CONTENTS

ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

The standard of work in this examination was pleasing, especially after the difficulties experienced by candidates last year in the first examination of this new style Paper and Syllabus. There was a definite improvement on the ability of candidates to cope with some of the "new" topics, particularly on this Paper with **Questions 6** and **8** on matrices and sets. Unfortunately the same cannot be said for **Questions 2** and **4** on surds and relative velocity – for there were relatively few correct solutions. Overall there were some very good scripts, but although the standard was higher than last year, there were still a significant number of candidates who should not have been entered for the examination.

Comments on specific questions

Question 1

The majority of candidates were able to score the method marks available, realising the need to obtain a quadratic equation in either x or y and to set the discriminant, $b^2 - 4ac$, to 0. Unfortunately the subsequent algebra proved too much for most candidates. The two most common errors were in expressing $(kx)^2$ as kx^2 or $-16(x^2 + 1)$ as $-16x^2 + 16$. Approximately a half of all attempts used $b^2 - 4ac = 0$, instead of $\geqslant 0$. The answers "$k = 0$ and $k > 0$" were both seen as were a few solutions in which the candidates obviously thought they had made an error in obtaining a value of 0.

Answer: $k \geqslant 0$.

Question 2

This was not well answered and it was obvious that a large proportion of candidates had not met the technique of rationalising the denominator by multiplying top and bottom by $\sqrt{3} - \sqrt{2}$. Most of the more successful candidates reduced $\sqrt{12}$ and $\sqrt{18}$ to $2\sqrt{3}$ and $3\sqrt{2}$ respectively and collected terms in $\sqrt{2}$ and $\sqrt{3}$. Unfortunately many of these then left the answer as $2\sqrt{2} - \sqrt{3}$ rather than expressing as $\sqrt{8} - \sqrt{3}$ as requested.

Answer: $\sqrt{8} - \sqrt{3}$.

Question 3

This was well answered by the majority of candidates. The binomial expansion in part **(i)** was nearly always correct, though errors over the "–" occurred when the term in x was given as "$+80x$", instead of "$-80x$". Again, the majority of candidates realised the need to consider two terms in part **(ii)** and most obtained the correct equation "$32 - 80k = -8$".

Answers: **(i)** $32 - 80x + 80x^2$; **(ii)** 0.5 .

Question 4

At least half of all candidates made no significant attempt at the question. Most of the others realised the need to calculate a distance of 54 km from a speed of 36 km h^{-1} travelled in 1.5 hours or to calculate a speed of 60 km h^{-1} from a distance of 90 km. The resulting distance or velocity triangle with an included angle of 45° was only occasionally correct, though an included angle of 135° was seen on a number of occasions. Candidates obtaining the correct triangle, or the one with 135°, confidently used the cosine formula to evaluate the speed of the lifeboat. Some candidates struggled with the notation of 0600 hours and 0730 hours and several scripts were seen in which a time of 130 hours was considered.

Answer: 42.9 km h^{-1}.

Question 5

This was well answered and a source of high marks. Candidates had little trouble in establishing a correct quadratic (usually in x) and in obtaining the x-coordinates of the points of intersection. The majority then obtained the y-coordinates and used a correct formula to obtain the distance. Some weaker candidates took the distance as being the difference between the x-coordinates or used the incorrect formula, usually either $\sqrt{\left((x_2+x_1)^2+(y_2+y_1)^2\right)}$ or $\sqrt{\left((x_2-x_1)^2-(y_2-y_1)^2\right)}$.

Answer: 10.3 units.

Question 6

This was well answered by most candidates. Surprisingly, evaluation of $\mathbf{A}^{-1}$ presented fewer problems than $\mathbf{A}^2$, though (2 x 1) – (0 x – 3) often appeared as 5. The more serious error, occurring in many scripts, was to express $\begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}^2$ as $\begin{pmatrix} 4 & 9 \\ 0 & 1 \end{pmatrix}$ without any evidence of working. The majority of candidates realised the need to subtract $4\mathbf{A}^{-1}$ from $\mathbf{A}^2$ for the last step.

