ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key Messages

Candidates should be reminded of the importance of reading the rubric on the front of the question paper, especially the instructions concerning accuracy. Work within a solution should always to be done to an accuracy greater than 3 significant figures, then giving the final answer correct to 3 significant figures unless otherwise instructed within the question.

Candidates should always check that they have completed the demands of the question and that their work is set out in a clear and concise way within the space provided on the question paper. If extra space is needed it is beneficial to make an appropriate comment by the question for which extra space is needed, informing the Examiner where this extra work may be found.

General Comments

There did not appear to be any time issue with the paper set and many candidates produced scripts of a good standard showing a clear understanding of the syllabus aims and objectives. However, it was also clear that there were many candidates who were not sufficiently prepared for the examination, evidenced by scripts which left many questions not attempted and those that were showed a complete lack of understanding of some of the syllabus content.

Comments on Specific Questions

Question 1

Most candidates attempted to form a three-term quadratic equation, but some were unable to cope with the fact that the coefficient of x was $2k-8$. The subsequent use of the discriminant of the three-term quadratic equation obtained was common, but many candidates did not simplify the resulting linear inequality correctly. The correct answer was rarely seen, with many candidates omitting to deal with the multiplication of both sides of an inequality by a negative quantity correctly.

Answer: $x < 2$

Question 2

Candidates found this a challenging question, and many did not realise that they needed to integrate the given derivative and introduce an arbitrary constant which needed to be found, before repeating the process a second time. Those that did integrate often omitted to introduce an arbitrary constant. Many candidates appeared to misunderstand the question and started working with an equation of a straight line. Very few completely correct solutions were seen.

Answer: $y = \frac{5}{2} - \frac{5}{2}x^2 - 3x$

Question 3

Many completely correct solutions were seen, with candidates showing a good understanding of the use of trigonometric identities and the algebraic skills needed to manipulate them in order to obtain the required result. However, there were many candidates who showed a complete lack of algebraic understanding e.g. incorrect manipulation of the square roots initially. Candidates need to realise that the basic laws of algebra also apply to trigonometric terms.

Question 4

(a) (i) Most candidates realised that the solution involved combinations and found the correct answer.

(ii) Very few correct solutions were seen. Candidates did not appear to realise that the clock and its position had no effect on the situation with the solution simply being one of permutations of 6 objects from 8 objects.

(iii) Again, very few correct solutions were seen, with candidates again not appreciating that the problem involved permutations and that the three music books can also be arranged differently within the set of three music books and also that they can be positioned either side of the clock.

(b) Candidates appeared to cope much better with this part of the question which made use of combinations. Many completely correct solutions were seen with most candidates opting to consider the three possible cases (1 woman, 2 women and 3 women) separately rather than consider the total number of possible teams and a subtraction of the number of teams which had no women.

Answer: **(a)(i)** 28 **(ii)** 20 160 **(iii)** 720 **(b)** 203

Question 5

(i) Most candidates recognised that they needed to differentiate the given equation making use of the product rule. Many completely correct solutions were seen. However, some candidates were unable to differentiate $\ln(2x^2+1)$ correctly, omitting to make use of $\frac{\mathrm{d}}{\mathrm{d}x}(\ln \mathrm{f}(x)) = \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}$. It appeared that some candidates were unsure how to deal with the differentiation of the natural logarithm of a polynomial function rather than that of a linear function. Candidates should be careful to give their numerical answer correct to 3 significant figures in a question of this type. An exact answer was also acceptable in this case, as the form of the answer had not been specified in the question.

(ii) This was very well done by most candidates who had attempted the first part of this question. Most candidates showed a good understanding of approximate changes and made use of their answer to part **(i)** and multiplied it by 0.03. Again, there was an expectation of the correct level of accuracy.

Answer: **(i)** 1.31 **(ii)** 0.0393

Question 6

The key to this question was to list all the elements in each of the given sets and work from there. Most candidates showed a good understanding of the set notation used as evidenced by many correct parts. Only part **(v)** caused any real problems. It was expected that answers were given in the form of a set using the correct notation.

(v) Many candidates were unsure of the subset notation. The question had been intended to test this knowledge together with the fact that there were several possible answers, hence the need to read the question carefully. 'Write down **a** set *E*….'. Many candidates mistakenly gave set *E* as being the same as set *D*.

Answer: **(i)** {3} **(ii)** {1, 3, 5, 6, 7, 9, 11, 12} **(iii)** {1, 5, 7, 11} **(iv)** {1, 9} **(v)** e.g. {2, 4}

Question 7

(i) Most candidates were able to make use of their knowledge of straight line graphs and find the gradient and the intercept on the vertical axis correctly. Whilst most were able to correctly equate the gradient to *b*, many were not able to deal with the intercept being equal to $\ln A$. Some chose to make use of simultaneous equations but this often reduced to the same error in not being able to deal with the constant *A*.

(ii) Candidates were given credit for using a correct method with their responses to part **(i)**, with many being able to gain the method mark available for this.

Answer: **(i)** $A = 233$, $b = 0.25$, **(ii)** 348

Question 8

Many good attempts were seen to this question. Most candidates realised they had to differentiate the given equation making use of the product rule. Problems occurred with the differentiation of $\left(x^2+5\right)^{\frac{1}{2}}$ and the subsequent algebraic manipulation and substitution of $x = 2$. Similar problems occurred for those candidates who chose to re-write the given equation and make use of the product rule. This question was an example of a case where some candidates did not check to see that they had done what was required, stopping their solution at the point where they had a numerical value for the gradient of the tangent. For those that did carry on, most were able to pick up marks for a correct method of finding the equation of the tangent required.

Answer: $9y = 4x + 1$

Question 9

(i) Many candidates realised that the integral was of the form $k(4+x)^{\frac{3}{2}}$. Some errors in the simplification of k were evident with $k = \frac{3}{2}$, rather than the required $\frac{2}{3}$ being common.

(ii) There was a lot of uncertainty as to which area was required. Many candidates chose to integrate their answer to part **(i)** usually making use of the correct limits, but often forgetting to take into account the area of the trapezium involved. For those candidates who had not scored in part **(i)**, they had the opportunity to pick up marks for the area of the trapezium. Marks were not awarded for integration that was not of the form $k(4+x)^{\frac{3}{2}}$. Some candidates chose to consider the area under the line *AB*, by finding the equation of the line *AB* and making use of integration. This was usually just as successful as finding the area of the trapezium.

Answer: **(i)** $\frac{2}{3}(4+x)^{\frac{3}{2}} + c$ **(ii)** $\frac{1}{6}$

Question 10

(i) It was expected that a reference to each of the sides being a radius of a circle of radius 10 cm would be made. Many candidates did just this, but many also just gave a definition of an equilateral triangle. Perhaps more attention should be paid to the meaning of the word ‘explain’.

(ii) Many candidates were unable to write down the required angle in terms of π. It is evident that candidates are uneasy with the use of radians and this is a syllabus area that could be concentrated upon more. Answers of $\pi - \frac{\pi}{3}$ were also common seeming to imply that candidates find it difficult to deal with π algebraically.

(iii) Most candidates were able to use a correct method to find the arc length *CE*, but answers which made use of a combination of degrees and radians were not condoned. Marks were given for correct work resulting from a change from radians to degrees. Correct methods were usually applied to find the length of the line *DE* and a method mark was awarded if use of an incorrect angle from part **(ii)** was made, provided it was in a valid form. Errors due to premature rounding were common, hence the recommendation to always work with figures that are more accurate than 3 significant figures.

(iv) This part was usually done more successfully than part **(iii)**. Most candidates were able to find the area of the required sector and triangle, although some candidates chose to subtract one from the other rather than add the two areas. Again errors due to premature rounding were common.

