ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

This paper produced a wide range of marks. Some candidates appeared to lack the knowledge required in certain questions (especially the use of vectors, in both **Question 6** and as a means of finding point *P* from points *A* and *B* in **Question 3**); others had the knowledge and even knew the processes, but were unsure of the application. Most candidates seemed to have time to do all they could but some spent time on overlong efforts (especially in **Question 3**) with little apparent regard for exam technique/timing. Some candidates misquoted formulae (especially differentiation of quotients and products) and others produced answers without showing full working. This applied mainly to solution of quadratic equations and definite integration. In these cases an incorrect answer cannot gain any available method marks. The question paper rubric states 'The use of an electronic calculator is expected, where appropriate'. The use of the more complex inbuilt calculator functions sometimes does not give candidates the opportunity to show what they know since working is omitted. This session saw more than usual disregard for the rubric requirement for 3 significant figures (or 1 decimal place for angles in degrees). Furthermore, even some of those candidates adhering to this were not consistent in working to one figure more before correcting. Standards of presentation remain variable, both in legibility and in organisation, e.g. not labelling question or part numbers.

Comments on specific questions

Question 1

This proved to be difficult for many candidates with few fully correct solutions seen. A large number shaded just $A \cap C'$ in part **(i)**. These were a reasonable number of candidates who managed part **(ii)** but fewer who were successful in part **(iii)** with many employing a great variety of shading with no key to this shading.

Answer: (a)**(ii)** $A \cap B' \cap C$.

Question 2

A surprisingly large number of candidates failed to differentiate at all while others applied either product or quotient rule, wrongly in many cases. Of those who applied them correctly some made an error in not having the $(2x + 4)$ in brackets. The majority of candidates who (somehow) obtained a value for the gradient of the tangent did know how to use this to find the gradient of the normal. This question was one of the better attempted questions.

Answer: $2y = x + 8$.

Question 3

Finding points *A* and *B* by removal of one variable and solution of the subsequent quadratic equation was done well by most candidates (even those who took the harder route by forming a quadratic in *y*). Surprisingly few candidates tried either a grid/graph method of solution or a vector move method. The two main approaches seemed to be (*a*) finding the equation of the line *AB* and solving with the original line equation not realising that the two are (or should be) the same and therefore that this method is of no use, (*b*) using Pythagoras' Theorem to compare any of *AP*, *PB* and *AB*.

In the latter case some of those who used the given ratio correctly forgot to square the 2 (or 3) from the ratio when trying to form an equation. Even those who did all this correctly still came down to just one equation in two variables and many ground to a halt. The more persistent then formed a second equation and attempted to solve these together. Few who used the Pythagoras approach realised that their first quadratic could be solved with the original equation of the line. Many candidates seemed to spend a great deal of time on this question, often to no avail. To make matters worse a worryingly large number of candidates translated $AP:PB = 1:2$ as meaning $AP = 2PB$.

Answer: (4, –3).

Question 4

Part **(i)** was well done by most candidates, although a minority confused ascending/descending.

Part **(ii)** was not so successfully done. Some candidates failed to see the connection with part **(i)** and embarked on the expansion of $(2 + 2x - 5x^2)^5$. The substitution given was, in a number of cases, applied only to one of the two necessary terms but a large number of candidates did manage the substitution correctly and combined the two resulting x^2 terms to get the answer required.

Answers: **(i)** $32 + 80u + 80u^2$; **(ii)** –80.

Question 5

A sizeable number of candidates decided to combine the two given terms and then to differentiate as a quotient twice. This tended to lead to extra work in all three parts of the question. Many of these candidates quoted or used the quotient rule incorrectly. Those candidates who attempted part **(i)** term by term made a much better job of the question.

Part **(ii)** asked the candidates to show (not prove) there was a stationary value at $x = 9$ and many simply put the value 9 into their first differential and showed this was equal to 0, which was quite acceptable. Others put their first differential equal to 0 and solved to find the x-value. It should be noted that, where a question states ‘show that', candidates should present the Examiner with every step of working.

Part **(iii)** was mainly tackled by looking at the sign of the second derivative, but as this was the sum of a negative and a positive, full and clear evaluation was expected. Some candidates seemed to try this part by looking at gradients or even y-values, but all too rarely showed the Examiner the necessary detail to show what they were doing in order to justify their conclusion.

