CONTENTS

ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

The paper achieved a key aim of enabling candidates to demonstrate what they had been taught. There were many excellent scripts, but also many from candidates who were inadequately prepared for the examination. The standard of numeracy and algebra was good and the scripts generally were easy to mark. Once again it is worth pointing out to Centres that candidates should *not* fold a page into two columns and work separately down each half.

Comments on specific questions

Question 1

This did not prove to be an easy starting question. Most candidates were able to obtain expressions for two of the vectors, $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{BC}$ and it was rare to see the error of taking $\overrightarrow{AB}$ = **a** + **b**. The finding of k presented more problems with $\overrightarrow{AB}$ **=** $\lambda\overrightarrow{OC}$ being a common error. Of candidates correctly stating that $\overrightarrow{AB}$ **=** $\lambda\overrightarrow{AC}$ (or equivalent) many scripts were seen in which λ was taken to be k and a quadratic expression was subsequently obtained for k. Equating gradients was a common method and proved to be more successful than the more obvious ratio method. Another reasonably successful method was to find the equation of the line ($y = -3x + 12$) and then to substitute (k, $3k$).

Answer: $k = 2$.

Question 2

Achievement on this question varied considerably from Centre to Centre. It was obvious that some candidates were totally unfamiliar with the topic, whilst others produced correct answers. The diagrams were generally well drawn, with only a few candidates not realising the need to draw three intersecting circles. Since it was given that all members play at least one sport, there were a variety of acceptable answers; e.g. $P \cap D' \cap T'$ or $(D \cup T)'$ etc. for part **(i)**.

Answers: **(i)** $P \cap D' \cap T'$; **(ii)** $P \cap D' \cap T$.

Question 3

This question was well answered. The majority of candidates had little difficulty in forming two correct simultaneous equations in x and y by expressing all quantities as either powers of 2 or 3. The most common errors were to express $2^{3x} \div 2^y = 2^6$ as $(3x) \div y = 6$ or to express $3^{-2(y-1)}$ as either 3^{-2y-2} or as 3^{-2y+1}.

Answer: $x = 5$, $y = 9$.

Question 4

The majority completed this question without too much difficulty but untidy arithmetic often led to unnecessary time wasting. The basic formulae for area and arc length were well known though such offerings as $A = r^2\theta$ or $A = \pi r^2\theta$ or $l = \pi r\theta$ were all seen. A few could not resist the temptation of converting to degrees, thereby lengthening the calculation. The most common error came from candidates misinterpreting the question and starting by assuming that $\frac{1}{2}r^2\theta = 48$. A considerable number of candidates made the calculation more difficult by forming a quadratic equation in x (where $x = BC = AD$).

Answer: 32 cm.

Question 5

Again performance on this question varied from Centre to Centre. Most candidates realised the need to expand $(1-2x)^n$ but there were many errors in the expansion. The binomial coefficients were omitted, $\binom{n}{2}$ was expressed as $\frac{1}{2}n$ and $(-2x)^2$ was often replaced by either $-2x^2$ or $2x^2$ or $-4x^2$. Most candidates realised that the value of a was 3, but only about a half of all attempts realised that both the coefficients of x and x^2 came from 2 terms in the product of $(a + x)$ and $(1-2x)^n$.

Answers: $a = 3$, $n = 7$, $b = 238$.

Question 6

This was very poorly answered by most candidates and it was rare to see a completely correct solution. Very few candidates knew the meaning of the terms 'amplitude' and 'period'. Whilst many more realised that the maximum and minimum values were 8 and 2 respectively, it was very rare to see the maximum point associated with an x-value of $\frac{1}{2}\pi$ and two minimum points with x-values at $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$.

Answers: Minimum at $(\frac{1}{4}\pi, 2)$ and at $(\frac{3}{4}\pi, 2)$. Maximum at $(\frac{1}{2}\pi, 8)$.

Question 7

Again the responses to this question were variable. Some Centres had obviously spent considerable time on the topic and their candidates achieved full marks very easily. Others struggled to get started and there was considerable confusion on whether to use combinations or permutations.

(a) Most candidates attempted the question by looking directly at 8 x 8!, though the alternative of using 9! − 8! was also widely attempted.