Answer: $\begin{pmatrix} 2 & -15 \\ 0 & -3 \end{pmatrix}$.

Question 7

Very few candidates realised the meaning of "amplitude" or "period" and there were very few marks gained for part **(i)**. The sketches in part **(ii)** were much better and it was pleasing to see that most candidates realised that there were two complete cycles and that the curve oscillated between 3 and 5. Common errors were to use lines instead of curves and to omit evidence of the 3 and the 5. Of the candidates who correctly sketched the curve, most then stated the coordinates of the two maximum points, though some failed to spot this part of the question and others surprisingly quoted the coordinates of the minimum points.

Answers: **(i)** Amplitude 1, Period 180°; **(ii)** Sketch, (90, 5) and (270, 5).

Question 8

Parts **(i)** and **(ii)** were correctly answered by most candidates. The sets were clearly labelled, though to avoid confusion candidates are advised to put labels on the boundaries of sets rather than inside the circles. Most candidates confidently placed the elements 34, 35, 36 and 37 in the correct places on the Venn diagram. Solutions to part **(ii)** suffered mainly through not understanding that the notation n(A) refers to the number of elements in A, but also through giving answers based on the subset {34, 35, 36, 37} rather than on the Universal set. It was also apparent that very few candidates realised that the set $O \cup S$ referred to "all odd numbers together with even square numbers".

Answers: **(i)** Sketch; **(ii)** Sketch; **(iii)** $n(O \cap S) = 4$, $n(O \cup S) = 54$.

Question 9

Solutions varied significantly in standard. In part **(i)** just under a half of all attempts were completely correct, usually by recognising that $\log_9(2x + 5) = 1.5$. Others preferred to convert all the logarithms to base 10 and following considerable work, some arrived at $\lg(2x + 5) = 1.431$. Weaker candidates failed to recognise the need to either evaluate two of the terms or endeavoured to produce incorrect work such as $\log_9(2x + 5) = \log_9 2x + \log_9 5$ etc. Many such errors over the laws of logarithms were common.

Solutions to part **(ii)** followed one of two paths. Either the candidates attempted to take logarithms of each term and scored zero or attempted to express the given equation as an equation in 3^y. This usually led to the correct quadratic, though $9^y = 3.3^y$ leading to a linear equation was common. The solution of $3^y = k$ was accurately manipulated, though a large number of extra solutions appeared from the negative root of the equation.

Answers: **(i)** 11; **(ii)** 1.46 (or 1.47).

Question 10

Virtually all candidates knew how to draw the graph and generally obtained a straight line graph. Choice of scale led to some errors however, particularly when a scale of 1 cm = 3 units was chosen for the *xy*-axis. A few candidates also plotted the x^2-axis with equal intervals between 0, 1, 4, 9, etc. Less than half of all attempts realised that the equation linking *x* and *y* was $xy = mx^2 + c$ and not $y = mx + c$. Values of *m* and *c* were generally accurate. A few solutions were seen in which x^2 was plotted on the vertical axis and *xy* on the horizontal axis. This was perfectly acceptable providing that the equation $x^2 = mxy + c$ was then applied. Many candidates failed to read the question fully in not making *y* the subject. Solutions to the last part were split between those who read directly from the graph at $xy = 45$ and those who substituted into the equation of the line. Answers were generally accurate but a large number arrived at $x = 21$ instead of $x^2 = 21$.

Answers: Graph; **(i)** $y = 1.6x + \frac{12}{x}$; **(ii)** *x* between 4.5 and 4.6.

Question 11

Attempts at this question tended to vary from Centre to Centre. Some excellent answers were seen, but the majority of candidates scored very badly. A significant number of candidates failed to realise the need to differentiate xe^{2x} as a product, for although the differential of e^{2x} was generally correct, results such as $\frac{dy}{dx} = 2xe^{2x}$ or $\frac{dy}{dx} = 2e^{2x}$ occurred in more than half of the solutions seen. Most candidates realised the need to set the differential to 0 for stationary points and to look at the sign of the second differential to determine the nature of the stationary point.