Answer: **(ii)** $\frac{2\pi}{3}$ **(iii)** 58.2 or 58.3 **(iv)** 148

Question 11

The syllabus area on functions is one that some Centres need to concentrate on as it is evident that many candidates have problems with this topic.

(a) (i) A correct attempt at completing the square was made by most candidates.

(ii) Unfortunately, many candidates did not check their range by considering the given domain and incorrect answers of $y \geqslant -5$ were seen. It is essential that candidates use correct notation when dealing with domains and ranges as incorrect notation will be penalised.

(iii) Many candidates found the inverse of the given function difficult because it was a quadratic function. It was often the inability to deal with the algebra of the situation that let candidates down. This was another question part where many candidates did not answer the question completely, forgetting to write down the domain of their inverse. Many candidates may have been able to pick up an extra mark here as it was given if a candidate had used their answer to part **(ii)** correctly.

(b) This was poorly done by most who attempted it and very few correct solutions were seen. Most candidates were aware of the correct order of operations but most were unable to deal with h^2, with many being under the misapprehension that $\mathrm{h}^2(x) = (5x+2)^2$. For those candidates who did deal with the composite functions correctly, many were then let down by the inability to solve $\mathrm{e}^x = 1$.

Answer: **(i)** $(x+3)^2 - 5$ **(ii)** $\mathrm{f} \geqslant 4$ **(iii)** $\mathrm{f}^{-1}(x) = \sqrt{x+5} - 3, \quad x \geqslant 4$ **(iv)** $x = 0$

Question 12

This was a completely unstructured question. Many candidates were able to gain at least the first 4 marks by finding the coordinates of the points of intersection between the line and the curve. Very few errors were seen. Problems then arose when candidates had not read the question carefully and did not find the midpoint of the two points they had just found. This was the main problem encountered, with most candidates making use of their incorrect line in a correct fashion to complete the question, thus being able to gain some of the available method marks. Many completely correct solutions were seen showing that many candidates were able to use their problem solving skills in a logical well thought out fashion.

Answer: 125

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key Messages

Candidates should be reminded of the importance of reading the rubric on the front of the question paper, especially the instructions concerning accuracy. Work within a solution should always to be done to an accuracy greater than 3 significant figures, then giving the final answer correct to 3 significant figures unless otherwise instructed within the question.

Candidates should always check that they have completed the demands of the question and that their work is set out in a clear and concise way within the space provided on the question paper. If extra space is needed it is beneficial to make an appropriate comment by the question for which extra space is needed, informing the Examiner where this extra work may be found.

General Comments

There did not appear to be any time issue with the paper set and many candidates produced scripts of a good standard showing a clear understanding of the syllabus aims and objectives. However, it was also clear that there were many candidates who were not sufficiently prepared for the examination, evidenced by scripts which left many questions not attempted and those that were showed a complete lack of understanding of some of the syllabus content.

Comments on Specific Questions

Question 1

Most candidates attempted to form a three-term quadratic equation, but some were unable to cope with the fact that the coefficient of x was $2k-8$. The subsequent use of the discriminant of the three-term quadratic equation obtained was common, but many candidates did not simplify the resulting linear inequality correctly. The correct answer was rarely seen, with many candidates omitting to deal with the multiplication of both sides of an inequality by a negative quantity correctly.

Answer: $x < 2$

Question 2

Candidates found this a challenging question, and many did not realise that they needed to integrate the given derivative and introduce an arbitrary constant which needed to be found, before repeating the process a second time. Those that did integrate often omitted to introduce an arbitrary constant. Many candidates appeared to misunderstand the question and started working with an equation of a straight line. Very few completely correct solutions were seen.

Answer: $y = \frac{5}{2} - \frac{5}{2}x^2 - 3x$

Question 3

Many completely correct solutions were seen, with candidates showing a good understanding of the use of trigonometric identities and the algebraic skills needed to manipulate them in order to obtain the required result. However, there were many candidates who showed a complete lack of algebraic understanding e.g. incorrect manipulation of the square roots initially. Candidates need to realise that the basic laws of algebra also apply to trigonometric terms.

Question 4

(a) (i) Most candidates realised that the solution involved combinations and found the correct answer.

(ii) Very few correct solutions were seen. Candidates did not appear to realise that the clock and its position had no effect on the situation with the solution simply being one of permutations of 6 objects from 8 objects.

(iii) Again, very few correct solutions were seen, with candidates again not appreciating that the problem involved permutations and that the three music books can also be arranged differently within the set of three music books and also that they can be positioned either side of the clock.

(b) Candidates appeared to cope much better with this part of the question which made use of combinations. Many completely correct solutions were seen with most candidates opting to consider the three possible cases (1 woman, 2 women and 3 women) separately rather than consider the total number of possible teams and a subtraction of the number of teams which had no women.

Answer: **(a)(i)** 28 **(ii)** 20 160 **(iii)** 720 **(b)** 203

Question 5

(i) Most candidates recognised that they needed to differentiate the given equation making use of the product rule. Many completely correct solutions were seen. However, some candidates were unable to differentiate $\ln(2x^2+1)$ correctly, omitting to make use of $\frac{\mathrm{d}}{\mathrm{d}x}(\ln \mathrm{f}(x)) = \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}$. It appeared that some candidates were unsure how to deal with the differentiation of the natural logarithm of a polynomial function rather than that of a linear function. Candidates should be careful to give their numerical answer correct to 3 significant figures in a question of this type. An exact answer was also acceptable in this case, as the form of the answer had not been specified in the question.

(ii) This was very well done by most candidates who had attempted the first part of this question. Most candidates showed a good understanding of approximate changes and made use of their answer to part **(i)** and multiplied it by 0.03. Again, there was an expectation of the correct level of accuracy.

Answer: **(i)** 1.31 **(ii)** 0.0393

Question 6

The key to this question was to list all the elements in each of the given sets and work from there. Most candidates showed a good understanding of the set notation used as evidenced by many correct parts. Only part **(v)** caused any real problems. It was expected that answers were given in the form of a set using the correct notation.

(v) Many candidates were unsure of the subset notation. The question had been intended to test this knowledge together with the fact that there were several possible answers, hence the need to read the question carefully. ‘Write down **a** set *E*….’. Many candidates mistakenly gave set *E* as being the same as set *D*.

Answer: **(i)** {3} **(ii)** {1, 3, 5, 6, 7, 9, 11, 12} **(iii)** {1, 5, 7, 11} **(iv)** {1, 9} **(v)** e.g. {2, 4}

Question 7

(i) Most candidates were able to make use of their knowledge of straight line graphs and find the gradient and the intercept on the vertical axis correctly. Whilst most were able to correctly equate the gradient to *b*, many were not able to deal with the intercept being equal to $\ln A$. Some chose to make use of simultaneous equations but this often reduced to the same error in not being able to deal with the constant *A*.

(ii) Candidates were given credit for using a correct method with their responses to part **(i)**, with many being able to gain the method mark available for this.

Answer: **(i)** $A = 233$, $b = 0.25$, **(ii)** 348

Question 8

Many good attempts were seen to this question. Most candidates realised they had to differentiate the given equation making use of the product rule. Problems occurred with the differentiation of $\left(x^2 + 5\right)^{\frac{1}{2}}$ and the subsequent algebraic manipulation and substitution of $x = 2$. Similar problems occurred for those candidates who chose to re-write the given equation and make use of the product rule. This question was an example of a case where some candidates did not check to see that they had done what was required, stopping their solution at the point where they had a numerical value for the gradient of the tangent. For those that did carry on, most were able to pick up marks for a correct method of finding the equation of the tangent required.

Answer: $9y = 4x + 1$

Question 9

(i) Many candidates realised that the integral was of the form $k\left(4 + x\right)^{\frac{3}{2}}$. Some errors in the simplification of k were evident with $k = \frac{3}{2}$, rather than the required $\frac{2}{3}$ being common.