(b) The majority realised the need to look at two cases i.e. '2 girls and 1 boy' or '3 girls'. Common errors however were to use ${}_5C_2 + {}_3C_1$ instead of ${}_5C_2 \times {}_3C_1$, or to use permutations or to omit the case of '3 girls'. A few candidates preferred to find the total number of ways and then to subtract the cases of '2 boys and 1 girl' and '3 boys'.

Answers: **(a)** 322 560; **(b)** 40.

Question 8

This question was well answered by the majority of candidates who showed a good understanding of the techniques and manipulation involved with functions.

(i) Obtaining a correct expression for f^{-1} was excellently done, with virtually all candidates obtaining a correct answer. Only a few however realised that because both f and f^{-1} were the same function and that f and f^{-1} are symmetrical about the line $y = x$, then this line must be a line of symmetry of the graph of $y = f(x)$.

(ii) This part was also well answered with the majority of candidates correctly finding $g^{-1}(x)$ and forming a correct quadratic equation for $f(x) = g^{-1}(x)$. The solution of this was generally correct though it was common to see incorrect factorisation or to see $x(x-3) = 10 \Rightarrow x = 10$ or $x - 3 = 10$.

(iii) It was hoped that the more able candidates would realise directly that $gf(x) = -2$ implies $x = -2$. However, only a handful of candidates were able to spot this. The others went to great lengths finding an expression for $gf(x)$ and then solving the resultant equation.

Answers: **(i)** $f^{-1}(x) = \frac{3x+11}{x-3}$, $y = f(x)$ symmetrical about the line $y = x$; **(ii)** −2, 5, **(iii)** −2.

Question 9

(a) This was very well answered with most candidates substituting $\cos^2 x = 1 - \sin^2 x$ and correctly solving the resulting quadratic equation.

(b) This part presented major problems for many of the candidates. A very large number of candidates showed a complete lack of understanding of trigonometric functions by replacing cot $2y$ by $\frac{2y}{\tan}$ and then saying that $2y = \tan 0.25$. Others, whilst recognising that $\tan 2y = 4$, then replaced this by $\tan y = 2$. A similar error was to state that since $\cot 2y = 0.25$, $\cot y = 0.125$ and that $\tan y = 8$. Even when candidates did correctly use $\tan 2y = 4$, there were major errors in coping with radian measure and in appreciating that there were three values of y in the range $0 < y < 4$. A minority of candidates were able to obtain full marks for this part.

Answers: **(a)** 210°, 330°; **(b)** 0.66, 2.23, 3.80.

Question 10

There were many very good solutions, particularly to parts **(i)** to **(iii)** but in general candidates struggled with part **(iv)**.

(i) It was pleasing that most candidates recognised this as a product and obtained a correct answer, though $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = (\ln x)^{-1}$ was a common error. Similarly, many candidates wrote $3x^2\ln x$ as $3\ln x^4$ or as $3\ln x^3$ – a mistake which obviously affected later parts.

(ii) Virtually all candidates recognised that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and many solved correctly to obtain $\ln x = -\frac{1}{3}$. Unfortunately, weaker candidates were unable to cancel the 'x^2' or finished with $\ln x = \frac{1}{3}$.

(iii) Most candidates understood the 'small increase' formula, though several failed to realise that $\delta x = p$ and many others left $\frac{\mathrm{d}y}{\mathrm{d}x}$ in algebraic form.

(iv) This was very poorly answered with only a small proportion of candidates realising the need to use the result in part **(i)**. Even among these candidates, the integration of x^2 or the division through by '3', was often omitted.

Answers: **(i)** $x^2(3\ln x + 1)$; **(ii)** $-\frac{1}{3}$; **(iii)** $4e^2p$ or $29.6p$; **(iv)** $\frac{x^3}{9}(3\ln x - 1)$.

Question 11

Despite the fact that this unstructured question involved many ideas, the vast majority of candidates coped well and scored highly. Virtually all candidates realised the need to solve two simultaneous equations to obtain the coordinates of Q and the algebra involved was generally accurate. Most candidates were able to obtain the gradient of the line perpendicular to PQ but many misunderstood the term 'perpendicular bisector' and failed to realise this went through M, the mid-point of PQ. Most candidates realised the need to solve the simultaneous equations formed by their equation of the bisector and the line $y = 4x$ and approximately a half of all attempts at R were correct. Most candidates preferred to use the 'matrix' method for the area of triangle PQR; others had few problems in evaluating PQ and MR and hence the area from '$A = \frac{1}{2}bh$'.

