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ADDITIONAL MATHEMATICS **0606/13**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

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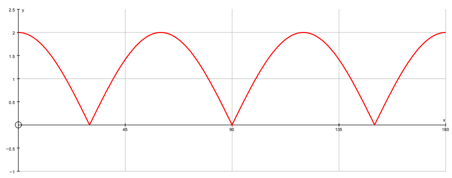
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Part Marks
1		B1 B1 B1	for symmetrical shape as in the diagram with curved maxima of equal height and cusps on the *x*-axis for a complete 'curve' with all low points on the *x*-axis and all high points on $y=2$ for a complete 'curve' meeting the *x*-axis at $x=30^\circ,\ 90^\circ,\ 150^\circ$ only.
2	$= \dfrac{4m^2-9}{2m+3}$	M1	for multiplying each term by $\sqrt{m}$, using a common denominator of $\sqrt{m}$ or for multiplying numerator and denominator by $2\sqrt{m}-\dfrac{3}{\sqrt{m}}$
	$= \dfrac{(2m-3)(2m+3)}{2m+3}$	A1	for a correct expression that will cancel $\dfrac{(2m-3)(2m+3)}{2m+3}$, $\dfrac{(4m^2-9)(2m-3)}{(4m^2-9)}$, $\dfrac{(2m-3)(2m+3)(2m-3)}{(2m+3)(2m-3)}$, or equivalents
	$= 2m-3$	A1	for $2m-3$ or $A=2,\ B=-3$
	Alternative Method $(4m\sqrt{m}-\dfrac{9}{\sqrt{m}})$ $= (2\sqrt{m}+\dfrac{3}{\sqrt{m}})\,(Am+B)$	M1	for correct expansion
	Comparing coefficients $2A=4,\ 3A+2B=0,\ 3B=-9$	A1 A1	for correct comparisons to obtain A and B for $2m-3$ or $A=2,\ B=-3$

Question	Answer	Marks	Part Marks
3 (i)	$3x^2 - 2xp + (p+3) = 0$ $(-2p)^2 - 4\times3\times(p+3) \geqslant 0$ oe	M1	for obtaining a 3-term quadratic in the form $ax^2+bx+c(=0)$
		DM1	for correct substitution of *their* a, b and c into 'b^2-4ac'and use of discriminant.
	$p^2 \geqslant 3(p+3)$ or $4p^2-12p-36 \geqslant 0$ $p^2-3p-9 \geqslant 0$	A1	for full correct working, $\geqslant$ the only sign used, $\geqslant$ used before division by 4 and $\geqslant$ used in answer line and penultimate line.
(ii)	Correct method of solution $p^2-3p-9=0$ leading to critical values	M1	for correct substitution in the quadratic formula or for correct attempt to complete the square. (allow 1 sign error in either method)
	$p = \dfrac{3\pm3\sqrt{5}}{2}$	A1	for both correct critical values
	$p \leqslant \dfrac{3-3\sqrt{5}}{2}$, $p \geqslant \dfrac{3+3\sqrt{5}}{2}$	A1	for correct range
4 (i)	$64 - 48x + 15x^2$	B3	for each correct term
(ii)	$(4\times'64')+(2\times'-48')+(3\times'15')$	M1	for correctly obtaining three products using *their* coefficients in (i)
		A1	for two correct out of three products (unsimplified) cao
	= 205 cao	A1	for 205 selected as final answer
5 (i)	$\log_9 xy = \log_9 x + \log_9 y$	M1	for use of $\log AB = \log A + \log B$
	$= \dfrac{\log_3 x}{\log_3 9} + \dfrac{\log_3 y}{\log_3 9}$	M1	for correct method for change of base. Division by $\log_3 9$ should be seen and not implied.
	$= \dfrac{\log_3 x}{2} + \dfrac{\log_3 y}{2} \quad = \dfrac{5}{2}$		
	$\log_3 x + \log_3 y = 5$	A1	for dealing with 2 correctly and 'finishing off'
	Alternative method $\log_9 xy = \dfrac{5}{2}$	M1	for obtaining xy as a power of 3
	$xy = 9^{\frac{5}{2}} = 3^5$	M1	for correct use of $\log_3$
	$\log_3 xy = 5$ $\log_3 x + \log_3 y = 5$	A1	for using law for logs and arriving at correct answer