Answers: **(i)** $\frac{1}{2\sqrt{x}} - \frac{9}{2\left(\sqrt{x}\right)^3}$, $-\frac{1}{4\left(\sqrt{x}\right)^3} + \frac{27}{4\left(\sqrt{x}\right)^5}$; **(iii)** Minimum.

Question 6

This was by far the least well done question on the paper. Many candidates ignored it, some wrote perhaps two lines of (usually incorrect) working and few fully correct answers were seen. As mentioned earlier ‘show that...' meant that candidates could use the 1.8 and arrive at the situation where the two vertical displacements (from the bottom of the screen) were identical. Using the 1.8 again and considering horizontal displacements (from the left of the screen) led to a value for k. Problems arose, however, with the half second delay before the missile was fired. Many ignored this altogether, others thought this meant 1.8 and 2.3 as times (rather than 1.8 and 1.3)...others used 0.5 seconds as the time. The other common attempt involved finding the position vector of the spacecraft after ‘t' seconds and the position vector of the missile after $t - \frac{1}{2}$ seconds. An equation involving vertical displacements led to $t = 1.8$ then an equation for horizontal displacements led to k. Common errors included misusing the $\frac{1}{2}$ second and ignoring the initial positions. Some candidates attempted to use relative velocity and relative displacement, but all too often failed to appreciate that the relative displacement must relate to the time when both spacecraft and missile were moving. The relatively few candidates who made a decent attempt suggests that many candidates are just not familiar enough with position vectors, or are just not confident in tackling such questions.

Answer: **(ii)** 20.

Question 7

Better candidates had little trouble with part **(i)** although too often they gave only a 2-figure answer. Weaker candidates often failed to recognise $5u$ on the left or $4u^{-1}$ on the right. Some candidates who reached something such as $5^x = k$ could not solve to find x while others produced an answer with no visible working and hence gained no credit for method if the answer was wrong. Part **(ii)** was well done by better candidates but poorly done by weaker ones. Several produced the final line '$p - q = p - q$'. Others had complicated equations with logarithms everywhere and some fortuitously came to the correct answer by making two successive blunders in the manipulation of logarithms. The most common error was to write the right-hand side of the equation as $\frac{\log p}{\log q}$ and to then cancel the word 'log'.

Answers: **(a)** $x = 0.431$; **(b)** $p = \frac{q^2}{q-1}$.

Question 8

In part **(a)** the majority of candidates realised that $\cos 3x = -0.2$ was required but not all were able to deal with the minus sign. Some worked in degrees and then converted but others ignored radians altogether. Many candidates gave just one solution, failing to realise that they needed to consider values of $3x$ up to 6 radians.

Part **(b)** proved straightforward for a majority of candidates who used correct trigonometry, solved the resultant quadratic and found solutions. Some candidates however started off by stating that '$\sec x = \frac{1}{\sin x}$' or that '$\sec x = 1 + \tan x$'. Others managed the first stage (obtaining an equation with just two trigonometric functions) then used an incorrect relationship such as $\cos x = 1 - \sin x$ or $\cos^2 x = 1 + \sin^2 x$. All too often the trigonometric work was correct but the 5 and the 3 from the original expression were lost somewhere along the way.

Answers: **(a)** 0.59 radians, 1.50 radians; **(b)** 19.5°, 160.5°.

Question 9

Part **(i)** was quite well done with only the weakest candidates not able to do what was asked. The use of 'unsuitable' scales, however, led to some misplotting and/or misreading of points.

The five points were on, or very close to, a perfect straight line, a fact used by most in part **(ii)**, but there were several candidates who had one (or more) points misplotted and whose graph was not a straight line but who, nevertheless, carried on to parts **(ii)** and **(iii)** as if it were.

Because the intercept on the vertical axis was negative, some candidates could not find this from their graph, but using the graph to find the gradient and hence the equation of the line removed the need to find 'c'. All too often however the straight line equation was $y = mx + c$ rather than $\frac{1}{y} = \frac{m}{x} + c$. Some of the better candidates did everything properly other than answering the question 'express y in terms of x'', or wrongly inverted each term.

