CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

INTERNATIONAL GCSE

MARK SCHEME

MAXIMUM MARK : 80

SYLLABUS/COMPONENT : 0606/2

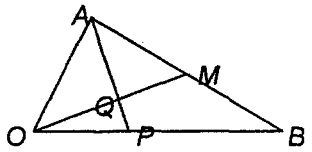
ADDITIONAL MATHEMATICS

(Paper 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 [4]	Inverse = $\begin{pmatrix} 6 & -3 \\ -7 & 4 \end{pmatrix}$ $\quad \times \frac{1}{3}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 & -3 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} -7 \\ -16 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$	B1 B1 M1 A1
2 [4]	$2^6 \quad + 6 \times 2^5 \times x \quad + \frac{6 \times 5}{1 \times 2} \times 2^4 \times x^2$ $= 64 + 192x + 240x^2$ Replace x by $x - x^2$ $\Rightarrow$ coefficient of x^2 = $-192 + 240 = 48$	B2, 1, 0 (-1 each incorrect or missing term) M1 A1 c.s.o.
3 [5]	(i) Either $\dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \times \dfrac{1 + \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$ or $\dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$ Simplify $\Rightarrow$ $2 + \sqrt{3}$ (ii) $\dfrac{1}{p} = \dfrac{1}{2 + \sqrt{3}} \times \dfrac{2 - \sqrt{3}}{2 - \sqrt{3}} = \dfrac{2 - \sqrt{3}}{4 - 3}$ $p - \dfrac{1}{p} = 2 + \sqrt{3} - (2 - \sqrt{3}) = 2\sqrt{3}$ Or $p - \dfrac{1}{p} = 2 + \sqrt{3} - \dfrac{1}{2 + \sqrt{3}} = \dfrac{6 + 4\sqrt{3}}{2 + \sqrt{3}}$ Multiply by $\dfrac{2 - \sqrt{3}}{2 - \sqrt{3}}$ $\Rightarrow 2\sqrt{3}$	M1 A1 M1 A1√ A1 B1√ M1 A1
4 [6]	Solving inequalities: $A \quad x < 3.5$ $B \quad x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = -1, 2$ $\quad\ x^2 - x - 2 > 0 \Rightarrow x < -1,\ x > 2$ Required values $\quad -5 < x < -1$ $\quad 2 < x < 3.5$	B1 M1 A1 A1 M1 A1

5 [6]	**(a)** Either ${}_5C_3 = \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3}$ or ${}_4C_2 = \dfrac{4 \times 3}{1 \times 2}$ Product = $10 \times 6 = 60$ **(b)** Either, ending in 1 (or 3) $\Rightarrow$ $2 \times 5 \times 4$ or, ending in 5 (or 7) $\Rightarrow$ $3 \times 5 \times 4$ Adding all 4 cases $\Rightarrow$ $40 + 40 + 60 + 60 = 200$	B1 M1 A1 B1 M1 A1
6 [6]	**(i)** $f(x) = -(x-1)(x-2)(x-k)$ $f(3) = -2 \times 1 \times (3-k) = 8 \Rightarrow k = 7$ **(ii)** $f(-3) = -(-4)(-5)(-10) = 200$	M1 A1 M1 A1 M1 A1
7 [6]	**(i)** $\dfrac{d}{dx}(x \sin x) = \sin x + x \cos x$ **(ii)** $\int x \cos x \, dx = x \sin x - \int \sin x \, dx$ $\int \sin x \, dx = -\cos x$ $x \sin x + \cos x$ $\dfrac{\pi}{2} - 1 \approx 0.571$	M1 A1 M1 DM1 A1 A1 c.s.o.
8 [6]	**(i)**  [$\rightarrow -\infty$ as $x \rightarrow 0$; thro' (1,0); $\rightarrow \infty$ as $x \rightarrow \infty$] **(ii)** Take logs $\ln x^2 + \ln e^{x-2} = \ln 1$ $\Rightarrow 2 \ln x + x - 2 = 0$. Make $\ln x$ the subject $\Rightarrow$ $\ln x = -\frac{1}{2}(x-2)$ $\Rightarrow$ line is $y = 1 - x/2$	B2,1,0 M1 A1 M1 A1
9 [7]	**(a)** Correct combination of indices Either $(a^{2/3} - a^{1/3}b^{2/3} + b^{4/3}) \times a^{1/3} = a - a^{2/3}b^{2/3} + a^{1/3}b^{4/3}$ Or $(a^{2/3} - a^{1/3}b^{2/3} + b^{4/3}) \times b^{2/3} = a^{2/3}b^{2/3} - a^{1/3}b^{4/3} + b^2$ Sum = $a + b^2$ **(b)** $2^{2x+2} = 4 \times 2^{2x}$ or $2^2 \times 4^x$ $5^{x-1} = 5^x \div 5$ $8^x = 2^{3x}$ $10^x = 4/5$	M1 A1 A1 B2,1,0 (-1 each incorrect or missing term) M1 A1

