CAMBRIDGE INTERNATIONAL EXAMINATIONS

CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the November 2003 question papers

0606 ADDITIONAL MATHEMATICS

0606/01	**Paper 1, maximum raw mark 80**
0606/02	**Paper 2, maximum raw mark 80**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2003 examination.

	maximum mark available	minimum mark required for grade:		
		A	C	E
Component 1	80	63	31	21
Component 2	80	67	36	26

Grade A* does not exist at the level of an individual component.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

- The following abbreviations may be used in a mark scheme or used on the scripts:

  AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

  BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

  CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

  ISW Ignore Subsequent Working

  MR Misread

  PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

  SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

- MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
- OW −1, 2 This is deducted from A or B marks when essential working is omitted.
- PA −1 This is deducted from A or B marks in the case of premature approximation.
- S −1 Occasionally used for persistent slackness.
- EX −1 Applied to A or B marks when extra solutions are offered to a particular equation.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

INTERNATIONAL GCSE

MARK SCHEME

MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 0606/01

ADDITIONAL MATHEMATICS
Paper 1

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1. $x + 3y = k$ and $y^2 = 2x + 3$ Elimination of x or y $\rightarrow y^2 + 6y - (2k+3) = 0$ or $\rightarrow x^2 - (2k + 18)x + (k^2 - 27) = 0$ Uses $b^2 - 4ac$ $\rightarrow k < -6$	 M1 A1 M1 A1 [4]	 x or y must go completely, but allow for simple arithmetic or numeric slips co Any use of b^2-4ac, even if =0 or >0 co
2. $8^{-x} = 2^{-3x}$ $4^{½x} = 2^x$ Attempts to link powers of 2 $\rightarrow x - 3 - (-3x) = 5 - (x)$ $\rightarrow x = 1.6$ or 8/5 etc [$\log 8^{-x} = -3x\log 2$, $\log 4^{½x} = x\log 2$ equate coefficients of log 2]	B1 B1 M1 A1 [4] [B1B1 M1A1]	Wherever used Needs to use $x^a \div x^b = x^{a-b}$ co
3. $x^3 + ax^2 + bx - 3$ Puts x=3 $\rightarrow 27+9a+3b-3=0$ Puts x=−2 $\rightarrow -8+4a-2b-3=15$ $(9a+3b=-24$ and $4a-2b=26)$ Sim equations $\rightarrow$ a = 1 and b = −11	 M1A1 M1A1 A1 [5]	 Needs x=3 and =0 for M mark Needs x=−2 and =15 for M mark (A marks for unsimplified) co
4. $(\sqrt{3}-\sqrt{2})^2 = 5 - 2\sqrt{6}$ or $5-2\sqrt{2}\sqrt{3}$ Divides volume by length² $\frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$ Denominator = 1 Numerator = $20\sqrt{2}-15\sqrt{3}+8\sqrt{12}-6\sqrt{18}$ But $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{18} = 3\sqrt{2}$ $\rightarrow 2\sqrt{2} + \sqrt{3}$	B1 M1 M1 M1 A1 [5]	Co anywhere V÷l² used × by denominator with sign changed Correct simplification somewhere with either of these co
5 y=0 when $3x + ¼\pi = \pi$ $\rightarrow x = ¼\pi$ $\int 6\sin(3x+\pi/4)dx = -6\cos(3x+\pi/4) \div 3$ Between 0 and π/4 $\rightarrow 2 + \sqrt{2}$ or 3.41	 B1 M1 A2,1 DM1 A1 [6]	 Co. Allow 45° Knows to integrate. Needs "cos". All correct, including ÷3, ×6 and −ve Uses limits correctly – must use x=0 In any form – at least 3sf
6 Wind 50**i**− 70**j** V(still air) = 280**i** −40**j** (i) Resultant velocity = $\mathbf{v}_{air} + \mathbf{w}$ $\rightarrow$ 330**i** − 110**j** $\tan^{-1}(110/330) = 18.4°$ $\rightarrow$ Bearing of Q from P = 108° (ii) Resultant speed = $\sqrt{(330^2+110^2)}$ Time = 273 ÷ resultant speed = 47 minutes Scale drawings are ok.	 M1 A1 DM1 A1 M1 A1√ [6]	 Connecting two vectors (allow −) Co (Could get these 2 marks in (ii)) For use of tangent (330/110 ok) co Use of Pythagoras with his components For 273 ÷ $\sqrt{(a^2+b^2)}$