Part **(iii)** was more commonly done by algebra, using the equation from part **(ii)** but more success was gained by those who used the graph although even here some candidates went straight to 0.15 on the vertical axis rather than $\frac{1}{0.15}$.

Answers: **(ii)** $y = \frac{x}{2.2 - 2x}$; **(iii)** 0.25.

Question 10

Many candidates showed a disregard for the accuracy required in calculations. In part **(i)** the question asked candidates to show angle *BCE* was equal to 1.287 radians correct to 3 decimal places. Various methods were employed, most of which had a final line of working that angle $BCE = \pi -$ angle ACB. However many gave the latter angle as 1.86 or 1.85 or 1.854 but not as 1.8546, the number of figures necessary. It is possible that candidates held more figures in their calculators than they wrote down but it was necessary to show Examiners clear working here.

Accuracy aside, part **(ii)** required the ability to find two arc lengths (usually done well) and a straight line length, and add these together. Some candidates found the arc length involving a radius of 10 m. Others managed to mismatch radii and angles.

Part **(iii)** required some complex thought but credit was obtained for finding the area of a sector and for finding the area of triangle *ABC* even if the candidate could not cope with the correct linking of the three areas necessary.

Some candidates tackled parts **(ii)** and **(iii)** in degrees, turning 73.7398° into 1.287 radians in part **(i)** and then continuing by using their angles as fractions (with denominator 360) of the circumference and of the area of the full circle. There were some candidates who used $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ with θ in degrees.

Answers: **(ii)** 13.6 m; **(iii)** 7.50 m^2.

Question 11 EITHER

For candidates who were familiar with calculus linked to trigonometry this was a very straightforward and rewarding question. Setting the first differential equal to zero and equating this to zero was well within the capabilities of most who tried this question but, again, many threw away a mark by giving an answer which was not to the required accuracy. The need to integrate to find the area was recognised by most candidates but some lost marks through sign errors, or through errors in substituting $\frac{1}{2}\pi$, or by omitting to substitute the lower limit of zero.

Answer: **(i)** (0.644, 5); **(ii)** 7 square units.

Question 11 OR

On the whole, this was less well done than the other alternative. Many candidates omitted the '3' from the differentiation of 'function of a function' and, likewise, omitted to divide by 3 when integrating. That said, the majority of candidates realised the need to differentiate to find the equation of the tangent, and to find where this crossed the *x*-axis. There were some strange attempts in part **(ii)** at finding the area, with rectangles and trapezia appearing in some solutions. Most candidates realised the need to integrate but some omitted to evaluate this at the bottom limit. Not all of those who got this far realised that they needed to subtract the area under the line. Once again some candidates reached a final answer which was not accurate enough.

Answers: **(i)** $\frac{1}{3}$; **(ii)** $\frac{1}{6}$ square unit.

ADDITIONAL MATHEMATICS

Paper 0606/02
Paper 2

General comments

The general standard of work was similar to that on the corresponding paper of 2006, but there was still a large number of candidates who appeared ill-prepared for this examination and who failed to reach a reasonable standard.

Comments on specific questions

Question 1

Some candidates took the area of the triangle to be $x(13 - 2x)$ m^2 and others failed to make any connection with the given area of 3 m^2. The handling of inequalities was poor in many cases with errors such as $13x - 2x^2 - 6 > 0 \Rightarrow 2x^2 - 13x + 6 > 0$ and $(2x - 1)(x - 6) < 0 \Rightarrow x < \frac{1}{2}$ or $x < 6$. The critical values of $\frac{1}{2}$ and 6 were found by the majority of candidates.

Answer: $\frac{1}{2} < x < 6$.

Question 2

Although the value of c was usually correct, very few candidates were able to find the values of a and/or b. The work offered was usually nonsensical with answers often given as multiples of π.

Answers: **(i)** 3; **(ii)** 1; **(iii)** 2.

Question 3

Candidates who substituted p and/or q for x, or eliminated p or q from $p + q = \sqrt{28}$, $pq = 2$ were usually unable to make any progress. Those who proceeded from $\frac{1}{2}(\sqrt{28} \pm \sqrt{20})$ to $\sqrt{7} \pm \sqrt{5}$ generally had little difficulty in completing the question. The process of rationalising the denominator was well known but candidates using $\frac{1}{2}(\sqrt{28} \pm \sqrt{20})$ sometimes made arithmetical errors or were unable to reduce $\sqrt{560}$ to $4\sqrt{35}$.