Answers: **(i)** $x = -0.5$; **(ii)** $k = 4$; **(iii)** Minimum.

Question 12 EITHER

This was the less popular of the two alternatives, yet candidates generally scored more than half marks. Most candidates realised the need to differentiate and to set the differential to 0 to find the *x*-coordinate of *B*. The standard of differentiation was good, though only about a half of all attempts realised that $\frac{dy}{dx} = 0 \Rightarrow \tan x = 0.5$. Common errors were to take $\tan x = \pm 2$ or -0.5 or to fail to recognise that the equation reduced to a single tangent. Most candidates left *x* in degrees but this lost no marks providing a value in radians was used for the very last part. Unfortunately the area of the rectangle was all too often expressed as 26.6 x 4. The integration of *y* was generally accurate, though $\int 2\sin x\,dx = 2\cos x$ was a common error. Most candidates correctly attempted to use the limits 0 to $\tan^{-1} 0.5$ but a large proportion automatically assumed that they could ignore the value at $x = 0$.

Answer: 0.144 .

Question 12 OR

This was the more popular alternative and generally, like **Question 12 EITHER**, a source of reasonable marks. Candidates realised the need to differentiate $y = \sqrt{1+4x}$ to find the gradient of the curve and hence the equation of the tangent. Differentiation was generally correct, though $\sqrt{1+4x}$ was often taken as $(1 + 4x)^{-1}$ or $(1 + 4x)^{-0.5}$ or even as $1 + 2\sqrt{x}$. Many candidates failed to include the “x 4” from the differential of the bracket. It was pleasing that virtually all candidates realised the need to obtain a numerical value for the gradient before using $y = mx + c$ or an equivalent equation. Integration of $\sqrt{1+4x}$ was again well done, and surprisingly fewer candidates omitted the “$\div$4” than had omitted the “x 4” earlier. Division by $\frac{3}{2}$ was often incorrect but the main error, even from more able candidates, was to assume that the lower limit of 0 could be ignored. Candidates were divided in their methods for finding the area under the line between using the area of a trapezium or using integration.

Answer: $\frac{1}{3}$.

Paper 0606/02

Paper 2

General comments

The length and difficulty of the Paper seemed to be appropriate. Few candidates failed to attempt all the questions. Average candidates were able to find sufficient questions where standard techniques allowed them to score reasonably well. There were also some sections containing rather more searching material which tested the ability of the better candidates.

Comments on specific questions

Question 1

(i) Most candidates understood how to proceed but algebraic errors sometimes prevented them reaching the required equation legitimately e.g. $4\left(\frac{-b}{2}\right)^4 .... - b^2\left(\frac{-b}{2}\right)^2 .... = -\frac{4b^4}{16} .... + \frac{b^4}{4} ....$ A few replaced b by $-2x$ and some tried long division but were rarely successful. Some weaker candidates thought that demonstrating that $b = -1$ or -2 was a root established the validity of the equation.

(ii) There were many correct solutions. Nearly all candidates were able to determine that either $b + 1$ or $b + 2$ was a factor. Those using synthetic division to find the quadratic factor were usually successful but those attempting long division were sometimes defeated by the absence of a term in b. Again, candidates taking the quadratic factor to be of the form $pb^2 + qb + r$ were usually able to evaluate p, q and r correctly. Some weaker candidates obtained the values -1, -2 fortuitously by taking $b^2(3b + 7) = 4$ to imply $b^2 = 4$ or $3b + 7 = 4$. It was illuminating to see that many candidates felt it necessary to change $3b^3 + 7b^2 - 4 = 0$ into $3x^3 + 7x^2 - 4 = 0$ in order to deal with it.

Answer: **(ii)** $-2, -1, \frac{2}{3}$.

