MARK SCHEME for the May/June 2013 series

0606 ADDITIONAL MATHEMATICS

0606/22   Paper 2, maximum raw mark 80

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Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1, 2 This is deducted from A or B marks when essential working is omitted.

PA –1   This is deducted from A or B marks in the case of premature approximation.

S –1   Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1   Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1			$m = \dfrac{18-3}{4-1}$ or 5 soi	**M1**	or $18 = 4m + c$ and $3 = m + c$ subtracting/substituting to solve for m or c, condone one error
			$Y - 3 = their\ 5(X - 1)$ or $Y - 18 = their\ 5(X - 4)$ or $3 = their\ 5 + c$ or $18 = their\ 5 \times 4 + c$	**M1**	or using *their m* or *their c* to find *their c* or *their m*, without further error
			$\sqrt{y} = (their\ m)\,x^2 + (their\ c)$ or $\sqrt{y} = (their\ m)(x^2 - 1) + 3$ or $\sqrt{y} = (their\ m)(x^2 - 4) + 18$	**M1**	*their m* and *c* must be validly obtained
			$y = (5x^2 - 2)^2$ or $y = (5(x^2 - 1) + 3)^2$ or $y = (5(x^2 - 4) + 18)^2$ cao, isw	**A1**	
2	(a)		$(p + 1)\ln 3 = \ln 0.7$	**M1**	or $p + 1 = \log_3 0.7$ or $p\ln 3 = \ln\left(\dfrac{0.7}{3}\right)$
			$p = \dfrac{\ln 0.7}{\ln 3} - 1$ or $p = \dfrac{\lg 0.7}{\lg 3} - 1$	**M1**	or $p = \log_3 0.7 - 1$ or $p\ln 3 = \ln\left(\dfrac{0.7}{3}\right) \div \ln 3$
			-1.32 cao	**A1**	allow **M2** for $p = \log_3\left(\dfrac{0.7}{3}\right)$ correct answer only scores **B3**
	(b)		$2^{\frac{5}{2}} \times x^6 \times y^{-\frac{1}{2}}$ or $a = \dfrac{5}{2}$, $b = 6$, $c = -\dfrac{1}{2}$	**B3**	**B1** for each component
3	(a)	(i)	A and E	**B2**	1 mark for each **B1** for 1 extra, **B0** if 2 or more extras
		(ii)	C and D	**B2**	1 mark for each **B1** if 1 extra, **B0** if 2 or more extras
	(b)			**B2**	$(-1, 0)$, $(1, 3)$, $(3, 4)$ or **B1** for two points correct and joined or for three points correct but clearly not joined

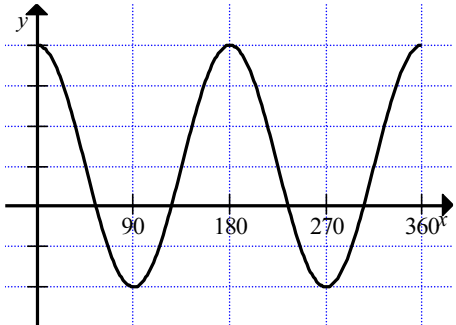| 4	(i)	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ or	**B1**	or $3\overrightarrow{AC} = 3(c_1 - 4)\mathbf{i} + 3(c_2 + 21)\mathbf{j}$
		$\overrightarrow{OB} - \overrightarrow{OA} = 3(\overrightarrow{OC} - \overrightarrow{OA})$ soi		o.e. soi
		$\pm(18\mathbf{i} - 9\mathbf{j})$ o.e. or $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA} + \frac{1}{3}\overrightarrow{OB}$	**B1**	
		$4\mathbf{i} - 21\mathbf{j} + \frac{1}{3}(their(18\mathbf{i} - 9\mathbf{j}))$ o.e. or	**M1**	or $3(c_1 - 4) = their\ 18$ and $3(c_2 + 21) = their\ (-9)$
		$\frac{2}{3}(4\mathbf{i} - 21\mathbf{j}) + \frac{1}{3}(22\mathbf{i} - 30\mathbf{j})$		
		$10\mathbf{i} - 24\mathbf{j}$ cao	**A1**	
	(ii)	$\left|\overrightarrow{OC}\right| = \sqrt{their10^2 + their(-24)^2}$ soi	**M1**	condone $\left|\overrightarrow{OC}\right| = \sqrt{their10^2 + their(24)^2}$
		$\frac{1}{13}(5\mathbf{i} - 12\mathbf{j})$ or $\frac{1}{26}(10\mathbf{i} - 24\mathbf{j})$ isw	**A1 FT**	FT their $x\mathbf{i} + y\mathbf{j}$ o.e.
5		$AX = \sqrt{45}$	**B1**	may be implied by $3\sqrt{5}$
		$AX = 3\sqrt{5}$	**B1**	may be seen later
		$\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times their\ \sqrt{45}$ soi	**M1**	may be implied by e.g. summation of rectangle and two triangles
		$15(\sqrt{5} + 2) = \frac{1}{2}(4 + \sqrt{5} + 2 + x) \times their\ \sqrt{45}$ or better	**M1**	
		Correctly divide *their* equation by *their* $\sqrt{5}$ or *their* $\sqrt{45}$ and rationalise denominator	**M1**	or correctly multiply both sides of *their* equation by *their* $\sqrt{5}$ or *their* $\sqrt{45}$ and obtain a rational coefficient of x soi
		completion to $4 + 3\sqrt{5}$ www	**A1**	answer only does not score