Answer: $6 + \sqrt{35}$.

Question 4

(i) Candidates giving an incorrect answer almost invariably derived it from $^{10}P_4$. Some candidates considered the five possible combinations, calculated the number of selections corresponding to each and then added; such candidates were usually able to obtain the required answer.

(ii) Although this part produced fewer correct solutions than part **(i)**, there were still many correct answers. A few candidates interchanged the operations of addition and multiplication but most errors were caused by the omission of one of the two possible combinations, most often 4 watercolours and 0 oils. Candidates attempting the more roundabout method of subtracting from 210 frequently overlooked the case of 2 watercolours and 2 oils.

Some of the weakest candidates thought that the number of combinations was required and so gave their answers to parts **(i)** and **(ii)** as 5 and 2 respectively.

Answers: **(i)** 210; **(ii)** 95.

Question 5

This proved to be one of the easiest questions on the paper in that the laws of indices were clearly well understood. Some candidates failed to deal with part **(i)** correctly, in effect giving a power of $\frac{1}{2}$, $\frac{1}{2^{5/2}}$, rather than $2^{-5/2}$; such candidates almost always used $2^{-5/2}$ in part **(iii)**. Conversely, some candidates stating $2^{-5/2}$ in part **(i)**, then used $\frac{1}{2^{-5/2}}$ in part **(iii)**, leading to – 4 or + 1.5. These values also arose from incorrect factorisation with candidates arguing that $2x^2 - 5x - 12 \equiv 2x^2 + 8x - 3x - 12 \equiv 2x(x + 4) - 3(x + 4)$.

Answers: **(i)** $2^{-5/2}$; **(ii)** $2^{6/x}$; **(iii)** –1.5, 4.

Question 6

(i) Differentiation of a product was attempted by most candidates. A few of these were unable to differentiate $\ln x$ whilst others committed errors after differentiating correctly, e.g. $(\ln x).(2x)$ became $\ln 2x^2$, or $x + 2x\ln x$ became $3x\ln x$ or the meaningless $x(1 + 2\ln)$.

(ii) Only the very best candidates could reverse the differentiation of part **(i)**. Many simply transferred the x, so that $\int (2x\ln x + x)\mathrm{d}x = x^2$ became $\int 2x\ln x\,\mathrm{d}x = x^2\ln x - x$; this transfer was facilitated by the almost universal absence of the differential dx. Because the answer was given, many candidates contrived to obtain $e^2 + 1$, but few achieved this in a valid manner.

Answer: **(i)** $x + 2x\ln x$.

Question 7

(i) Apart from simple numerical mistakes the most common error was to assume that, as $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix}$, then $\mathbf{A}^2 = \begin{pmatrix} 4 & 9 \\ 4 & 1 \end{pmatrix}$ or even $\begin{pmatrix} 4 & 9 \\ -4 & -1 \end{pmatrix}$. The addition of matrices was clearly understood.

(ii) The adjoint of **A** was sometimes incorrect, e.g. $\begin{pmatrix} -2 & -2 \\ 3 & 1 \end{pmatrix}$ and the discriminant of **A** was occasionally found to be – 8 from – 2 – 6, but the most common error in finding **Y** by using the inverse matrix was to pre-multiply **B** by $\mathbf{A}^{-1}$ rather than post-multiply. A surprising feature was the number of candidates who, having taken $\mathbf{A}^2$ in part **(i)** to be $\begin{pmatrix} 4 & 9 \\ 4 & 1 \end{pmatrix}$, showed, in part **(ii)**, that they were able to correctly multiply two matrices. Many candidates answered part **(ii)** correctly by taking **Y** to be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and then solving simultaneous equations. Some, however, took **Y** to be $\begin{pmatrix} y & y \\ y & y \end{pmatrix}$ so that $2y = 8$, $3y = 10$, $-2y = -4$ and $-y = 2$ then led to $\begin{pmatrix} 4 & \frac{10}{3} \\ 2 & -2 \end{pmatrix}$. This result was also obtained by those candidates who assumed it was possible to divide one matrix by another i.e. **Y** = **B**/**A**, by dividing each element of **B** by the corresponding element of **A**.