Question 2

(i) Most candidates knew how to find $\overrightarrow{AB}$, although errors did occasionally arise through misreading e.g. 6**i** + 3**j**, absence of brackets e.g. ... – 6**i** – 3**j**, or misuse of brackets e.g. ... – (6**i** – 3**j**) = ... – 6**i** – 3**j**. Weak candidates simply added the position vectors of A and B. The idea of a unit vector was unclear to many of the weaker candidates: some stated that 9**i** + 12**j** was the unit vector whilst others, having calculated $\sqrt{(9^2+12^2)}$, took 15 to be the unit vector or did not know how to make use of it.

(ii) A common error by those candidates attempting to express $\overrightarrow{AC}$ as a fraction of $\overrightarrow{AB}$ was to take $\overrightarrow{AC}$ to be $\frac{1}{3}\overrightarrow{AB}$. Candidates proceeding from the equation $\overrightarrow{AC} = 2\overrightarrow{CB}$ via $\overrightarrow{AO} + \overrightarrow{OC} = 2(\overrightarrow{CO} + \overrightarrow{OB})$ to $3\overrightarrow{OC} = \overrightarrow{OA} + 2\overrightarrow{OB}$ were usually successful, although there were some errors in the removal of brackets, e.g. $2(\overrightarrow{CO} + \overrightarrow{OB}) = 2\overrightarrow{CO} + \overrightarrow{OB}$, and with the signs and directions of vectors. Some candidates took the position vector of C to be $\overrightarrow{AC}$ rather than $\overrightarrow{OC}$.

Answers: **(i)** 0.6**i** + 0.8**j**; **(ii)** 12**i** + 5**j**.

Question 3

A few weak candidates failed to perform any integration, a few differentiated rather than integrated and a few made errors in the coefficient e.g. $\int 3x^{1/2}dx = \frac{3x^{1/2}}{3\times 3/2}$ and/or $\int 2x^{-1/2}dx = \frac{2x^{1/2}}{2\times 1/2}$, but overall the integration was very well done as was the use of limits. However, a large number of candidates failed to obtain full marks; this was almost always due to an inability to express $2x^{3/2}$, when $x = 8$, as a multiple of $\sqrt{2}$.

Answer: $-6 + 40\sqrt{2}$.

Question 4

Probably no more than half the candidates were able to obtain full marks. Careless errors were rife, e.g. $2^{4(x+1)} = 2^{4x+1}$, $2^{3(x+2)} = 2^{3x+2}$, $20(4^{2x}) = 80^{2x}$ and $2^{x-3}8^{x+2} = 16^{2x-1}$. A few candidates took logs whilst others dispensed with the base number so that, e.g, $\frac{2^{4x+4}+5(2^{4x+2})}{2^{4x+3}}$ became $\frac{4x+4+5(4x+2)}{4x+3}$. Cancellation was sometimes incorrect, e.g. $\frac{2^{4x+4}+20(2^{4x})}{2^{4x+3}}$ becoming $2 + 20(2^{4x})$. Some candidates, having achieved the value 4.5, went on to solve $2^x = 4.5$.

Answer: 4.5.

Question 5

(i) Most candidates attempted to find an algebraic expression for $f^2(x)$ before substituting $x = 0$, with only a small minority taking the easier route of finding $f(0) = \frac{1}{2}$ and then evaluating $f(\frac{1}{2})$. Expressing $f^2(x)$ algebraically proved too difficult for some candidates and others took $f^2(x)$ to be $\{f(x)\}^2$.

(ii) Weaker candidates were unable to make a sensible attempt at f^{-1}. Those who appreciated that logarithms were involved frequently took $e^x = 4y - 1$ to imply $x = \ln 4y - \ln 1 = \ln 4y$. Another common error was to omit brackets, so that $e^x = 4y - 1$ became $x = \ln 4y - 1$ rather than $x = \ln(4y - 1)$.

(iii) Very few candidates understood the relationship between the range and domain of f and the domain and range of f^{-1} and thus completely correct answers were rare. The domain of f^{-1} was frequently omitted but the candidates attempting to find it usually gave the answer $x > \frac{1}{4}$, from considering $4x - 1 > 0$. The range of f^{-1} was stated correctly more often than the domain of f^{-1}.