7 $(0.6 \quad 0.2 \quad 0.5)\times\begin{pmatrix}8&6&6&5\\5&4&3&2\\3&3&2&2\end{pmatrix}\times\begin{pmatrix}40\\50\\50\\60\end{pmatrix}$	B2,1,0	Wherever 3 matrices come – as row or column matrices – as 3 by 4 or 4 by 3 – independent of whether they are compatible for multiplication or not.
$= (7.3 \quad 5.9 \quad 5.2 \quad 4.4)\times\begin{pmatrix}40\\50\\50\\60\end{pmatrix}$	M1 A1	Correct method for multiplying any 2 of the 3 - co for A mark.
or $(0.6 \quad 0.2 \quad 0.5)\times\begin{pmatrix}1220\\670\\490\end{pmatrix}$	M1	Correct method for remaining two.
→ $1111	B1 [6]	Co – even if from arithmetic.
8 (i) d/dx(lnx) = 1/x	B1	Anywhere, even if not used in "u/v"
$\frac{dy}{dx}=\frac{(2x+3)\times\frac{1}{x}-(\ln x)\times 2}{(2x+3)^2}$	M1A1√	Uses correct formula. All ok. Could use product formula. A mark unsimplified.
(ii) δy = (dy/dx) × δx = 0.2p	M1A1	Allow if δy mixed with dy/dt. M mark given for algebraic dy/dx × p.
(iii) dy/dt = dy/dx × dx/dt	M1	Allow if dy/dt mixed with δy
→ dx/dt = 0.6	A1√ [7]	√ for 0.12 ÷ his dy/dx. Condone use of δx etc
9 (a) Uses $\sec^2x = 1+\tan^2x$ → quad in sec		
or $\times c^2$ then uses $s^2+c^2=1$ → quad in cos		
→ $4\sec^2x+8\sec x-5=0$	B1	Co.
→ $-5\cos^2x+8\cos x+4=0$	M1	Sets to 0 and uses correct method for solution of a 3 term quadratic in sec or cos.
→secx = −2.5 (or0.5) or cosx=−0.4 (or2)		
→ x = 113.6° or 246.4°	A1A1√	A1 co. A1√ for 360°−"first ans" only.
(b) tan(2y+1) = 16/5 = 3.2		
Basic angle associated with 3.2 = 1.27	B1	Anywhere (allow 72.6°)
Next angle = π + 1.27 and 2π + 1.27	M1	Realising the need to add on π and/or 2π
(Value − 1) ÷ 2 → 3.28	M1A1	Correct order used ie −1, then ÷2 for any correct value. Allow if all 3 values are given, providing none are over 4. (degrees – max 2/4 B1, M0, M1, A0)
(others are 0.134 and 1.705)	[8]	