(ii) There was a lot of uncertainty as to which area was required. Many candidates chose to integrate their answer to part **(i)** usually making use of the correct limits, but often forgetting to take into account the area of the trapezium involved. For those candidates who had not scored in part **(i)**, they had the opportunity to pick up marks for the area of the trapezium. Marks were not awarded for integration that was not of the form $k\left(4 + x\right)^{\frac{3}{2}}$. Some candidates chose to consider the area under the line *AB*, by finding the equation of the line *AB* and making use of integration. This was usually just as successful as finding the area of the trapezium.

Answer: **(i)** $\frac{2}{3}\left(4 + x\right)^{\frac{3}{2}} + c$ **(ii)** $\frac{1}{6}$

Question 10

(i) It was expected that a reference to each of the sides being a radius of a circle of radius 10 cm would be made. Many candidates did just this, but many also just gave a definition of an equilateral triangle. Perhaps more attention should be paid to the meaning of the word ‘explain’.

(ii) Many candidates were unable to write down the required angle in terms of π. It is evident that candidates are uneasy with the use of radians and this is a syllabus area that could be concentrated upon more. Answers of $\pi - \frac{\pi}{3}$ were also common seeming to imply that candidates find it difficult to deal with π algebraically.

(iii) Most candidates were able to use a correct method to find the arc length *CE*, but answers which made use of a combination of degrees and radians were not condoned. Marks were given for correct work resulting from a change from radians to degrees. Correct methods were usually applied to find the length of the line *DE* and a method mark was awarded if use of an incorrect angle from part **(ii)** was made, provided it was in a valid form. Errors due to premature rounding were common, hence the recommendation to always work with figures that are more accurate than 3 significant figures.

(iv) This part was usually done more successfully than part **(iii)**. Most candidates were able to find the area of the required sector and triangle, although some candidates chose to subtract one from the other rather than add the two areas. Again errors due to premature rounding were common.

Answer: **(ii)** $\frac{2\pi}{3}$ **(iii)** 58.2 or 58.3 **(iv)** 148

Question 11

The syllabus area on functions is one that some Centres need to concentrate on as it is evident that many candidates have problems with this topic.

(a) (i) A correct attempt at completing the square was made by most candidates.

(ii) Unfortunately, many candidates did not check their range by considering the given domain and incorrect answers of $y \geqslant -5$ were seen. It is essential that candidates use correct notation when dealing with domains and ranges as incorrect notation will be penalised.

(iii) Many candidates found the inverse of the given function difficult because it was a quadratic function. It was often the inability to deal with the algebra of the situation that let candidates down. This was another question part where many candidates did not answer the question completely, forgetting to write down the domain of their inverse. Many candidates may have been able to pick up an extra mark here as it was given if a candidate had used their answer to part **(ii)** correctly.

(b) This was poorly done by most who attempted it and very few correct solutions were seen. Most candidates were aware of the correct order of operations but most were unable to deal with h^2, with many being under the misapprehension that $\mathrm{h}^2(x) = (5x+2)^2$. For those candidates who did deal with the composite functions correctly, many were then let down by the inability to solve $\mathrm{e}^x = 1$.

Answer: **(i)** $(x+3)^2 - 5$ **(ii)** $\mathrm{f} \geqslant 4$ **(iii)** $\mathrm{f}^{-1}(x) = \sqrt{x+5} - 3, \quad x \geqslant 4$ **(iv)** $x = 0$

Question 12

This was a completely unstructured question. Many candidates were able to gain at least the first 4 marks by finding the coordinates of the points of intersection between the line and the curve. Very few errors were seen. Problems then arose when candidates had not read the question carefully and did not find the midpoint of the two points they had just found. This was the main problem encountered, with most candidates making use of their incorrect line in a correct fashion to complete the question, thus being able to gain some of the available method marks. Many completely correct solutions were seen showing that many candidates were able to use their problem solving skills in a logical well thought out fashion.

Answer: 125

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key Messages

Candidates should be reminded of the importance of reading the rubric on the front of the question paper, especially the instructions concerning accuracy. Work within a solution should always to be done to an accuracy greater than 3 significant figures, then giving the final answer correct to 3 significant figures unless otherwise instructed within the question.

Candidates should be able to use their calculator effectively and should work with as many figures as possible in their calculations to avoid errors from premature rounding.

Candidates should be aware of when to use radians and to work in radians where appropriate.

Candidates would benefit from practice in differentiating and integrating trigonometric functions and from familiarising themselves with the standard results. These are not included in the list of formulae.

General Comments

Most candidates were able to attempt a high proportion of the questions and the standard of presentation was very good. There was some carelessness in candidates' responses, particularly in miscopying their own figures and in rearranging algebraic expressions. Errors were also made in some simple calculations where a calculator had not been considered necessary.

When giving non-exact answers correct to three significant figures, candidates should be aware that, for a decimal number less than one, the leading zeroes are not counted as significant figures. They should also be aware that if the third significant figure happens to be a zero it should be included in their answer.

Candidates should be aware that radians are used to measure angles in questions concerning trigonometric differentiation and integration and also when finding arc lengths and areas in circular measure questions. Radians should also be used in the solution of trigonometric equations if a range for the answers is given in terms of π. To attempt to convert between degrees and radians in these questions invariably leads to confusion and inaccuracy and candidates should be encouraged to work in radians and to set their calculators in the appropriate mode if necessary.

Comments on Specific Questions

Question 1

This question was generally done well, with most candidates making their intention clear in all parts. Some candidates shaded $A \cap B \cap C$ for parts **(i)** and **(ii)** and shaded extra regions in part **(iii)** suggesting that set notation was not always fully understood. Candidates who assigned numbers to the different regions or drew 'working' diagrams at the side seemed to be the most successful as a systematic approach was required.

Question 2

Most candidates obtained $\cos\left(3x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ but some overlooked $\cos\left(3x+\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$. Very few candidates realised that $3x+\frac{\pi}{4}=\frac{\pi}{4}$ would lead to a solution of $x = 0$ radians, but a good number obtained both $x=\frac{\pi}{6}$ and $x=\frac{\pi}{3}$. Candidates should be aware that if $\frac{\pi}{6}$ is expressed as a decimal it is 0.524 to 3 significant figures and if using decimals they needed to work with more figures before dividing by 3 to obtain this value. Candidates were expected to work with radians throughout this question and those who switched between degrees and radians made the question more difficult.

Answers: 0, $\frac{\pi}{6}$, $\frac{\pi}{3}$

Question 3

Candidates would benefit from improving their knowledge of matrix terminology.

(a) The majority of candidates correctly multiplied to obtain a 2 by 3 matrix, but some candidates made careless errors in their arithmetic.

(b) Although there were many good solutions, not all candidates understand what is meant by an Identity matrix, which meant that they could not progress beyond squaring the matrix **X**. Not all candidates were familiar with the addition of matrices and so were unable to form equations in *m* and *n*.

(c) Many good solutions were seen, but some candidates are unclear about the definition of the determinant of a matrix. Candidates should be aware that $a^2 = 6$ has a positive and a negative solution.

Answers: **(a)** $\begin{pmatrix} 12 & 16 & 4 \\ 30 & 32 & 10 \end{pmatrix}$ **(b)** $m = -4$, $n = 44$ **(c)** $a=\pm\sqrt{6}$

Question 4

Candidates responded well to the request not to use a calculator and many carefully presented solutions were seen.

(i) Candidates demonstrated a good knowledge of rationalisation of the surd in the denominator. Not all candidates realised that very little arithmetic was necessary as the numerator did not require expansion and if left as $47\left(4\sqrt{3}-1\right)$ it would cancel with 47 in the denominator.

(ii) Candidates responded well to this part and many showed full expansions of the two squared terms as required, leading to $\sqrt{98}$ and the correct final answer. Candidates should be aware that they were not expected to handle multiplication of large numbers without a calculator and that the expression brought forward from the first part should have been a simple one.