Answers: **(i)** 0.662; **(ii)** $\ln(4x - 1)$; **(iii)** $x \geqslant \frac{1}{2}$, $f^{-1}(x) \geqslant 0$.

Question 6

(i) Very few candidates were unable to obtain the critical values 2, 6. The inequality $(x - 2)(x - 6) > 0$ almost always led to $x > 2$, $x > 6$, which weaker candidates frequently took to be the answer whilst better candidates explored the situation to arrive correctly at $x < 2$ or $x > 6$, often stated as $2 > x > 6$.

(ii) This proved to be slightly easier than part **(i)** with most candidates obtaining $0 < x < 8$, although weak candidates divided through by x, obtaining merely $x < 8$.

(iii) Some candidates, perhaps intuitively, assumed that the answer to this part was a combination of the answers to parts **(i)** and **(ii)** and proceeded to the correct answer. Others ignored the modulus signs to obtain, as in part **(ii)**, $x^2 - 8x < 0$ and some tried to remove the modulus signs by squaring, e.g. $x^2 - 8x + 6 < 36$. The inequality $| x^2 - 8x + 6 | < 6$ was often taken to imply $x^2 - 8x + 6 < 6$ and, erroneously, $x^2 - 8x + 6 < -6$. The latter gives $x^2 - 8x + 12 < 0$ and should lead to $2 < x < 6$ but quite often led to the solution set of part **(i)**.

Answers: **(i)** $\{x : x < 2\} \cup \{x : x > 6\}$; **(ii)** $\{x : 0 < x < 8\}$; **(iii)** $\{x : 0 < x < 2\} \cup \{x : 6 < x < 8\}$.

Question 7

(i) This part was almost always answered correctly.

(ii) 5! was usually present, but this was sometimes multiplied by 6 to give 720.

(iii) 4! was usually present, but this was frequently multiplied by 1×1 to give 24 or by 2×2 to give 96.

(iv) This was often answered correctly. A commonly occurring incorrect answer was 360 derived from ${}_6P_4$.

(v) Correct answers to this part were less frequent than to part **(iv)**, with 60 often occurring either from ${}_5P_3$ or from ${}_6P_1 \times {}_5C_3$.

Answers: **(i)** 720; **(ii)** 120; **(iii)** 48; **(iv)** 15; **(v)** 10.

Question 8

(a) Many candidates were able to answer this correctly. A few made careless errors, arriving at $\sin x = \pm \cos x$, while $\frac{3(\sin x - \cos x)}{\sin x + \cos x} \times \frac{\sin x - \cos x}{\sin x - \cos x}$ was sometimes employed, invariably leading to failure. Some candidates introduced extraneous answers by proceeding from $\sin x - 5\cos x$ to $\sin x (1 - 5\cot x) = 0$ or $\cos x (\tan x - 5) = 0$. Answers were sometimes inaccurate with 78.69 … truncated to 78.6 rather than rounded to 78.7.

(b) Here again there were many correct solutions. Some candidates succeeded after taking the rather odd initial step of replacing 1 by $\sin^2 y + \cos^2 y$. A few weak candidates came to the conclusion that the only solution was 180° by arguing that $\sin y (3 \sin y + 4) = 4$ implies $\sin y = 4$ or $3 \sin y + 4 = 4$. Other candidates were unable to obtain any solutions within the stated range through error in dealing with the quadratic equation, e.g. $\sin y = -\frac{2}{3}$ or $+ 2$. Accuracy proved difficult for some candidates in that they believed that angles in *radians* should only be given correct to 1 decimal place.

Answers: **(a)** 78.7°, 258.7°; **(b)** 0.730, 2.41 .