10 $f(x) = 5-3e^{½x}$		
(i) Range is <5	B1	Allow ≤ or <
(ii) $5-3e^{½x} = 0 \rightarrow e^{½x} = 5/3$ Logs or calculator $\rightarrow$ x = 1.02	M1A1	Normally 2,0 but if working shown, can get M1 if appropriate
(iii) (1.02, 0) and (0, 2)	B1 B1√	Shape in 1st quadrant. Both shown or implied by statement.
(iv) $e^{½x} = (5 - y)\div3$ $x/2 = \ln[(5-y)/3]$ $f^{-1}(x) = 2\ln[(5-x)/3]$	M1 M1 A1	Reasonable attempt $e^{½x}$ as the subject. Using logs. All ok, including x, y interchanged.
	[8]	
11 (i) y=½x and y=3x-15 $\rightarrow$ C(6,3)	M1 A1	Soln of simultaneous eqns Co (or step method if B done first)
OB=OC+CB	M1	Vectors, step or soln of y=½x+5 and y=3x-15
$\rightarrow$ B(8,9)	A1√	From his C
m of OC = ½, m of AD = −2 eqn of AD is y−6=−2(x−2) or y=−2x+10	M1 A1	use of $m_1m_2=-1$ (M0 if perp to y=3x) Co – unsimplified.
Soln of y=½x and eqn of AD $\rightarrow$ D(4,2)	M1A1	Sol of simultaneous eqns. co.
(ii) Length OC = √45, OA = √40 Perimeter of OABC = 2(√45+√40) $\rightarrow$ 26.1	M1 M1 A1	Once. Adding OA,AB,BC,CO Co.
	[11]	
12 **EITHER**		
(i) $125 = \pi r + 2x + 2(5r/4)$ $\rightarrow x = ½(125 - \pi r - 5r/2)$	M1 A1	Attempt at 4/5 lengths. Co.
h = 3r/4	M1	Anywhere in the question – independent of any other working
Area of triangle = ½ × 2r × 3r/4 = 3r²/4	M1	Use of ½bh with h as function of r
$A = ½\pi r^2 + 2rx + \ldots$ $= 125r - ½\pi r^2 - 7r^2/4$	B1 A1	Correct $½\pi r^2 + 2rx$. Answer given – beware fortuitous ans.
(ii) $dA/dr = 125 - \pi r - 7r/2$	M1A1	Any attempt to differentiate. Co.
Solved = 0 to give	DM1	Setting his differential to 0.
$\rightarrow r = 250 / (2\pi + 7)$ or 18.8	A1	Any correct form.
	[10]	

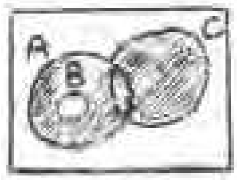
12 **OR**		
(i) $h / (12-r) = 30 / 12$	M1	Use of similar triangles – needs ¾ lengths correct.
$\rightarrow h = 5(12-r) / 2$	A1	Correct in any form – needs h as subject
Uses $V=\pi r^2 h$ to give	M1	Needs correct formula
$\rightarrow V = \pi(30r^2 - 5r^3/2)$	A1	Beware fortuitous answers (AG)
(ii) $dV/dr = \pi(60r - 15r^2/2)$	M1A1	Any attempt to differentiate. co
$= 0$ when $r = 8 \rightarrow h = 10$	DM1	Setting his dV/dr to 0 + attempt.
$\rightarrow V = 640\pi$ or 2010	A1	Correct to 3 or more sig figures
Volume of cone $= \frac{1}{3}\pi\times 12^2\times 30$ $\rightarrow 1440\pi$ or 4520	M1	Anywhere
Ratio of 4 : 9 or 1 : 2.25 (3 sf)	A1 [10]	Exactly 4:9 or 2.25 to 3 sig figures
DM1 for quadratic equation (1) Formula. Sets the equation to 0 Formula must be correct and correctly used. Condone simple slips in sign.		(2) Factors Sets the equation to 0 Attempts to obtain brackets Solves each bracket to 0.