Answer: 25 unit^2.

Question 12 EITHER

This was the more popular of the two alternatives, but marks varied considerably from Centre to Centre.

(a)(i) The majority of candidates were able to substitute n = 10 and obtain a correct answer.

(ii) A minority of attempts were correct. Candidates often failed to realise the need to make $e^{-0.05n}$ the subject prior to taking logarithms, and there were some poor attempts from candidates attempting to take logarithms. A significant minority of candidates obtaining n = 46 failed to give the answer as 2006.

(b) More able candidates realised the need to put $y = 3^x$ to obtain a quadratic in 3^x and the solution of $3^x = 6$ was generally correct. Weaker candidates tended to take logarithms and made no progress. Division by either 3^{x-1} or 3^{x+1} led to the similar equations of $3^{x-1} = 2$ or $3^{x+1} = 18$.

Answers: **(a)(i)** 12 100, **(ii)** 2006; **(b)** 1.63.

Question 12 OR

The majority of candidates recognised the basic principles of the question; knowing that $\frac{dy}{dx} = 0$ at the stationary point, that the sign of $\frac{d^2y}{dx^2}$ can be used to determine its nature and that the area under the curve is obtained from integrating y. Unfortunately the basic differentiation and integration and the solution of the exponential equation proved to be beyond most candidates.

(i) There were very few completely correct solutions. Differentiation of $e^{\frac{1}{2}x}$ presented problems with $e^{\frac{1}{2}x}$, $\frac{1}{2}xe^{\frac{1}{2}x}$ and $\frac{1}{2}e^{-\frac{1}{2}x}$ being the most common errors. Even when $\frac{dy}{dx}$ was correct, many candidates had problems in solving $\frac{dy}{dx} = 0$ and it was rare to see $e^x = 3$ and even rarer for candidates to correctly manipulate this to obtain $y = 2\sqrt{3}$. Most candidates used decimals and obtained no credit.

(ii) Similar errors affected the differentiation of $\frac{dy}{dx}$ to find $\frac{d^2y}{dx^2}$, though candidates looked at the sign of $\frac{d^2y}{dx^2}$ to determine the nature of the stationary point.

(iii) Candidates knew to integrate and to use the limits 0 to 1 correctly and it was pleasing that very few candidates ignored the value at x = 0. Unfortunately the integration suffered as in parts **(i)** and **(ii)** and it was rare to see e^{ax} integrated as $\frac{e^{ax}}{a}$.

Answers: **(ii)** Minimum; **(iii)** 3.66 unit^2.

Paper 0606/02

Paper 2

General comments

Marks gained by candidates were generally lower than in last year's examination and there was a widespread inability to deal with the topic of relative velocity as posed by **Question 8**. Unfortunately, a significant number of candidates were entered who lacked the basic knowledge and mathematical skill necessary to achieve a reasonable standard of performance at this level.

Comments on specific questions

Question 1

The majority of candidates had little difficulty in writing down the required inverse matrix. Many of the weaker candidates ignored the word 'hence' and so gained nothing for their solution of the simultaneous equations by elimination. Some candidates post-multiplied by the inverse matrix when pre-multiplying was appropriate.

Answer: $\frac{1}{23}\begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}$; $x = 1$, $y = -2$.

Question 2

Better candidates easily applied the two processes of squaring and rationalising the denominator, in either order, and thus obtained the correct answer. Less able candidates were often only able to apply one of the two processes, reaching either $4 - \sqrt{3}$ or $\frac{169}{19 + 8\sqrt{3}}$. The weakest candidates were unable to rationalise a denominator or were guilty of elementary errors e.g. $(4 + \sqrt{3})^2 = 16 + 3$.

Answer: $a = 19$, $b = -8$.