Question 9

In general this was answered poorly with very many candidates unable to distinguish between the *variable* acceleration of the first period and the *constant* deceleration of the final period. Thus graphs consisting of three straight line segments were extremely common, with the total distance travelled being calculated as the area of the trapezium so formed. Frequently candidates producing answers in this way had earlier arrived at, and deleted, $s = 6t^2 - \frac{1}{3}t^3$. Some candidates found t to be 12 from $12t - t^2 = 0$, drew the parabola $v = 12t - t^2$ from $t = 0$ to $t = 12$ and, integrating over this interval, found the total distance to be 288 m. Others found the total time to be 21s, either from $t_3 = \frac{36}{4}$ or from $4 = \frac{36}{t - 12}$, and evaluated the integral over the interval $t = 0$ to $t = 21$. The time, t_3, for deceleration was frequently taken to be 4s or 8s by taking the variable acceleration, $12 - 2t$, of the first part of the motion to be equal to ± 4. Some candidates appreciated that the graph corresponding to the first part of the motion was a curve but a few of these took it to be a section of a minimum curve rather than a maximum.

Answer: **(i)** 522 m.

Question 10

(i) Very few candidates misquoted or misused the quotient rule so that differentiation was usually correct: relatively few candidates opted to use the product rule on $(2x + 4)(x - 2)^{-1}$. Most errors in evaluating k were caused by carelessness e.g. as in the following:

$2(x - 2) = 2x + 4$, $2(x - 2) - (2x + 4) = 2(x - 2) - 2x + 4$, and $2(x - 2) - (2x + 4) = 2x - 2 - 2x - 4$.

Those candidates arriving at $k = 0$ were not unduly alarmed but continued blithely onwards.

(ii) Some candidates thought the curve crossed the x-axis when $x = 0$. Others took $\frac{2x+4}{x-2} = 0$ to imply $2x + 4 = 0$ or $x - 2 = 0$ leading to $x = -2$ or 2, although the latter value was quickly abandoned; again, $\frac{2x+4}{x-2} = 0$ sometimes led to $2x + 4 = x - 2$ and $x = -6$. Few candidates failed to proceed from the gradient, m, of the tangent to the gradient, $-\frac{1}{m}$, of the normal, although in some cases this was left as a function of x.

(iii) Most candidates knew how to use the chain rule to find $\frac{dy}{dt}$ and rarely was the relationship between the rates of change misquoted or misused. Occasionally miscalculations arose through taking x, rather than y, to be 6.

Answers: **(i)** -8; **(ii)** $y = 2x + 4$; **(iii)** -0.1 units per second.

Question 11 EITHER

This was by far the less popular of the two alternatives and very few candidates scored full marks, although many obtained the coordinates of B correctly. Those failing to do so generally understood the principles involved but were unable to avoid some elementary arithmetical error. A few very weak candidates became confused as to which equations they were solving and, for instance, solved the equations of AC and BC to find B, or attempted to solve equations of the parallel lines AC and BD. Hardly any candidate spotted the similar triangles AEC and ABD and used $\frac{AC^2}{BD^2}$ to give the ratio of their areas; those who did calculate this ratio, $\frac{1}{4}$, usually took it to be the ratio of the area of the quadrilateral $ABDC$ to the area of the triangle EBD. Some candidates calculated the lengths of AC and BD but only as a step, when combined with the length of BC, in finding the area of the quadrilateral $ABDC$. These candidates then had to embark on the rather lengthy, and usually unsuccessful, process of finding the coordinates of E, a process made difficult by the rather awkward equation of DC, $13y = 14x - 46$. Having found E a few successfully employed the array method to find the area of triangle EBD, but others assumed that this area was given by $\frac{1}{2}ED \times BC$.

Answers: **(i)** (5.5, 7), **(ii)** 3 : 4.

Question 11 OR

Even weaker candidates usually scored reasonably well on this alternative. Pythagoras' theorem was almost always correctly applied resulting in $r = 6$. Most candidates knew how to find angle AOB with only a few careless errors, e.g. $\sin AOB = \frac{5}{7}$. The expressions for arc length and area of sector were used by virtually every candidate although in a few cases the angle used was in degrees rather than radians. The most frequent loss of marks was due to premature approximation in taking angle AOB to be 0.39 or 0.4 radians.

Answers: **(i)** 2.37 cm; **(ii)** 22.9 cm^2.