Answers: **(i)** $4\sqrt{3}-1$ **(ii)** $7\sqrt{2}$

Question 5

Candidates should be aware that $\frac{\pi}{4}$ is a value measured in radians and that radians had to be used when obtaining the value of y at $\frac{\pi}{4}$ and when substituting in the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Candidates should be aware that they should know the derivative of $\tan x$, which is not given in the formulae list. Most candidates were aware of the relationship between the gradient of the normal and the gradient of the curve but should realise that differentiation and substitution of $\frac{\pi}{4}$ was necessary to obtain the gradient of the curve at that point.

Answer: $10y + x - 20 - \frac{\pi}{4} = 0$

Question 6

(i) Candidates should be aware that a sketch graph was required and that detailed plotting was unnecessary if there is an awareness of the basic shape and orientation of the graph of a quadratic function and the effect of the modulus on this graph. For the graph to be relevant for the later parts of the question, it had to be symmetrical with a maximum point to the right of the y-axis and arms that extended above the maximum point.

(ii) Few candidates used the symmetry of the graph to obtain a maximum point at $x = 2$ and hence $y = 16$. Most successful candidates used the lengthier method of using calculus to find the minimum point of the quadratic graph and hence the coordinates of the reflected maximum point.

(iii) Candidates should be aware that this part could be easily answered from inspection of the graph and the maximum point found in part **(ii)** and that no further calculations were required.

Answers: **(ii)** (2, 16) **(iii)** $k = 0$ $k > 16$

Question 7

Candidates, in general, found this question challenging. They needed to be aware that two constants of integration had to be found successively and that the first constant had to be integrated to form part of the final expression for y. Candidates often misunderstood the question and thought that a straight line equation had to be found. Candidates who were unaware of the results for integration of $\cos kx$ and $\sin kx$ could not obtain marks for this question and would benefit from practice with integration of trigonometric functions.

Otherwise, good solutions were sometimes marred by carelessness in the substitution of $\frac{\pi}{9}$ and in the rearrangements to find the constants of integration.

Answer: $y = -\frac{2}{3}\cos 3x + 3\sqrt{3}x - \frac{\sqrt{3}}{3}\pi$

Question 8

(a) This question was well answered. Most candidates obtained $1024k$, $1792k^2$ and $1792k^3$ but some slips were made in rearranging to obtain k and some candidates had k instead of k^2 and k^3.

(b) Although there were many good solutions, there were a significant number of candidates who ignored the negative sign. Some candidates obtaining 84 x 8 went on to calculate their final answer incorrectly. Candidates who calculated $r = 3$ or fourth term should be aware that the actual term was required.

Answers: **(a)** $k = \frac{1}{4}$ $p = 112$ $q = 28$ **(b)** – 672

Question 9

(a) (i) Most candidates made a good start by obtaining 12 or 24. Of those obtaining 24 not all realised that if the Mathematics books were treated as one item a multiplication by 2 was required. Of those obtaining 12 not all realised that they had to multiply by 4.

(ii) Most candidates calculated 5! but not all realised that their answer to part **(i)** had to be subtracted. Very few obtained a correct answer independently of their answer to part **(i)** using 6 x 2 x 3!.

(b) (i) Many correct answers were seen. Those that were incorrect usually used permutations rather than combinations.

(ii) There were many good solutions from candidates using products of combinations relating to the four possibilities. '5 men and no women' was not always included and there was some carelessness in the addition of the four results. Candidates who attempted to subtract from 3003 were not usually successful in identifying what had to be subtracted. A very few candidates used permutations instead of combinations and some added combinations rather than multiplying.

Answers: **(a) (i)** 48 **(ii)** 72 **(b) (i)** 3003 **(ii)** 2862

Question 10

(i) A good number of candidates realised that they needed to show a further figure after 1.970 in their working which could then be rounded to three decimal places. Most candidates substituted correctly in the cosine rule but many also successfully used sine to find the half angle.

(ii) Many candidates correctly obtained $6\left(\dfrac{\pi - 1.970}{2}\right)$ but working with prematurely rounded figures often led to a loss of accuracy. A high proportion of candidates had difficulty obtaining the length of *XY* and many did not attempt to include the length *XY* in their plan. Candidates would benefit from gaining a fuller understanding of what is required for a perimeter calculation.

(iii) Most candidates tried to find the area of a sector and the area of a triangle. They should be aware that the angle *A* or *C* should be used for the area of the required sector using a radius of 6. Not all candidates using a correct sector and triangle knew how to combine them to obtain a final answer and the final mark was often lost because rounded or truncated values were being used in the calculation.

Answers: **(ii)** 9.03 **(iii)** 4.50

Question 11

Most candidates made a good attempt at this long multi-stage question, setting out their working clearly and persisting until a final answer was reached. Many good attempts were marred at an early stage by mistakes with signs in simplification of algebraic expressions and later through miscopying of candidates' own figures. Candidates who correctly obtained points *A* and *B* did not always realise that the perpendicular bisector would go through the mid-point of *AB* and so did not obtain a mid-point to use when forming their straight line equation. Candidates who obtained the coordinates of *C* and *D* as decimals received credit for those answers but were precluded from finding a final answer in terms of $\sqrt{5}$.

It was a feature of this question that candidates were often able to obtain 'correct' answers from incorrect working. These fortuitous answers were not awarded accuracy marks.

Answer: $8\sqrt{5}$

Question 12

(a) Most candidates knew that the best approach was to change each term in the first equation into powers of 2 and the second equation into powers of 3. A majority of candidates successfully obtained a linear equation in *x* and *y* from the first equation, but were less successful with the second where the 1 was not always dealt with correctly and mistakes with the algebra were made. Particular difficulty was found with the change of sign when subtracting $3x$ –12. Candidates often went on to make further careless algebraic errors; particularly, miscopying their own work.

(b) Not all candidates recognised the given equation as a quadratic in 5^z and received no credit for premature use of logs. Candidates should be aware that $5^z = -1$ cannot be solved. Most candidates who solved the quadratic equation correctly went on to use logs correctly for $5^z = \frac{1}{2}$. Candidates should be aware that the final answer should be given to three significant figures.

Answers: **(a)** $x = 4$, $y = -4$ **(b)** $z = -0.431$

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key Messages

Candidates are reminded that if a question requests a specific method no credit will be given for an attempt using a different method.

Candidates should remember to give their answers to the accuracy specified in the rubric on the front cover.

General Comments

Many candidates were well-prepared for this examination and scored high marks, but there were some for whom the paper proved too difficult.

Candidates need to give their answers to the required degree of accuracy, especially angles to one decimal place.

Examiners are often unable to give a method mark particularly when solving an equation as sufficient working is not always seen. The other main loss of marks which could be easily rectified is poor presentation which makes reading of written work difficult.

Comments on Specific Questions

Question 1

Candidates generally knew that the substitution of $x = -2$ would give the desired result of zero in part **(i)** but many did not show the evaluation of the four terms which was necessary to be awarded the mark. Only a few substituted incorrectly and there were some correct attempts at long division. The quadratic factor was generally found very well with a mixture of inspection, synthetic division and most frequently long division applied. There were a large number of candidates who completed the question as instructed but a larger number who ignored the 'hence' and worked separately with the linear and quadratic factors thus never showing a complete factorisation. Some candidates did fully factorise but did not show solutions. There was also a clear implication that some candidates found the roots on their calculators then tried to make the factors fit their values leading to incorrect factors being shown.

Answers: **(ii)** $(x+2)(2x-3)(2x-3)$ $x = -2$, 1.5

Question 2

(i) Most candidates were able to find, in the simplest form, the first three terms of the expansion. A few then divided each term by a common factor, usually 16, thinking they needed to "simplify" it. There were some sign errors and some incorrect attempts to deal with either the minus sign or the 3 or both and some expansions were left unsimplified. However, the majority got at least the first term correct.