6	(i)	arc $AB = r\left(\dfrac{\pi}{3}\right)$	B1	
		chord $AB = r$ with justification and summation and completion to given answer	B1	$r\left(\dfrac{3+\pi}{3}\right)$
	(ii)	$r = 12.7$	B1	must be seen; accept awrt 12.7
		$\dfrac{1}{2} \times their\ r^2 \times \left(\dfrac{\pi}{3} - \sin\left(\dfrac{\pi}{3}\right)\right)$	M3	may be implied for example $84.45\ldots - 69.84\ldots$ or **M1** for $\dfrac{1}{2} \times their\ r^2 \times \dfrac{\pi}{3}$ or $84.45\ldots$ **and** **M1** for $\dfrac{1}{2} \times their\ r^2 \times \sin\dfrac{\pi}{3}$ o.e. or $69.84\ldots$ **and** **M1** for Area Sector – Area triangle attempted
		awrt 14.6	A1	
7	(i)	$k(3-5x)^{11}$	M1	
		$5 \times 12(3-5x)^{11}$ or better, isw	A1	
	(ii)	$x^2(their \cos x) + (their\ 2x)\sin x$	M1	clearly applies correct form of product rule
		$x^2 \cos x + 2x \sin x$ isw	A1	
	(iii)	Quotient rule attempt:		Product rule attempt:
		$\dfrac{d}{dx}(\tan x) = \sec^2 x$	B1	$\dfrac{d}{dx}(\tan x) = \sec^2 x$
		$\dfrac{d}{dx}\left(1 + e^{2x}\right) = 2e^{2x}$	B1	$\dfrac{d}{dx}(1 + e^{2x})^{-1} = -2e^{2x}(1 + e^{2x})^{-2}$
		clearly applies correct form of quotient rule $\dfrac{(1 + e^{2x})(their \sec^2 x) - (their\ 2e^{2x})\tan x}{(1 + e^{2x})^2}$	M1	$\tan x\ (their - 2e^{2x}(1 + e^{2x})^{-2}) + (1 + e^{2x})^{-1}(their \sec^2 x)$
		$\dfrac{(1 + e^{2x})\sec^2 x - 2e^{2x}\tan x}{(1 + e^{2x})^2}$ isw	A1	$\tan x\ (-2e^{2x}(1 + e^{2x})^{-2}) + (1 + e^{2x})^{-1}(\sec^2 x)$

