}{dt} = -1000ke^{-kt}$ when $t = 0$, $\frac{dN}{dt} = -20$ leading to $k = \frac{1}{50}$	M1 DM1 A1 [3]	M1 for differentiation DM1 for use of $\frac{dN}{dt} = \pm 20$
(iii)	$500 = 1000e^{-kt}$ $t = -50\ln\frac{1}{2}$ leading to 34.7 mins	M1 M1 A1 [3]	M1 for attempt to formulate equation using half life M1 for a correct attempt at solution (beware of fortuitous answers)
9 **(i)**	$20 \times -2(1 - 2x)^{19}$	B1,B1 [2]	B1 for 20 and $(1 - 2x)^{19}$ B1 for -2 provided $(1 - 2x)^{19}$ is present
(ii)	$x^2\frac{1}{2} + 2x \ln x$ ISW	M1 B1 A1 [3]	M1 for attempt to differentiate a product. B1 for $\frac{1}{x}$ A1 all other terms correct
(iii)	$\frac{x(2\sec^2(2x+1)) - \tan(2x+1)}{x^2}$ ISW	M1 B1 A1 [3]	M1 for attempt to differentiate a quotient. B1 for differentiation of tan $(2x + 1)$ A1 all other terms correct

10 **(i)** $\frac{dy}{dx} = 9x^2 - 4x + 2$ at P grad = 7 tangent $y - 3 = 7(x - 1)$	M1 A1 DM1 A1 [4]	M1 for differentiation A1 for gradient = 7 and $y = 3$ DM1 for attempt to find tangent equation.
(ii) at Q, $7x - 4 = 3x^3 - 2x^2 + 2x$ leading to $3x^3 - 2x^2 - 5x + 4 = 0$ $(x - 1)(3x^2 + x - 4) - 0$ $(x - 1)(3x + 4)(x - 1) = 0$ leading to $x = -\frac{4}{3}, y = -\frac{40}{3}$	M1 B1 DM1 DM1 A1 [5]	M1 for equating tangent and curve equations B1 for realising $(x - 1)$ is a factor DM1 attempt to factorise cubic DM1 for attempt to solve quadratic A1 for both
11 **(a)** $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{1}{\cos\theta\sin\theta}$ $= \text{cos ec}\,\theta\sec\theta$	B1 B1 B1 [3]	B1 for attempt to obtain one fraction B1 for use of an appropriate identity B1 for simplification Scheme follows for alternative proofs
(b) **(i)** $\tan x = 3\sin x$ $\frac{\sin x}{\cos x} = 3\sin x$ $\sin x - 3\sin x\cos x = 0$ leading to $\cos x = \frac{1}{3}$, $\sin x = 0$ $x = 70.5°, 289.5°$ and $x = 180°$	M1 A1√A1 B1 [4]	M1 for use of $\tan x = \frac{\sin x}{\cos x}$ and correct attempt to solve √A1 on their $x = 70.5°$ B1 for x = 180°
(ii) $2\cot^2 y + 3\,\text{cosec}y = 0$ $2(\text{cosec}^2 y - 1) + 3\,\text{cosec}y = 0$ $2\,\text{cosec}^2 y + 3\,\text{cosec}y - 2 = 0$ $(2\,\text{cosec}y - 1)(\text{cosec}y + 2) = 0$ leading to $\sin y = -\frac{1}{2}, y = \frac{7\pi}{6}, \frac{11\pi}{6}$ allow $y = 3.67, 5.76$	M1 M1 M1 A1,A1 [5]	M1 for use of correct identity M1 for attempt to solve quadratic M1 for dealing with cosec/cot Scheme follows for alternative solutions

12 EITHER		
(i) $\pi r^2 h = 1000$, leading to	M1	M1 for attempt to use volume
$h = \dfrac{1000}{\pi r^2}$	A1 [2]	
(ii) $A = 2\pi rh + 2\pi r^2$ leading to given answer	B1	B1 for $A = 2\pi rh + 2\pi r^2$ GIVEN ANSWER
$A = 2\pi r^2 + \dfrac{2000}{r}$	A1 [2]	
(iii) $\dfrac{dA}{dr} = 4\pi r - \dfrac{2000}{r^2}$	M1 A1	M1 for attempt to differentiate given A A1 all correct
when $\dfrac{dA}{dr} = 0$, $4\pi r = \dfrac{2000}{r^2}$	DM1	DM1 for solution = 0
leading to $r = 5.42$	A1 [4]	
(iv) $\dfrac{d^2 A}{dr^2} = 4\pi + \dfrac{4000}{r^3}$	M1	M1 for second derivative method or gradient method'
+ ve when $r = 5.42$ so min value	A1	A1 for minimum, must be from correct work
$A_{min} = 554$	B1 [3]	
12 OR		
(i) $y = x + \cos 2x$	M1	M1 for attempt to differentiate
$\dfrac{dy}{dx} = 1 - 2\sin 2x$	A1	
when $\dfrac{dy}{dx} = 0$, $\sin 2x = \dfrac{1}{2}$	DM1	DM1 for setting to 0 and attempt to solve
leading to $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	DM1 A1,A1 [6]	DM1 for correct order of operations
(ii) Area = $\displaystyle\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} x + \cos 2x.dx$	M1	M1 for attempt to integrate
$= \left[\dfrac{x^2}{2} + \dfrac{1}{2}\sin 2x\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$	A1,A1 DM1	A1for each term correct DM1 for correct use of limits – must be in radians
$= \dfrac{\pi^2}{12}$	A1 [5]	(Trig terms cancel out)