(ii) Those candidates who had produced simplified terms in part **(i)** were usually able to correctly find the coefficient in this part. A few candidates left this part blank and there were a number of miscopies of the candidate's coefficients from part **(i)** in their answering of part **(ii)**.

Answers: **(i)** $64 - 576x + 2160x^2$ **(ii)** 1008

Question 3

There were a number of very good solutions to this question. In both parts it was most common to use Pythagoras' Theorem immediately to find the distance between two points rather than to find the displacement vector first, although both methods were used frequently. Finding the velocity vector from the displacement vector then applying Pythagoras was also seen on a number of occasions. In part **(i)** a common error was to divide by 20 resulting in 0.85 although this was frequently recovered to gain 40 minutes in part **(ii)**. It was good to see very few multiplying by $\frac{1}{3}$ or using an inaccurate decimal when dividing. Some candidates were confused by the idea of vector quantities and tried to work with the lengths of $\overrightarrow{OA}, \overrightarrow{OB}$ etc. or calculated $\overrightarrow{AB}$ and $\overrightarrow{BC}$ incorrectly.

Answers: **(i)** $51\,\text{kmh}^{-1}$ **(ii)** 40 mins

Question 4

(a) Those candidates who produced a 2×2 matrix usually had the correct elements in it, with just a few candidates making the odd slip. It was, however, quite common to see 3×3 matrices, formed from taking the product the wrong way around, and matrices with just 2 elements, namely 48 and 34. Only a few candidates omitted to multiply by the 2 with an occasional candidate multiplying both of the original matrices by 2 before performing the matrix multiplication.

(b) (i) Most candidates correctly found the inverse matrix, with the most common error being a determinant of 4 rather than 8, caused by a sign error.

(ii) Candidates found this part more challenging, often forgetting that matrix multiplication is not commutative. Some candidates correctly subtracted **D** from **I** but then did not pre-multiply by $\mathbf{C}^{-1}$. Errors were often seen in the subtraction, with some candidates using an incorrect matrix for **I**. Some candidates, in trying to rearrange the original equation for **X**, began by trying to multiply each side of the equation by $\mathbf{C}^{-1}$, but often omitted to include **D**. There were a number of candidates who did not appreciate that the word hence in the question means that they were to use the result established in the previous part of the question in their working.

Answers: **(a)** $\begin{pmatrix} 48 & 10 \\ 10 & 34 \end{pmatrix}$ **(b)(i)** $\frac{1}{8}\begin{pmatrix} 6 & -2 \\ 1 & 1 \end{pmatrix}$ **(ii)** $\frac{1}{8}\begin{pmatrix} -10 & 18 \\ -3 & -1 \end{pmatrix}$

Question 5

(a) Many candidates gave fully correct solutions. Most candidates knew how to approach the question but a lack, or incorrect use, of brackets caused some to give incorrect indices. Another common error was to write 9^{p-4} as $3^{2(p-4)}$. Some candidates, having correctly given the equations in terms of indices got the two variables confused when it came to solving the equations.

(b) This proved more difficult than part **(a)** with some candidates making little or no attempt. Quite a few candidates added the brackets instead of multiplying them. Of those candidates who correctly multiplied the brackets many were unable to simplify the right hand side. Where candidates arrived at a quadratic equation it was usually solved correctly whether by formula or factorisation. There were however, some very good solutions but only the strongest candidates remembered that a negative solution is not possible.

Answers: **(a)** $p = 5, q = 2$ **(b)** $x = 4$

Question 6

(i) Many candidates correctly identified that *a* should be 3, although an incorrect answer of 5 was quite common. Candidates found obtaining *b* and *c* more difficult, with *c* causing the most difficulty. The value for *c* was sometimes given as a multiple of π or a multiple of $90°$.

(ii) Many candidates obtained a multiple of $\cos cx$ when differentiating, but often forgot to multiply their *b* by *c* for the correct multiple of $\cos cx$ or had an incorrect minus sign in their answer.

(iii) The gradient of the tangent at $x = \frac{\pi}{2}$ was often incorrectly evaluated, with many candidates appearing to have their calculator in degrees mode instead of radian mode. Some candidates used $\frac{1}{\mathrm{d}y/\mathrm{d}x}$ rather than $\frac{-1}{\mathrm{d}y/\mathrm{d}x}$ for the gradient of the normal or omitted to substitute for the value of *x* in their expression and thus did not obtain a numeric value for the gradient. Quite a number of candidates did not appreciate that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ gives the gradient of the tangent to a curve and continued to use this value for the equation of the line omitting to change it to the gradient of the normal.

Answers: **(i)** $a = 3, b = 2, c = 4$ **(ii)** $8\cos 4x$ **(ii)** $\frac{y-3}{x-\frac{\pi}{2}} = \frac{-1}{8}$

Question 7

Many candidates found parts **(i)** and **(ii)** to be the most challenging questions in the whole paper resulting in no attempts made to these parts by a large number of candidates. Many candidates spent a considerable amount of time doing calculations involving lengths and areas which contributed nothing to the solution. Despite the question clearly indicating that similar triangles should be used many ignored this instruction. Many who adopted the ratio approach got the ratios wrong although if the ratios were stated correctly rearranging to find *h* was nearly always correct. As the answer was given for part **(ii)** solutions needed to be fully correct with extra care being required over the use of brackets and including all components of each term throughout. There were many creative attempts, often using an incorrect formula for the volume of a cylinder, but which nevertheless arrived at the given expression for V. It should also be realised that 'working backwards' from the given V and quoting a correct expression for *h* without substantiation will gain no credit. In some cases, candidates who had struggled with the first two parts of the question did not attempt part **(iii)** using the equation for *V* given in part **(ii).** The majority did however, and there were many good solutions with well executed differentiation and solution of the resulting quadratic. A significant number seemed to forget to find the value of V for $r = 4$ and some also omitted to attempt to determine the nature of the stationary value. As usual there were also a number of occasions where V or the second derivative were equated to zero..

Answers: **(i)** $h = \frac{4}{3}(6-r)$ **(iii)** $r = 4,\ V = \frac{128}{3}\pi,$ maximum

Question 8

Most candidates correctly began part **(i)** by finding the gradient of the line *AB*, although a few made evaluation errors. Some candidates stopped at this point, but most went on to correctly find the equation of the line and set $x = 0$. However, a large number of candidates quoted coordinates for the point *C* which was either on the *x*-axis or even not on either axis having set both $x = 0$ and $y = 0$ into their equation of *AB*.

Most candidates quoted correct coordinates in part **(ii)** for the midpoint of *AB* although occasionally candidates subtracted the coordinates instead of adding and sometimes the coordinates were interchanged.

In part **(iii)** most candidates correctly obtained the gradient of the perpendicular bisector but many then used the coordinates of either *A* or *B*, rather than the midpoint *D*, to obtain the equation.

Most candidates correctly put $x = 0$ into their equation for the perpendicular bisector in part **(iv)**.

Many candidates found part **(v)** challenging, with those finding the areas using the 'shoelace' method being the most successful. A number of candidates attempting this method did not multiply by the half whilst others did not repeat their starting point within the matrix and worked on a 2×3 matrix instead of the necessary 2×4 matrix. Those using other methods often incorrectly multiplied the lengths of sides which were not perpendicular to one another or lost accuracy in the calculations.

Answers: **(i)** $(0, 3.5)$ **(ii)** $(3, 5)$ **(iii)** $y = -2x + 11$ **(iv)** $(0, 11)$

Question 9

While many candidates managed to score on all parts of the question it was not unusual for a candidate to score on only one or possibly two parts and gain no credit on the other(s). There were also a significant number who left some or all of the parts blank. As always, candidates are advised to check whether a solution is required in degrees or radians. Additional values often appeared, often by quoting a 'working number' as an answer which led to a lost mark.