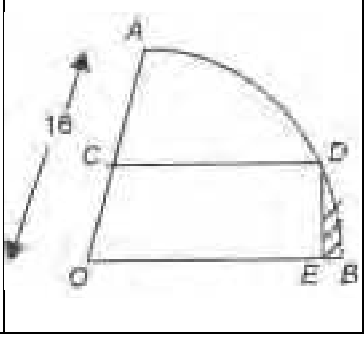
CAMBRIDGE
INTERNATIONAL EXAMINATIONS

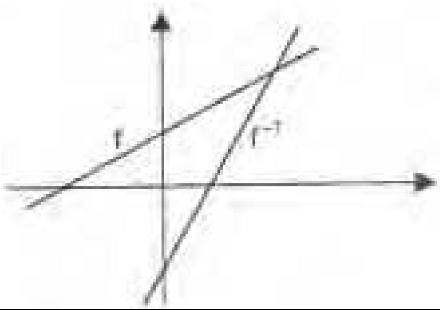
November 2003

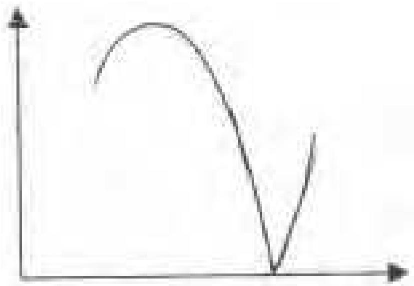
INTERNATIONAL GCSE

MARK SCHEME
MAXIMUM MARK: 80
SYLLABUS/COMPONENT: 0606/02 **ADDITIONAL MATHEMATICS** **Paper 2**

1 [4]		Eliminate x or y	M1
		$\Rightarrow y^2 - 8y + 15 = 0 \qquad x^2 - 10x + 9 = 0$	
		Factorise or formula $\Rightarrow$ (1, 3) and (9, 5)	DM1 A1
		Midpoint is (5, 4)	B1 √
2 [4]		$\cos\theta\left(\frac{1+\sin\theta-(1-\sin\theta)}{1-\sin^2\theta} = \cos\theta\left(\frac{2\sin\theta}{1-\sin^2\theta}\right) = \frac{2\sin\theta\cos\theta}{1-\sin^2\theta}\right)$	M1 A1
		Use of Pythagoras $\Rightarrow \frac{2\sin\theta\cos\theta}{\cos^2\theta} = 2\tan\theta \Rightarrow k = 2$	B1 A1
3 [4]		$\log_2 x = 2\log_4 x$ or $\log_4 (x-4) = \frac{1}{2}\log_2(x-4)$	B1
		$2\log_4 x - \log_4 (x-4) = 2$ or $\log_2 x - \frac{1}{2}\log_2(x-4) = 2$	
		Eliminate logs $\frac{x^2}{x-4} = 16$ or $\frac{x}{\sqrt{x-4}} = 4$	M1 A1
		Solve for x $\Rightarrow$ $x = 8$	A1
4 [4]	**(i)**		B2 B1 B1
	(ii)	$A \cap B' \cap C'$	
	(iii)	$B \cup (A \cap C)$	
5 [5]	**(i)**	$243x^5 \quad -405x^4 \quad +270x^3$	B1 B1 B1
	(ii)	Coefficient of $x^4 = (-405 \times 1) + (270 \times 2) = 135$	M1 A1
6 [6]		At B, $v = 40(e^{-t} - 0.1) = 0 \Rightarrow e^{-t} = 0.1 \Rightarrow t = \ln 10\ (=2.30)$	M1 A1
		$\int 40(e^{-t} - 0.1)dt = 40(-e^{-t} - 0.1t)$	M1 A1
		$AB = \int_0^{\log 10} = 40\left[\left(-\frac{1}{10} - \frac{\ln 10}{10}\right) - (-1)\right] = 4(9 - \ln 10) \approx 26.8$	DM1 A1

7 [7]		Dealing with elements $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$	M1
		$\mathbf{A}^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$ $\quad \mathbf{B}^{-1} = \frac{1}{8}\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$	A1 A1
	(i)	$\mathbf{C} = \mathbf{B} - 2\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 7 \end{pmatrix}$	M1 A1
	(ii)	$\mathbf{D} = \mathbf{B}^{-1}\mathbf{A} = \frac{1}{8}\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 9 & 5 \\ 14 & 6 \end{pmatrix}$	M1 A1
8 [7]	(i)	$\frac{10!}{6!4!} = \frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4} = 210$	M1 A1
	(ii)	No pink selected i.e. any 6 from (5 + 2) = 7	B1
	(iii)	All selections contain at least 1 red	
		No yellow selected i.e. any 6 from (3 + 5) = $\frac{8!}{6!2!} = 28$	M1 A1
		At least 1 of each colour – 120 – (7 + 28) = 175	M1 A1
9 [8]	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{4x-3}\right) = (4x-3)^{-\frac{1}{2}} \times \frac{1}{2} \times 4$	M1 A1
		$\frac{\mathrm{d}}{\mathrm{d}x}\left\{(2x+3)\sqrt{4x-3}\right\} = (2x+3)\left(\frac{2}{\sqrt{4x-3}}\right) + 2\sqrt{4x-3}$	M1 A1 √
		$= \frac{12x}{\sqrt{4x-3}} \Rightarrow k = 12$	A1
	(ii)	$\int \frac{x}{\sqrt{4x-3}}\,\mathrm{d}x = (2x+3)\sqrt{4x-3} \times \frac{1}{12}$	M1 A1
		$\int_1^7 = \frac{1}{2}(85-5) = 6\frac{2}{3}$	A1
10 [10]		(i) $\angle AOB$ = 19.2 ÷ 16 = 1.2	M1 A1
		(ii) DE = 8 sin 1.2 ≈ 7.46	M1 A1
		(iii) $\angle DOE = \sin^{-1}$ (7.46 ÷ 16) ≈ 0.485 (AG)	M1 A1
		(iv) Sector DOB = $\frac{1}{2}$ x 16^2 x 0.485 = 62.08	M1
		Length OE = √ ($16^2 - 7.46^2$) ≈ 14.2	M1
		ΔDOE = $\frac{1}{2}$ x 7.46 x 14.2 ≈ 52.97	M1
		Shaded area ≈ 9.1 – 9.3 (9.275)	A1