Question 3

Most candidates understood that the integrals of $\sin 2x$ and $\cos x$ involved $\cos 2x$ and $\sin x$ respectively but common errors were to attach incorrect signs to these functions or to use 3 or 6, rather than $\frac{3}{2}$, as the numerical value of the coefficient of $\cos 2x$. Other mistakes involved, for instance, $\cos x$ becoming $\frac{1}{2}\sin x^2$ or $4\cos x$ becoming $4x\cos x$. Not all candidates who integrated correctly were able to find the correct area, in that trigonometric functions involving π or $\frac{\pi}{2}$ radians were frequently evaluated with the calculator in degree mode. Also, the value of the integral at the lower limit, $x = 0$, was often ignored or taken to be zero and $[\cos 2x]_{x=\pi/2}$ was often taken to be [cos(2 radians)] $\times \frac{\pi}{2}$.

Answer: 7 units^2.

Question 4

Virtually all candidates appreciated that one of the variables, almost always y, had to be eliminated. Most then attempted to arrange their equation in the form $ax^2 + bx + c$, although weaker candidates were often unable to avoid errors in the simplification and collection of terms. Again, most candidates tried to consider the discriminant, with relatively few attempting to solve the quadratic equation in x. Many obtained $16k - 4k^2$ but this was usually equated to 0 leading to $k = 0$ or $k = 4$. Only the best candidates were able to handle the inequality $16k - 4k^2 \geqslant 0$ properly and arrive at $0 \leqslant k \leqslant 4$. The use of $<$, rather than $\leqslant$, was condoned.

Answer: $0 \leqslant k \leqslant 4$.

Question 5

Full marks were often obtained by better candidates employing a variety of methods. Many candidates stated $\log_{16}(3x - 1) = \dfrac{\log_4(3x-1)}{\log_4 16}$ but then took this to be $\log_4\left(\dfrac{3x-1}{16}\right)$. Others omitted the formal statement and proceeded directly from $\log_{16}(3x - 1)$ to some erroneous form e.g. $2\log_4(3x - 1)$, $\log_4 2(3x - 1)$, $\log_4(3x - 1)^2$ or $\frac{1}{4}\log_4(3x - 1)$. The terms of $\log_4(3x) + \log_4(0.5)$ were usually combined as $\log_4(1.5x)$, although some preferred to proceed using $\log_4(0.5) = -0.5$, and weaker candidates simply took the sum of the logs to obtain $\log_4(3x + 0.5)$. Some of the poorest candidates simply 'cancelled' log throughout. Squaring $1.5x$ sometimes resulted in $2.25x$ or $1.5x^2$ and, in applying the formula to the quadratic equation $2.25x^2 - 3x + 1 = 0$, 2×2.25 was occasionally taken to be 5.

Answer: $\frac{2}{3}$.

Question 6

(i) A few candidates arrived at $\sin\theta = \cos\theta$ but the vast majority reached $\sin\theta = 5\cos\theta$ although the weakest candidates could not proceed any further. Most who continued obtained $\tan\theta = 5$, a few took $\tan\theta$ to be $\frac{1}{5}$ and others succeeded via $\sin^2\theta = 25\cos^2\theta$, while some erroneously replaced $\cos\theta$ by $1 - \sin\theta$. The answer was usually given as 78.7°, but some candidates found 258.7° and then rejected this angle, while others gave both angles and a few actually selected 258.7° as the acute angle.

(ii) Some candidates interpreted the question as involving $x^2 = y^2$ or $x^2 + y^2 = 0$. Errors in expanding x^2 and y^2 were quite common amongst weaker candidates e.g. $3\sin^2\theta$, $-6\sin\theta\cos\theta$, $-4\cos^2\theta$ and also the omission of terms involving $\sin\theta\cos\theta$. Some candidates assumed that the substitution of specific values for θ, including the angle or angles found in part **(i)**, would be sufficient. Others reached $13\sin^2\theta + 13\cos^2\theta$ but could not continue or took this expression to be 1. A few deduced that $x^2 + y^2$ was constant by showing that $\dfrac{d}{d\theta}\left(13\sin^2\theta + 13\cos^2\theta\right) = 0$.

Answer: **(i)** 78.7° or 1.37 radians.