(i) A large number of candidates could find a value of $\tan 2x$ if not always the correct one. A number of candidates performed a further incorrect division by 2 to arrive at a value for $\tan x$ Of those who obtained –1.25 far too many lost accuracy in their solution by rounding early for $2x$ or by omitting one of the final values. A frequently seen poor method was to split the given equation into two, one in $\sin x$ and one in $\cos x$.

(ii) There were many fully correct answers. A significant number of candidates could form an equation in $\operatorname{cosec} y$ or $\sin y$ although a considerable number of candidates either did not know the correct identity or used it incorrectly. The resulting quadratic equation was usually solved correctly although many arrived at $y = 90°$ fortuitously from an incorrect quadratic.

(iii) This part provided some very concise and elegant answers from those who worked in both radians and multiples of π throughout. However, too many candidates who knew how to solve the equation including choosing the appropriate quadrants and using the correct order of operations ignored the requirement of how to give their answers and worked with decimal values instead. A number of candidates dabbled with degrees but very few of these got the correct final answers often leaving their answer in degrees or in a combination of degrees and radians.

Answers: **(i)** $64.3°, 154.3°$ **(ii)** $90°, 194.5°, 345.5°$ **(iii)** $\frac{5\pi}{12}, \frac{13\pi}{12}$

Question 10

Most candidates correctly tried to integrate the expression for the velocity in order to find the displacement in part **(i)**, but a few tried to differentiate. Many candidates were able to correctly integrate the exponential terms with respect to t, but some obtained $-x$ rather than $-t$ when integrating –1. Many candidates omitted the constant of integration altogether and it was quite rare to see a candidate attempt to calculate it. Those that did often incorrectly evaluated e^0 as 0.

Almost all candidates attempted to use the suggested substitution in part **(ii)** in the given expression for v setting it to zero. The most common error was for $-u$, rather than $\frac{1}{u}$, used to replace e^{-2t}. Some candidates having correctly solved their quadratic for u forgot to go back to the substitution used and find t.

Those that did attempt to find t usually did so successfully, correctly giving just the one solution. A number of candidates used their expression from part **(i)** instead of the given velocity function incorrectly solving displacement = 0.

Some candidates integrated, rather than differentiated, in part **(iii)**, but many correct expressions for the acceleration were seen. It was less common to see the correct value for the acceleration at this time, due to errors in part **(ii)** and errors in evaluating the expression. In both parts **(i)** and **(iii)** some candidates incorrectly changed the powers of e in their attempts at integration and differentiation.

Answers: **(i)** $s = \frac{1}{2}e^{2t} + 3e^{-2t} - t - 3.5$ **(ii)** $t = \frac{1}{2}\ln 3$ **(iii)** 10

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key Messages

Candidates are reminded that if a question requests a specific method no credit will be given for an attempt using a different method.

Candidates should remember to give their answers to the accuracy specified in the rubric on the front cover.

General Comments

Many candidates were well-prepared for this examination and scored high marks, but there were some for whom the paper proved too difficult.

Candidates need to give their answers to the required degree of accuracy, especially angles to one decimal place.

Examiners are often unable to give a method mark particularly when solving an equation as sufficient working is not always seen. The other main loss of marks which could be easily rectified is poor presentation which makes reading of written work difficult.

Comments on Specific Questions

Question 1

Candidates generally knew that the substitution of $x = -2$ would give the desired result of zero in part **(i)** but many did not show the evaluation of the four terms which was necessary to be awarded the mark. Only a few substituted incorrectly and there were some correct attempts at long division. The quadratic factor was generally found very well with a mixture of inspection, synthetic division and most frequently long division applied. There were a large number of candidates who completed the question as instructed but a larger number who ignored the 'hence' and worked separately with the linear and quadratic factors thus never showing a complete factorisation. Some candidates did fully factorise but did not show solutions. There was also a clear implication that some candidates found the roots on their calculators then tried to make the factors fit their values leading to incorrect factors being shown.

Answers: **(ii)** $(x+2)(2x-3)(2x-3)$ $\quad x = -2,\ 1.5$

Question 2

(i) Most candidates were able to find, in the simplest form, the first three terms of the expansion. A few then divided each term by a common factor, usually 16, thinking they needed to "simplify" it. There were some sign errors and some incorrect attempts to deal with either the minus sign or the 3 or both and some expansions were left unsimplified. However, the majority got at least the first term correct.

(ii) Those candidates who had produced simplified terms in part **(i)** were usually able to correctly find the coefficient in this part. A few candidates left this part blank and there were a number of miscopies of the candidate's coefficients from part **(i)** in their answering of part **(ii)**.

Answers: **(i)** $64 - 576x + 2160x^2$ **(ii)** 1008

Question 3

There were a number of very good solutions to this question. In both parts it was most common to use Pythagoras' Theorem immediately to find the distance between two points rather than to find the displacement vector first, although both methods were used frequently. Finding the velocity vector from the displacement vector then applying Pythagoras was also seen on a number of occasions. In part **(i)** a common error was to divide by 20 resulting in 0.85 although this was frequently recovered to gain 40 minutes in part **(ii)**. It was good to see very few multiplying by $\frac{1}{3}$ or using an inaccurate decimal when dividing. Some candidates were confused by the idea of vector quantities and tried to work with the lengths of $\overrightarrow{OA}, \overrightarrow{OB}$ etc. or calculated $\overrightarrow{AB}$ and $\overrightarrow{BC}$ incorrectly.

Answers: **(i)** $51\,\text{kmh}^{-1}$ **(ii)** 40 mins

Question 4

(a) Those candidates who produced a 2×2 matrix usually had the correct elements in it, with just a few candidates making the odd slip. It was, however, quite common to see 3×3 matrices, formed from taking the product the wrong way around, and matrices with just 2 elements, namely 48 and 34. Only a few candidates omitted to multiply by the 2 with an occasional candidate multiplying both of the original matrices by 2 before performing the matrix multiplication.

(b) (i) Most candidates correctly found the inverse matrix, with the most common error being a determinant of 4 rather than 8, caused by a sign error.

(ii) Candidates found this part more challenging, often forgetting that matrix multiplication is not commutative. Some candidates correctly subtracted **D** from **I** but then did not pre-multiply by $\mathbf{C}^{-1}$. Errors were often seen in the subtraction, with some candidates using an incorrect matrix for **I**. Some candidates, in trying to rearrange the original equation for **X**, began by trying to multiply each side of the equation by $\mathbf{C}^{-1}$, but often omitted to include **D**. There were a number of candidates who did not appreciate that the word hence in the question means that they were to use the result established in the previous part of the question in their working.

Answers: **(a)** $\begin{pmatrix} 48 & 10 \\ 10 & 34 \end{pmatrix}$ **(b)(i)** $\frac{1}{8}\begin{pmatrix} 6 & -2 \\ 1 & 1 \end{pmatrix}$ **(ii)** $\frac{1}{8}\begin{pmatrix} -10 & 18 \\ -3 & -1 \end{pmatrix}$

Question 5

(a) Many candidates gave fully correct solutions. Most candidates knew how to approach the question but a lack, or incorrect use, of brackets caused some to give incorrect indices. Another common error was to write 9^{p-4} as $3^{2(p-4)}$. Some candidates, having correctly given the equations in terms of indices got the two variables confused when it came to solving the equations.

(b) This proved more difficult than part **(a)** with some candidates making little or no attempt. Quite a few candidates added the brackets instead of multiplying them. Of those candidates who correctly multiplied the brackets many were unable to simplify the right hand side. Where candidates arrived at a quadratic equation it was usually solved correctly whether by formula or factorisation. There were however, some very good solutions but only the strongest candidates remembered that a negative solution is not possible.