Answers: **(i)** $\begin{pmatrix} 14 & 23 \\ -10 & -1 \end{pmatrix}$; **(ii)** $\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}$.

Question 8

(i) Only a small minority of candidates scored full marks. Nearly all were able to eliminate y. Some then attempted the solution of the quadratic but rarely continued further. Most were able to identify correctly the elements a, b and c of the discriminant. Many candidates simply considered $b^2 - 4ac = 0$, and the handling of the inequality, when present, was often poor e.g. $-8k > -20 \Rightarrow k > 2.5$. The most frequent answer was $k = 2.5$ with an inequality sign never in evidence and the information that k was an integer ignored. Many of those candidates who arrived at $k \varnothing 2.5$ failed to provide the correct answer and some candidates even wrote that '2.5 is the largest integer'. The value 2.5 was also arrived at by the relatively small number of candidates who considered gradients, via $\frac{dy}{dx} = x$ (for C) $= 3$ (for L) $\Rightarrow y = 6.5 \Rightarrow k = 2.5$.

(ii) The equation of the line L was occasionally taken to be $y = 3x - 2$, in effect taking k to be +2 rather than – 2. The elimination of y and solution of the resulting quadratic in x were understood by nearly all candidates but some could only find one point of intersection, believing that $x = 6$ was the only solution of $x^2 - 6x = 0$. Despite finding both points of intersection, (0, 2), (6, 20), candidates commonly failed to complete the question, being unable to appreciate the simple stratagem of showing that the coordinates, (3, 11), of the mid-point of the line joining the points of intersection of L and C, satisfied the equation $y = 2x + 5$. Many never found the coordinates of the mid-point. Others simply showed that $y = 3x + 2$ and $y = 2x + 5$ intersected at (3, 11), although some candidates having done this, belatedly showed that this was also the mid-point of the points of intersection of L and C. For many candidates the word 'bisected' indicated that the perpendicular bisector was involved and puzzlement followed when the equation of this line turned out to be $3y + x = 36$ rather than $y = 2x + 5$.

Answer: **(i)** 2.

Question 9

(i) Candidates usually combined vectors correctly e.g. $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$, although some thought that this combination gave $\overrightarrow{RP}$, whilst some of the weakest candidates thought that $\overrightarrow{PR}$ was obtained from $\overrightarrow{OP} + \overrightarrow{OR}$ The magnitudes of vectors were almost always correct but the use of decimals rather than surds sometimes led to unnecessary approximations. Most candidates used the Pythagoras theorem to establish that angle PQR was 90°, although a considerable number used the cosine rule. A few candidates ignored the instructions given in the question and used the gradients of PQ and QR.

(ii) This proved to be the most difficult part of the question with many candidates, mainly the weaker ones, seemingly unable to grasp the basic idea that the magnitude of a unit vector is unity.

(iii) Candidates answered this part of the question very well and there were many correct answers. A careless error, seen occasionally, was to obtain $n = 4\frac{1}{2}$ from $9n = 2$.

Answers: **(ii)** $0.8\mathbf{i} + 0.6\mathbf{j}$; **(iii)** $m = 3$, $n = \frac{2}{9}$.

Question 10

(i) Only the very weakest candidates interpreted f^{-1} and g^{-1} as $\frac{1}{f}$ and $\frac{1}{g}$. The only common error in finding f^{-1} occurred in proceeding from $x = 3y - 2$ to $x - 2 = 3y$. The method of finding g^{-1} by making y the subject of $\frac{7y - a}{y + 1}$ was clearly understood by most, but poorer candidates did not have the algebraic skill to succeed.

(ii) Some candidates carelessly attempted $f^{-1}g^{-1}$ (4) = 2 whilst others reversed the order of operations, in effect using gf^{-1} (4). Only the weakest candidates took $f^{-1}g$ to be $f^{-1} \times g$. The required algebra proved difficult with fairly frequent errors in the simplification of $\dfrac{\frac{28-a}{5}+2}{3}$ e.g. $\frac{3}{5}(28-a+10)$ or $\frac{1}{3}\left(\frac{28-a}{5}\right)+2$. It was pleasing to see that a few candidates solved g (4) = f (2).