Question 7

Judging by the number of completely correct solutions, this was possibly the easiest question on the paper. Most candidates substituted a for x and obtained the correct cubic equation in a, but those attempting long division by $x - a$ were usually unable to cope with the algebra involved. The root $a = -2$ was most frequently spotted, although a surprising number spotted $a = \frac{1}{2}$. Apart from numerical and sign errors, candidates were generally able to obtain the quadratic factor and then solve the equation. Weak candidates went from $6a^3 + 5a^2 - 12a = -4$ to $a(6a^2 + 5a - 12) = -4$ and hence to $a = -4$ and $6a^2 + 5a - 12 = 0$ or -4.

Answers: -2, $\frac{1}{2}$, $\frac{2}{3}$.

Question 8

This question was frequently omitted and the vast majority of candidates who did attempt it scored very few marks. The overriding error was to assume that the velocity triangle was right-angled, resulting in answers obtained from dividing one of 150, 200 or 250 by one of 3, 6, 9, $\sqrt{6^2+3^2}$ or $\sqrt{6^2-3^2}$; other variations were $\dfrac{200+150}{6+3}$ and $\dfrac{200}{6}+\dfrac{150}{3}$. Some candidates unnecessarily used the sine or cosine rule with a right-angled triangle. Those candidates who appreciated that the velocity triangle was not right-angled did not always have the 6ms^{-1} velocity in the correct position and some assumed that the point B was 150 m downstream of the point on the bank at which the boat was initially headed. A few took the 3ms^{-1} velocity to be acting upstream rather than downstream. Candidates producing a correct triangle of velocities were almost always able to reach the correct answer and it was pleasing to see a variety of methods and some elegant work among their solutions.

Answer: 34 s.

Question 9

(i) A common error was to take $\log(ab^x)$ to be $x \log ab$, giving $x \log a + \log b$. Surprisingly, many candidates obtaining $\log y = x \log b + \log a$ chose X to be $\log b$ and m to be x.

(ii) Most candidates obtained $Y = \log y$ and $X = \log x$ but, from $\log y = k \log Ax = k \log A + \log x$, these were sometimes accompanied by $m = 1$, $c = k \log A$.

(iii) Algebraic errors such as $\frac{p}{y} + \frac{q}{x} = 1 \Rightarrow \frac{y}{p} + \frac{x}{q} = 1$ and $y = \frac{px}{x - q} \Rightarrow y = \frac{px}{x} - \frac{px}{q}$ were common. Candidates able to obtain a useful form e.g. $y = \frac{qy}{x} + p$, frequently made the wrong choice, saying $Y = y$, $X = \frac{1}{x}$, $m = qy$, $c = p$. The weakest candidates attempted to apply logs.

Parts **(i)** and **(ii)** were probably more routine than part **(iii)**, so it was not uncommon to find that candidates who answered parts **(i)** and **(ii)** correctly were unable to deal with part **(iii)**. What was surprising was the number of candidates to which the reverse applied i.e. who were unable to answer parts **(i)** and **(ii)**, but answered part **(iii)** correctly.

Answers: **(i)** $Y = \log y$, $X = x$, $m = \log b$, $c = \log a$; **(ii)** $Y = \log y$, $X = \log x$, $m = k$, $c = \log A$; **(iii)** Various alternatives including $Y = y$, $X = \frac{y}{x}$, $m = q$, $c = p$ and $Y = \frac{1}{y}$, $X = \frac{1}{x}$, $m = -\frac{q}{p}$, $c = \frac{1}{p}$.

Question 10

(i) Most candidates missed the point of this part of the question and merely showed that $x \mapsto x^2 - 8x + 7$ has a minimum value at $x = 4$, usually via the calculus or occasionally by completing the square. Comments as to the nature of the stationary point of $f(x) = |x^2 - 8x + 7|$ were extremely rare.

(ii) Weaker candidates simply produced the graph of $y = x^2 - 8x + 7$. Those who did consider the modulus function sometimes used values of x outside the given domain or failed to produce that part of the curve beyond $x = 7$. Other errors on the sketch were a rounded minimum, rather than a cusp, at $x = 7$ and a curve, between $x = 7$ and $x = 8$, with incorrect curvature.

(iii) Those candidates with a correct graph usually found the range correctly.