Answers: **(a)** $p = 5, q = 2$ **(b)** $x = 4$

Question 6

(i) Many candidates correctly identified that *a* should be 3, although an incorrect answer of 5 was quite common. Candidates found obtaining *b* and *c* more difficult, with *c* causing the most difficulty. The value for *c* was sometimes given as a multiple of π or a multiple of 90°.

(ii) Many candidates obtained a multiple of $\cos cx$ when differentiating, but often forgot to multiply their *b* by *c* for the correct multiple of $\cos cx$ or had an incorrect minus sign in their answer.

(iii) The gradient of the tangent at $x = \frac{\pi}{2}$ was often incorrectly evaluated, with many candidates appearing to have their calculator in degrees mode instead of radian mode. Some candidates used $\frac{1}{{}^{\mathrm{d}y}\!/\!{}_{\mathrm{d}x}}$ rather than $\frac{-1}{{}^{\mathrm{d}y}\!/\!{}_{\mathrm{d}x}}$ for the gradient of the normal or omitted to substitute for the value of *x* in their expression and thus did not obtain a numeric value for the gradient. Quite a number of candidates did not appreciate that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ gives the gradient of the tangent to a curve and continued to use this value for the equation of the line omitting to change it to the gradient of the normal.

Answers: **(i)** $a = 3, b = 2, c = 4$ **(ii)** $8\cos 4x$ **(ii)** $\dfrac{y-3}{x-\frac{\pi}{2}} = \dfrac{-1}{8}$

Question 7

Many candidates found parts **(i)** and **(ii)** to be the most challenging questions in the whole paper resulting in no attempts made to these parts by a large number of candidates. Many candidates spent a considerable amount of time doing calculations involving lengths and areas which contributed nothing to the solution. Despite the question clearly indicating that similar triangles should be used many ignored this instruction. Many who adopted the ratio approach got the ratios wrong although if the ratios were stated correctly rearranging to find *h* was nearly always correct. As the answer was given for part **(ii)** solutions needed to be fully correct with extra care being required over the use of brackets and including all components of each term throughout. There were many creative attempts, often using an incorrect formula for the volume of a cylinder, but which nevertheless arrived at the given expression for V. It should also be realised that 'working backwards' from the given V and quoting a correct expression for *h* without substantiation will gain no credit. In some cases, candidates who had struggled with the first two parts of the question did not attempt part **(iii)** using the equation for *V* given in part **(ii).** The majority did however, and there were many good solutions with well executed differentiation and solution of the resulting quadratic. A significant number seemed to forget to find the value of V for $r = 4$ and some also omitted to attempt to determine the nature of the stationary value. As usual there were also a number of occasions where V or the second derivative were equated to zero..

Answers: **(i)** $h = \frac{4}{3}(6 - r)$ **(iii)** $r = 4,\ V = \frac{128}{3}\pi,$ maximum

Question 8

Most candidates correctly began part **(i)** by finding the gradient of the line *AB*, although a few made evaluation errors. Some candidates stopped at this point, but most went on to correctly find the equation of the line and set $x = 0$. However, a large number of candidates quoted coordinates for the point *C* which was either on the *x*-axis or even not on either axis having set both $x = 0$ and $y = 0$ into their equation of *AB*.

Most candidates quoted correct coordinates in part **(ii)** for the midpoint of *AB* although occasionally candidates subtracted the coordinates instead of adding and sometimes the coordinates were interchanged.

In part **(iii)** most candidates correctly obtained the gradient of the perpendicular bisector but many then used the coordinates of either *A* or *B*, rather than the midpoint *D*, to obtain the equation.

Most candidates correctly put $x = 0$ into their equation for the perpendicular bisector in part **(iv)**.

Many candidates found part **(v)** challenging, with those finding the areas using the 'shoelace' method being the most successful. A number of candidates attempting this method did not multiply by the half whilst others did not repeat their starting point within the matrix and worked on a 2×3 matrix instead of the necessary 2×4 matrix. Those using other methods often incorrectly multiplied the lengths of sides which were not perpendicular to one another or lost accuracy in the calculations.

Answers: **(i)** $(0, 3.5)$ **(ii)** $(3, 5)$ **(iii)** $y = -2x + 11$ **(iv)** $(0, 11)$

Question 9

While many candidates managed to score on all parts of the question it was not unusual for a candidate to score on only one or possibly two parts and gain no credit on the other(s). There were also a significant number who left some or all of the parts blank. As always, candidates are advised to check whether a solution is required in degrees or radians. Additional values often appeared, often by quoting a 'working number' as an answer which led to a lost mark.

(i) A large number of candidates could find a value of $\tan 2x$ if not always the correct one. A number of candidates performed a further incorrect division by 2 to arrive at a value for $\tan x$ Of those who obtained –1.25 far too many lost accuracy in their solution by rounding early for $2x$ or by omitting one of the final values. A frequently seen poor method was to split the given equation into two, one in $\sin x$ and one in $\cos x$.

(ii) There were many fully correct answers. A significant number of candidates could form an equation in $\operatorname{cosec} y$ or $\sin y$ although a considerable number of candidates either did not know the correct identity or used it incorrectly. The resulting quadratic equation was usually solved correctly although many arrived at $y = 90°$ fortuitously from an incorrect quadratic.

(iii) This part provided some very concise and elegant answers from those who worked in both radians and multiples of π throughout. However, too many candidates who knew how to solve the equation including choosing the appropriate quadrants and using the correct order of operations ignored the requirement of how to give their answers and worked with decimal values instead. A number of candidates dabbled with degrees but very few of these got the correct final answers often leaving their answer in degrees or in a combination of degrees and radians.

Answers: **(i)** $64.3°, 154.3°$ **(ii)** $90°, 194.5°, 345.5°$ **(iii)** $\frac{5\pi}{12}, \frac{13\pi}{12}$

Question 10

Most candidates correctly tried to integrate the expression for the velocity in order to find the displacement in part **(i)**, but a few tried to differentiate. Many candidates were able to correctly integrate the exponential terms with respect to t, but some obtained $-x$ rather than $-t$ when integrating –1. Many candidates omitted the constant of integration altogether and it was quite rare to see a candidate attempt to calculate it. Those that did often incorrectly evaluated e^0 as 0.

Almost all candidates attempted to use the suggested substitution in part **(ii)** in the given expression for v setting it to zero. The most common error was for $-u$, rather than $\frac{1}{u}$, used to replace e^{-2t}. Some candidates having correctly solved their quadratic for u forgot to go back to the substitution used and find t.

Those that did attempt to find t usually did so successfully, correctly giving just the one solution. A number of candidates used their expression from part **(i)** instead of the given velocity function incorrectly solving displacement = 0.

Some candidates integrated, rather than differentiated, in part **(iii)**, but many correct expressions for the acceleration were seen. It was less common to see the correct value for the acceleration at this time, due to errors in part **(ii)** and errors in evaluating the expression. In both parts **(i)** and **(iii)** some candidates incorrectly changed the powers of e in their attempts at integration and differentiation.

Answers: **(i)** $s = \frac{1}{2}e^{2t} + 3e^{-2t} - t - 3.5$ **(ii)** $t = \frac{1}{2}\ln 3$ **(iii)** 10

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key Messages

Working should always be shown so that marks for method can be awarded, even when an answer is incorrect. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

In “show that...” or “prove that...” questions, every step leading to the given answer must be clearly shown.

When a method adopted in solving a problem seems to be overlong, or unlikely to be successful, it is worth pausing to consider whether or not a different approach might be employed.

In longer multi-part questions candidates should be aware of the possibility of connections between the different parts.

General comments

There was wide variation in the quality of work seen. Some candidates produced excellent answers which displayed sound mathematical knowledge. Others clearly had insufficient knowledge for an examination of this standard.