Question	Answer	Marks	Part Marks
(ii)	$\log_3 x(5-\log_3 x)=-6$		
	$-(\log_3 x)^2+5\log_3 x=-6$	M1	for substitution, correct expansion of brackets and manipulation to get a 3 term quadratic
	$(\log_3 x)^2-5\log_3 x-6=0$	A1	for a correct quadratic equation in the form $ax^2+bx+c=0$
	leading to $\log_3 x=6$, $\log_3 x=-1$	A1	for both solutions
		DM1	for method of solution of $\log_3 x=k$ or $\log_3 y=k$
	$x=729$, $x=\frac{1}{3}$		
	$y=\frac{1}{3}$, $y=729$	A1	for all x and y correct
6 (i)	$\frac{6x}{3x^2-11}$	M1 A1	**M1** for $\frac{mx}{3x^2-11}$
(ii)	$p=\frac{1}{6}$	B1	**FT** for $p=\frac{1}{m}$
(iii)	$\frac{1}{6}\ln(3a^2-11)-\frac{1}{6}\ln 1=\ln 2$	M1	for correct use of limits in $p\ln(3x^2-11)$ May be implied by following equation
	$\ln(3a^2-11)=\ln 2^6$	DM1	for dealing with logs correctly
	$3a^2-11=64$	DM1	for solution of $3a^2-11=k$
	$a=5$ only	A1	for 5 obtained from an exact method

Question	Answer	Marks	Part Marks
7 (i)	$\ln y = \ln A + \frac{b}{x}$	B1	for equation, may be implied, must be using ln unless recovered
	Gradient: $b = -0.8$	B1	for $b = -0.8$ oe
	Intercept or use of equation: $\ln A = 4.7$	B1	for ln A = 4.7 oe, allow 4.65 to 4.75
	$A = 110$	B1	for A = 110, allow 105 to 116 Allow A in terms of e
	Alternative Method $3.5 = \ln A + 1.5b$ and $1.5 = \ln A + 4b$	B1	for one equation
	leading to $b = -0.8$	B1	for $b = -0.8$
	$\ln A = 4.7$	B1	for $\ln A = 4.7$
	and $A = 110$	B1	for $A = 110$ or $e^{4.7}$
	Alternative Method $e^{1.5} = Ae^{4b}$	B1	for $e^{1.5} = Ae^{4b}$ or $4.48 = Ae^{4b}$
	$e^{3.5} = Ae^{1.5b}$	B1	for $e^{3.5} = Ae^{1.5b}$ or $33.1 = Ae^{1.5b}$
	leading to $b = -0.8$	B1	for $b = -0.8$
	and $A = 110$	B1	for $A = 110$ or $e^{4.7}$
(ii)	When $x = 0.32, \frac{1}{x} = 3.125, \ln y = 2.2$	M1	for a complete method to obtain y, using either the graph, using *their* values in the equation for ln$y$ or using *their* values in the equation for y.
	$y = 9$ (allow 8.5 to 9.5) or $e^{2.2}$	A1	
(iii)	When $y = 20$, $\ln y = 3$, $\frac{1}{x} = 2.125$	M1	for a complete method to obtain x, using either the graph, using *their* values in the equation for ln$y$ or using *their* values in the equation for y.
	so $x = 0.47$ (allow 0.45 to 0.49)	A1	

Question	Answer	Marks	Part Marks
8 (a) (i)	$\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta}=\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}-\sin\theta}$	M1	for using $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$ and either attempt to multiply top and bottom by sin θ or an attempt to combine terms in denominator.
	$=\dfrac{1}{1-\sin^2\theta}$ or $=\dfrac{\frac{1}{\sin\theta}}{\frac{(1-\sin^2\theta)}{\sin\theta}}$	DM1	for correct use of $1-\sin^2\theta=\cos^2\theta$
	$=\dfrac{1}{\cos^2\theta}$ $=\sec^2\theta$	A1	for completing the proof
	Alternative Method using cosec $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta}=\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\frac{1}{\operatorname{cosec}\theta}}$		
	$=\dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta-1}$	M1	for using $\sin\theta=\dfrac{1}{\operatorname{cosec}\theta}$ and an attempt to combine terms in denominator.
	$=\dfrac{1+\cot^2\theta}{\cot^2\theta}$	DM1	for use of $1+\cot^2\theta=\operatorname{cosec}^2\theta$
	$=\tan^2\theta+1=\sec^2\theta$	A1	for completing the proof
(ii)	$\cos^2\theta=\dfrac{1}{4},\ \cos\theta=\pm\dfrac{1}{2}$ or $\tan^2\theta=3,\ \tan\theta=\pm\sqrt{3}$ or $\sin^2\theta=\dfrac{3}{4},\ \sin\theta=\pm\dfrac{\sqrt{3}}{2}$	M1	for using (i) to obtain a value for $\cos^2\theta$, $\tan^2\theta$ or $\sin^2\theta$ and then taking the square root.
	$\theta=60^\circ, 120^\circ, 240^\circ, 300^\circ$	A1 A1	for two correct values for two further correct values and no extras in range.
(b)	$\tan\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{3}}$	M1	for correct order of operations, can be implied by $x=-\dfrac{\pi}{12}$
	$x=\dfrac{\pi}{6}-\dfrac{\pi}{4},\ \dfrac{7\pi}{6}-\dfrac{\pi}{4},\ \dfrac{13\pi}{6}-\dfrac{\pi}{4}$		
	$x=\left(-\dfrac{\pi}{12}\right),\ \dfrac{11\pi}{12},\ \dfrac{23\pi}{12}$	A1,A1	**A1** for $x=\dfrac{11\pi}{12}$ **A1** for $x=\dfrac{23\pi}{12}$ If there are extra solutions in range in addition to the two correct ones then **A1A0**