(iv) Only the best candidates understood what was involved.

Answers: **(i)** Maximum; **(iii)** $0 \leqslant f(x) \leqslant 9$; **(iv)** 4.

Question 11

The gradients of lines perpendicular and parallel to $y = 3x$ were almost always correct and for some candidates this was as much as they could do. Others could also quote $\sqrt{x^2 + y^2} = \sqrt{250}$ but could not see that y should be replaced by $3x$. Incorrect work on the distance idea included $x^2 - y^2 = 250$, $\sqrt{x^2 + y^2} = \sqrt{250} \Rightarrow x + y = \sqrt{250}$ or 250 and $x^2 + (3x)^2 \equiv x^2 + 3x^2$. Some candidates obtained (5, 15) by mental arithmetic or trial and error but others arrived at (9, 13). Candidates finding the coordinates of A correctly almost invariably found the coordinates of B and C correctly and, whatever the coordinates of A, candidates who continued usually could demonstrate that they understood the principles required to complete the question. Some candidates employed trigonometry to find the coordinates of B before those of A, in effect using $OB = \frac{\sqrt{250}}{\cos(90° - \tan^{-1} 3)}$.

Answers: $A(5, 15)$, $B(0, 16\frac{2}{3})$, $C(-3\frac{1}{3}, 6\frac{2}{3})$.

Question 12 EITHER

All but the weakest candidates had a clear understanding of the relationships between a, v and s and integration was generally carried out well.

(i) The constant of integration was frequently taken to be zero, leading from $1.4t - 0.3t^2 = 0$ to $t = 4\frac{2}{3}$ which was then taken to be approximately 5, or, producing an argument which attempted to reconcile the initial speed of 0.5 ms^{-1} with the fact that $[1.4t - 0.3t^2]_{t=5} = -0.5$. The value of c was usually obtained from $v = 0.5$ when $t = 0$, but some candidates used $v = 0$ when $t = 5$ to find c, invalidating their later argument purporting to show the particle was instantaneously at rest when $t = 5$. Having obtained c via the correct method, the substitution of $t = 5$ was sufficient to establish instantaneous rest, but some candidates preferred to solve $1.4t - 0.3t^2 + 0.5 = 0$ i.e. $3t^2 - 14t - 5 = 0$, which occasionally resulted in the introduction of the spurious value, $t = \frac{1}{3}$.

(ii) The information given in part **(i)** was lost on some candidates who now found it necessary to solve the quadratic in t in order to find $t = 5$. Some candidates, having previously rearranged the expression for v in order to solve e.g. $3t^2 - 14t - 5$, mistakenly integrated this in order to find s. The better candidates had a clear idea, often illustrated by a diagram, of the motion of the particle and completed the question correctly; some succeeded despite employing $\int_0^{1/3}$, $\int_{1/3}^{5}$ and $\int_5^{10}$. Weaker candidates often merely considered $\int_0^{10}$. Some candidates calculated the displacements at $t = 1, 2, \ldots, 10$ and, by considering the sum of the differences, reached the correct answer; this method of attack sometimes failed due to merely adding the displacements or to ignoring the distance travelled between $t = 0$ and $t = 1$ resulting in an answer of 38.9m.

Answer: **(ii)** 40 m.

Question 12 OR

(i) Almost every candidate who attempted this question understood what was required, but success lay mainly in the candidate's ability to deal correctly with $\int \frac{2}{x^2}\mathrm{d}x$. Unsuccessful attempts included $\frac{2}{x}$, $-\frac{1}{2x}$ and various terms involving x^{-3}. Errors sometimes occurred in evaluation; these usually involved $\frac{2x^{-1}}{-1}$ or the non-use of brackets e.g. – 6 – 1 rather than – (6 – 1).

(ii) Most difficulty was caused by the inability to differentiate $\frac{b}{x^2}$ correctly. Although a few candidates failed to make use of the fact that (2, 3) satisfied the equation of the curve, most realised that two equations were necessary in order to evaluate a and b. The final demand, to determine the nature of the stationary point, was quite often overlooked.

Answers: **(i)** 18.5 units^2; **(ii)** $a = 1$, $b = 4$; minimum.