11 [10]		v: 5, 10, 15, 20, 25; R: 32, 96, 180, 290, 420; lg v: 0.70, 1.00, 1.18, 1.30, 1.40; lg R: 1.51, 1.98, 2.26, 2.46, 2.61 — **(i)** Plotting lg R against lg v	M1
		Accuracy of points: Straight line	A2, 1, 0
		(ii) $R = kv^{\beta} \Rightarrow \lg R = \lg k + \beta \lg v$	B1
		β = gradient $\approx$ 1.55 - 1.60	M1 A1
		lg k = lg R intercept $\approx 0.4 \Rightarrow k \approx 2.4 - 2.6$	M1 A1
	(iii)	lg R = lg 75 $\approx 1.88 \Rightarrow$ from graph lg $v \approx 0.92 - 0.96 \Rightarrow v \approx 8.3 - 9.1$	M1 A1
		[Or by solving e.g., $75 = 2.5v^{1.58}$ or $1.88 = 0.4 + 1.58 \lg v$]	
12 EITHER [11]	**(i)**	$\mathrm{gf}(x) = \dfrac{4}{2-(3x-2)}$	B1
		Solve $\dfrac{4}{4-3x} = 2$ [or solve $\mathrm{fg}(x) = 3\left(\dfrac{4}{2-x}\right) - 2 = 2$]	M1
		$\Rightarrow x = 2/3$	A1
	(ii)	$\mathrm{f}(x) = \mathrm{g}(x) \Rightarrow 3x - 2 = \dfrac{4}{2-x} \Rightarrow 3x^2 - 8x + 8 = 0$	
		Discriminant = 64 – 96 < 0 $\Rightarrow$ No real roots	M1 A1
	(iii)	$\mathrm{f}^{-1} : x \mapsto (x + 2) \div 3$	B1
		$y = 4 / (2 - x) \Rightarrow x = 2 - 4/y \Rightarrow \mathrm{g}^{-1} : x \mapsto 2 - 4/x$	M1 A1
	(iv)		B1 B1
		Lines intersect at (1, 1)	B1

12 OR [11]	**(i)**	$1 - x^2 + 6x \equiv a - (x + b)^2 = a - x^2 - 2bx - b^2 \Rightarrow a - b^2 = 1$ and $-2b = 6$	M1 A1
		[or $1 - x^2 + 6x \equiv 1 - (x^2 - 6x) \equiv 1 - \{(x-3)^2 - 9\}$]	
		$\Rightarrow b = -3,\ a = 10$	A1
	(ii)	$1 - x^2 + 6x \equiv 10 - (x - 3)^2 \quad \Rightarrow$ Maximum at (3, 10)	
		$\therefore$ Single-valued for $x \geqslant 3$ and hence for $x \geqslant 4$	M1 A1
	(iii)	$y = 10 - (x - 3)^2 \quad \Rightarrow \quad (x - 3)^2 = 10 - y \quad \Rightarrow \quad x - 3 = \sqrt{(10 - x)}$	M1
		$\Rightarrow f^{-1} : x \mapsto 3 + \sqrt{(10 - x)}$	A1
	(iv)	When $x = 2$, $g(x) = 9$ and when $x = 7$, $g(x) = -6$	B1
		Range of g is $-6 \leqslant g \leqslant 10$	B1
	(v)		B 2, 1, 0