Question	Answer	Marks	Part Marks
9 (a) (i)	${}^{18}C_5 = 8568$ mmm	B1	
(ii)	**Either**		
	${}^{10}C_4 \times {}^{8}C_1 = 1680$	B1	for a correct plan
	${}^{10}C_3 \times {}^{8}C_2 = 3360$	B2,1,0	**B2** 4 correct numbers with no extras **B1** 3 correct numbers (out of 3 or 4)
	${}^{10}C_2 \times {}^{8}C_3 = 2520$		
	${}^{10}C_1 \times {}^{8}C_4 = 700$		
	Total = 8260	B1	for correct total
	Or		
	their ${}^{18}C_5 - \left({}^{10}C_5 + {}^{8}C_5\right)$	B1	for correct plan
	$8568 - (252 + 56)$	B1	for 252 subtracted
		B1	for 56 subtracted
	Total =8260	B1	for correct total
(b) (i)	${}^{10}P_6 = 151200$	B1	
(ii)	$4 \times {}^{8}P_4 \times 3$	M1	for correct unsimplified
	= 20160	A1	for correct numerical answer
(iii)	Answer to (i) - ${}^{7}P_6$	M1	for correct plan
		A1	for correct unsimplified
	=146160	A1	for correct numerical answer
	Alternative:		
	1 symbol: 45360	B2,1,0	**B2** for all 3 correct **B1** for 2 correct (out of 2 or 3)
	2 symbols: 75600		
	3 symbols: 25200		
	Total: 146160	B1	for correct sum

Question	Answer	Marks	Part Marks
10 (i)	$\mathrm{f}(x)=3x^2-4\mathrm{e}^{2x}\ (+c)$	**M1** **A1** **A1**	for one correct term for one correct term $3x^2$ or $-4\mathrm{e}^{2x}$ for a second correct term with no extras
	passing through $(0,-3)$	**DM1**	for correct method to find c.
	$-3=3\times 0-4\mathrm{e}^0+c$ $\mathrm{f}(x)=3x^2-4\mathrm{e}^{2x}+1$	**A1**	for correct equation
(ii)	$\mathrm{f}'(0)=-8$	**B1**	for $m=\frac{1}{8}$
	Normal: $y+3=\frac{1}{8}x$	**M1**	for equation of normal using $m=\frac{1}{8}$
	$8y+24=x$ $y=2-3x$	**DM1**	for solving normal equation simultaneously with $y=2-3x$ to get a value of x
	leads to $x=\frac{8}{5}$ oe	**A1**	for $x=\frac{8}{5}$, 1.6 oe
	Area = $=\frac{1}{2}\times 3\times\frac{8}{5}=2.4$ oe	**B1**	**FT** for a numerical answer equal to $\left\lvert\frac{1}{2}\times 3\times \text{their } x\right\rvert$
11 (i)	$a=8t-8$ When $t=3$, $a=16$	**B1** **B1**	for $8t-8$ for 16
(ii)	0.5, 1.5	**B1,B1**	**B1** for each
(iii)	$s=\frac{4}{3}t^3-4t^2+3t$	**M1** **A1**	for at least two terms correct all correct
	when $t=\frac{1}{2}$, $s=\frac{2}{3}$	**DM1**	for calculating displacement when either $t=\frac{1}{2}$ or $t=\frac{3}{2}$
	when $t=\frac{3}{2}$, $s=0$	**DM1**	for calculating displacement at $t=\frac{1}{2}$ and doubling.
	total distance travelled = $\frac{4}{3}$	**A1**	for $\frac{4}{3}$ oe allow 1.33
	Alternative method	**M1A1** **DM1**	As before **DM1** for calculating displacement when $t=0.5$ or for calculating distance travelled between $t=0.5$ and $t=1.5$
		DM1	**DM1** for doubling distance travelled between $t=0.5$ and $t=1.5$ or for adding that distance to displacement at $t=0.5$
		A1	**A1** for $\frac{4}{3}$ oe allow 1.33

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