It is essential that the exact mathematical forms or expressions given in a question are used. The precise position of a bracket, a power, or a root should be very carefully noted (for example see **Question 4** below).

In solving quadratic equations the method should always be shown. If the equation is incorrect, a method mark can then be awarded. Method marks are not awarded for solutions to an incorrect equation taken directly from a calculator.

If the mathematics refers to a practical situation, the candidate should pause to consider if the answer obtained is a reasonable one for the given situation. If it is not, the work should be checked to find the error (for example see **Question 11** below).

Problems are still encountered by Examiners in trying to decipher answers consisting of first attempts at solutions overwritten with second attempts, something which makes the work presented very difficult to read. If the mathematics presented to the Examiner is unclear, credit for it cannot be given.

Comments on specific questions

Question 1

There were many fully correct answers to this question. Some candidates found the equation of the normal rather than the tangent.

Answer: $y = 19x - 35$

Question 2

The steps in the method of solving this type of problem were generally well known. When algebraic errors were made, marks for method were awarded provided this was clearly shown. Although the question was a good source of marks for many candidates, the last mark was very frequently lost, even when the correct critical values of *k* were obtained. The final inequality was often incorrect, or completely absent.

Answer: $k < -12$ or $k > 4$

Question 3

Good application of the quotient and product rules was seen in this question. The clearest answers were those which stated the formulae and included working out of the component parts at the side. Such answers earned credit for method if a mistake was made in finding one of the parts. Some candidates, whilst applying the product rule correctly in part **(b)**, had difficulty in manipulating the result into the given form.

Answers: **(a)** $\dfrac{6x^2 - x^4}{\left(2 - x^2\right)^2}$ **(b)** 6

Question 4

Almost all candidates were able to take the first step of eliminating one of the unknowns. Thereafter, however, answers were of variable quality. The strongest obtained an explicit expression for the remaining unknown, and then showed clearly the rationalisation step needed to obtain a solution in the form given in the question. The weakest made an early algebraic error, and either did not show any rationalisation step, or show any awareness that one was needed. Also, in a few instances, insufficient care was taken in reading the question, the first term in the second equation being taken as $\sqrt{3x}$.

Answer: $x = 4 + \sqrt{3}, \quad y = 1 - 2\sqrt{3}$

Question 5

Some very good answers to this question, occupying only a few lines, were seen. But there were also many, of little merit, which filled the whole of the answer space with fruitless working. Such usually began with two applications of the factor theorem, and showed no apparent appreciation of the fact that two equations were insufficient to solve for three unknowns. When it was realised that this approach was not going to be successful a common strategy was to *assume* that $c = -9$ (which was to be *shown*), and then to solve for *a* and *b*. Such answers had two very serious limitations: they assumed what was to be proved, and they made no use of the information given that the equation had a repeated root.

Answer: $a = -7$, $b = 15$

Question 6

Work seen on logarithmic equations continues to be of very variable quality. Those candidates knowing the laws of logarithms, and the manipulations which can and cannot be done, achieved success with this question very easily. Others continue to assume that, with logarithms, anything is possible: for example, that any multiplication can be converted into an addition, any addition into a multiplication, and that a bracket can be removed as though 'log' is an algebraic quantity. A great deal of totally invalid manipulation was seen, and so candidates would benefit from more experience of work with logarithms.

Answer: $x = \dfrac{5}{8}$ or $x = 3$

Question 7

In this question generally good knowledge was shown of the differential and integral relationships between displacement, velocity and acceleration. Many correct answers to part **(i)** were seen, but in part **(iii)** a mark was often lost through the omission of an evaluated constant of integration. Even when an answer to part **(iii)** was incorrect, credit could be earned in part **(iv)** provided substitution of the values of *t* into the expression for displacement could be seen clearly, and the results subtracted.

Answers: **(i)** –0.16 ms^{-2} **(ii)** because $\frac{10}{(t+2)^2}$ is never zero **(iii)** $-\frac{10}{t+2}+5$ **(iv)** 1m

Question 8

In part **(i)**, most candidates expressed the left side of the given relationship in terms of cos*x* and sin*x*, and after a few short steps completed the proof easily. Those choosing to use the identities given on the *Mathematical Formulae* page of the paper usually struggled much more. It is acceptable in such a question as this to work from the left side to the right side or vice-versa, but candidates need to make it absolutely clear which way they are working. Occasionally a candidate was so unclear about this themselves that after a few manipulations they ended at the same side as the one on which they had begun. Candidates need also to realise that in a proof, where the result is given, absolutely every step needs to be shown.

In part **(ii)**, success was most readily achieved by candidates who used the identity from part **(i)**, and rewrote the equation in terms of cos *x* and sin *x*. Several other (much longer) methods are possible and were seen. One chosen by many candidates involved use (again) of the identities given on the *Mathematical Formulae* page of the paper to rewrite the equation in terms of tan *x* and cot *x*. If developed correctly this can lead to a three term quartic equation in tan *x*, but few candidates were able to obtain this successfully without errors and, even if they did, to solve it correctly.

Both parts of this question have quite short solutions, but many candidates presented excessively long attempts.

Answer: **(ii)** 135°, 225°

Question 9

Whilst the method of completing the square, the subject of part **(i)**, is well known, algebraic errors in its application often occurred. The connection between parts **(i)** and **(ii)** was clear to many, but the connection between parts **(i)** and **(iii)** very much less so. Most candidates opted in part **(iii)** to start again with the function in its original form, substitute $x=\frac{1}{y}$, and obtain a three term quadratic in $\frac{1}{y}$, a much longer method than that using the form obtained in part **(i)**.

Answers: **(i)** a = 3, b = 2, c = –10 **(ii)** –10 at x = –2 **(iii)** –5.74, –0.26

Question 10

Candidates who understood clearly the connection between parts **(i)** and **(ii)** usually obtained full marks on the first three parts. Even with an incorrect answer in part **(ii)**, most knew the method of using this answer in part **(iii)**. As the question contained the word “hence”, marks were not awarded in part **(iii)** for a correct answer produced from the calculator when the answer in part **(ii)** was incorrect.

Few fully correct answers were seen in part **(iv)**. This was almost always a consequence of the product rule not being used when finding $\frac{dy}{dx}$, but occasionally also a consequence of omitting one of the stationary points.

Answers: **(i)** –2 **(ii)** $-\frac{3e^{2-x^2}}{2}+c$ **(iii)** 1.5(e – 1) **(iv)** $\left(\pm\frac{1}{\sqrt{2}},\ \pm\frac{3}{\sqrt{2}}e^{1.5}\right)$

Question 11

Most candidates obtained the correct relationship in part **(i)**, and many used the appropriate variables in part **(ii)**. Those using log*t* on one axis in part **(ii)**, or even simply plotting *N* against *t*, were unable to obtain marks in part **(iii)** because the relationship in these cases is non-linear. Such candidates should have observed this in the plotting of their points, and reviewed their previous work to find the error they had made.

General awareness was shown in part **(iii)** that gradient and intercept had to be used. But whilst the intercept was usually used correctly, a common error with the gradient was to set it equal to log*b* rather than –log*b*.

Some of the answers presented to parts **(iv)** and **(v)** were impossibly large or impossibly small, both positive and negative. When mathematics is applied to a physical situation such as this, there should be some awareness on the part of the candidate as to what constitutes a reasonable answer, and what is totally unrealistic.

Answers: **(i)** $\log N = \log A - t\log b$ **(iii)** $A = 2950$, $b = 1.50$ **(iv)** 51 **(v)** 14

Question 12

A clear diagram showing the relationship between the vectors is of crucial importance in solving problems on relative velocity, and a reasonable number of candidates produced such a diagram. Generally, when a good clear diagram was presented, full marks were earned. However there were also a number of much weaker attempts based on inaccurate or incomplete diagrams.

Answer: plane 271, windspeed 50.1