8	(i)	$y - 2 = \left(\dfrac{6-2}{2+6}\right)(x+6)$ o.e. soi	M1	or $y - 6 = \left(\dfrac{6-2}{2+6}\right)(x-2)$
		$y = \dfrac{1}{2}x + 5$ isw	A1	
	(ii)	Use of $m_1 m_2 = -1$ $y - 6 = (their\ -2)(x - 2)$ or better, isw	M1 A1 FT	or $y = (their\ -2)x + c$, $c = their\ 10$, isw
	(iii)	$(x + 6)^2 + (y - 2)^2 = 10^2$ o.e.	B1	or $(x-2)^2 + (y-6)^2 = (\sqrt{20})^2$ o.e. or $(\sqrt{80})^2 +$ $\left((x-2)^2 + (y-6)^2\right) = 10^2$
		Substitute $y = their\ (-2x + 10)$	M1*	or identifying one point by inspection from the length equation and testing it in the equation of BC or vice versa
		Solve their quadratic	M1 dep*	or identifying the second point by inspection from the length equation and testing it in the equation of BC or vice versa
		$(0, 10)$ and $(4, 2)$ o.e. only	A1	answer only does not score
9	(a)	$14 = k + c$ and $6 = \dfrac{k}{9} + c$ o.e.	M1	for two equations in k and c; may be unsimplified; condone one slip in one equation
		$c = 5$ $k = 9$	A1 A1	
	(b) (i)	79.2 or 79.158574 … rot to 4 or more sf	B1	
	(ii)	$e^{2x} + 5e^x - 24(= 0)$ or $(e^x)^2 + 5e^x - 24(= 0)$ o.e. factorise *their* 3 term quadratic	M1 M1	condone one error, but must be three terms or correct/correct ft use of formula or completing the square
		$e^x = 3$ $x = \ln 3$ or 1.1(0) or 1.0986122 … rot to 3 or more sf **as only answer from fully correct working**	A1 A1	ignore $e^x = -8$ do not allow final mark if value given from $e^x = -8$ if **M0M0** then **SC2** if $e^x = 3$ is seen www and leads to $x = \ln3$ or 1.1(0) or 1.0986122… rot to 3 or more sf

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10	(a)	(i)		B1	shape; cosine curve – ends must be approaching a turning point
				B1	be centred on $y = 1$
				B1	clear intent to have min at –2 and max at 4
				B1	2 cycles
		(ii)	3	B1	
		(iii)	180	B1	
	(b)		$\operatorname{cosec} x = \dfrac{1}{\sin x}$ soi	B1	or $1 + \tan^2 x = \dfrac{1}{\cos^2 x}$
			$\sin x = \sqrt{1 - \cos^2 x}$ or $\sqrt{1 - p^2}$	B1	or $\operatorname{cosec}^2 x = 1 + \dfrac{1}{1 - p^2 \big/ p^2}$ soi
			$\dfrac{-1}{\sqrt{1 - p^2}}$ o.e.	B1	or $-\sqrt{1 + \dfrac{p^2}{1 - p^2}}$ or better

11	(i)	$\dfrac{dy}{dx} = 3 - 3(x-4)^{-4}$ o.e. isw	B1 + B1	
		$\dfrac{d^2y}{dx^2} = (their\ 12)(x-4)^{their\ (-5)}$ o.e.	M1	
		$\dfrac{d^2y}{dx^2} = 12(x-4)^{-5}$ o.e. isw	A1	if **M0** then **SC1** for $12(x-4)^{-5}$ + one other term
	(ii)	Verifies $\dfrac{dy}{dx} = 0$ when $x = 3$ and $x = 5$	M1	if **M0** then **SC1** for verifying or correctly solving to find one x coordinate and showing that it gives rise to the corresponding y coordinate
		or solves $3 - \dfrac{3}{(x-4)^4} = 0$ to obtain 3 and 5		
		Shows that $x = 3 \Rightarrow y = 8$ and $x = 5 \Rightarrow y = 16$	A1	
	(iii)	$x = 5\ \dfrac{d^2y}{dx^2}\ (=12) > 0 \Rightarrow$ min or	M1	or, using first derivative e.g.

x	–	5	+
$\dfrac{dy}{dx}$		0	

min at $x = 5$

or

x	–	3	+
$\dfrac{dy}{dx}$		0	

max at $x = 3$

		$x = 3\ \dfrac{d^2y}{dx^2}\ (=-12) < 0 \Rightarrow$ max		
		Both correct cao	A1	
	(iv)	$\dfrac{3x^2}{2} - \dfrac{(x-4)^{-2}}{2} (+c)$ o.e. isw	B1 + B1	may be unsimplified
	(v)	*their* $\left[\left(\dfrac{3(6)^2}{2} - \dfrac{1}{2(6-4)^2}\right) - \left(\dfrac{3(5)^2}{2} - \dfrac{1}{2(5-4)^2}\right)\right]$	M1	
		16.875 to 3 or more sf or $\dfrac{135}{8}$ or $16\dfrac{7}{8}$ cao	A1	