(iii) Candidates usually obtained an equation in x, either quadratic or of a higher order, and attempted to solve.

Answers: **(i)** $\frac{x+2}{3}$, $\frac{a+x}{7-x}$; **(ii)** 8.

Question 11

This question proved to be difficult for very many candidates. Apart from those candidates who attempted to use constant acceleration formulae or 'speed = distance ÷ time', most understood that integration was required in each of the two stages of obtaining expressions for velocity and displacement. Unfortunately, in finding the velocity, many candidates either failed to include a constant of integration or, having introduced c, showed a lack of understanding of the first sentence of the question by concluding with 'when t = 0, v = 0 and therefore c = 0'. Part **(ii)** was frequently omitted and few candidates realised that stationary values of velocity occur when the acceleration is 0. Some candidates attempted a solution of part **(ii)** by trial and error, but in every case only integer values of t were considered. In part **(iii)** the positive and negative displacements occasionally caused difficulty and some candidates found the total distance travelled by the particle in moving from O to B.

Answers: **(i)** -5 ms^{-2}, 5 ms^{-2}; **(ii)** 6.25 ms^{-1}; **(iii)** $20\frac{5}{6}$ m.

Question 12 EITHER

(i) Finding the equation of two lines and solving these equations to find the coordinates of the point of intersection was clearly understood by the candidates attempting this alternative. The weakest candidate unfortunately found the equations of AB and AE or of AE and EB which, handled correctly - usually not the case - should have led to the coordinates, already given and used, of A or E respectively. Better candidates had little difficulty in arriving at the correct coordinates, (5, 8).

(ii) This part of the question proved to be an insuperable stumbling-block for the many candidates who assumed that the area of triangle EBC was given by $\frac{1}{2}$ $EB \times BC$. Few of the candidates who used the array method appeared to be aware of the significance of the modulus signs and, as most of them arrayed the coordinates in a clockwise direction, there were many incorrect solutions. Arriving at $|\ 12 - 4x\ | = 48$, candidates should have considered the possibilities $12 - 4x = \pm\ 48$, leading to $x = -\ 9$ or $x = 15$, of which only the latter was acceptable on consideration of the given diagram; most simply took $12 - 4x$ to be 48 giving the answer – 9, which was then usually fudged to + 9, a value also obtained by those who thought that $|\ 12 - 4x\ |$ was $12 + 4x$. The relatively simple idea that $x_C = x_E$ + length of EC, where $\frac{1}{2}$ (length of EC) $\times$ $(y_B - y_E)$ = 24 was only seen infrequently.

(iii) Employing the equation of AD and the x-coordinate of C in order to find the y-coordinate of D was well understood.

Answers: **(i)** (5, 8); **(ii)** (15, 4); **(iii)** (15, – 2).

Question 12 OR

This was the more popular alternative.

(a) There were few errors in principle but a surprisingly large number of candidates were unable to avoid arithmetical error. Some candidates made life more difficult for themselves by ignoring the structuring of the question, collecting four equations in the four unknowns *a*, *b*, *c* and the remainder *R* and then attempting the solution of these by obtaining an expression for *a*, say, in terms of *b*, *c* and *R* from one equation, substituting in other equations, etc.

(b) This was well done on the whole although it appeared that some candidates, by only giving the answers 0.73 and – 2.73, did not perhaps appreciate that $x = -1$ was also a solution of the equation, but merely regarded the factor $x + 1$ as a convenient way to obtain a quadratic equation. Perhaps because the given equation only contained three terms some candidates tried to use the quadratic formula directly. The three terms also caused difficulty for some candidates employing synthetic division, using 1 3 –2 rather than 1 3 0 –2. Some of the weakest candidates took $x(x^2 + 3x) = 2$ to imply $x = 2$ or $x^2 + 3x = 2$. A few candidates ignored the instruction to give 'answers to 2 decimal places' and either left their answers as surds or used only one decimal place.

Answers: **(a)(i)** – 4, **(ii)** – 1, **(iii)** – 2; **(b)** – 2.73, –1, 0.73 .