10 [9]	 **(i)** AP = **b**/3 – **a** OM = **a**/2 + **b**/2	B1 M1 A1
	(ii) $OQ = \lambda$ (**a**/2 + **b**/2)	B1√
	(iii) $OQ = OA + \mu AP$ = **a** + μ (**b**/3 – **a**)	M1 A1√
	(iv) Comparing coefficients $\lambda/2 = 1 - \mu$ and $\lambda/2 = \mu/3$	M1
	Solving $\lambda = \frac{1}{2}$ $\mu = \frac{3}{4}$	M1 A1
11 [11]	**(i)** $v = \int(\frac{3t}{2} - 6)\mathrm{d}t = \frac{3t^2}{4} - 6t \quad (+c)$	M1 A1
	$[v]_{t=0} = 20 \Rightarrow c = 20$ $[v]_{t=4} = 12 - 24 + 20 = 8$	A1 A1
	(ii) $\int(\frac{3t^2}{4} - 6t + 20)\mathrm{d}t = \frac{t^3}{4} - 3t^2 + 20t$	M1 A1√
	$AB = [\ ]_0^4 = 16 - 48 + 80 = 48$	A1
	(iii) $v_B = 8$, $v_C = 20 \Rightarrow t_{BC} = (20 - 8)/2 = 6$	M1 A1√
	(iv)  curve	B1
	straight line	B1√
12 [10] **Either**	$A = \pi r^2 + \pi r l \Rightarrow l = (120 - \pi r^2)/\pi r$	B1 M1
	$V = \frac{1}{2}\pi r^2$ (expression for l) = $60r - \frac{1}{2}\pi r^3$ (AG)	M1 A1
	$\mathrm{d}V/\mathrm{d}r = 60 - 3\pi r^2/2 = 0$ when $r^2 = 40/\pi$, $r \approx 3.57$	B1 M1 A1
	Stationary value of $V \approx 143$ (142.73)	A1
	$\mathrm{d}^2V/\mathrm{d}r^2 = -3\pi r < 0$ for $r > 0 \Rightarrow$ maximum [or any valid method]	M1 A1
Or	**(i)** $\mathrm{d}y/\mathrm{d}x = x^2 \times 1/x + 2x \ln x$	M1 A1
	At Q, $y = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$ $[\mathrm{d}y/\mathrm{d}x]_{x=1} = 1$ c.s.o.	B1 A1
	(ii) At P, $\mathrm{d}y/\mathrm{d}x = 0 \Rightarrow x(1 + 2\ln x) = 0 \Rightarrow \ln x = -\frac{1}{2}$	M1 A1
	$\Rightarrow x = e^{-\frac{1}{2}} = 1/\sqrt{e}$ (≈ 0.6065) (AG)	A1
	(iii) $\mathrm{d}^2y/\mathrm{d}x^2 = \mathrm{d}(x + 2x\ln x)/\mathrm{d}x = 1 + 2\ln x + (2x \times 1/x)$	M1 A!
	$= 3 + 2\ln x$	
	$[\mathrm{d}^2y/\mathrm{d}x^2]_{x=\frac{1}{\sqrt{e}}} = 3 + 2(-\frac{1}{2}) = 2$ c.s.o.	A1