ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key message

In order to ensure that candidates receive credit where possible for incorrect answers, they should remember to show all their working. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown.

General comments

The majority of candidates were sufficiently well prepared to make a reasonable attempt at most of the questions on the paper. Some candidates would have benefited from more practice in most topic areas.

As noted above, candidates should be reminded of the need to show all the steps in their working, particularly in questions where the answer is given, such as **Qs 1**, **6** and **12 OR**.

In order not to lose marks unnecessarily, candidates need to give non-exact numerical answers to 3 significant figures, which should require working to at least 4 significant figures during the solution of a question.

Comments on specific questions

Question 1

There were many excellent solutions starting from the left-hand side of the identity. Those starting from the right-hand side made little useful progress. Candidates need to understand that all significant steps in the argument should be shown in order to gain maximum credit.

Question 2

Few candidates managed to get full marks on this question. Many candidates obtained $\log ab^3$ but could not incorporate the 3 into a single logarithm.

Answer: $\log\left(\frac{ab^3}{1000}\right)$

Question 3

(a) Most candidates were able to identify and shade the required regions.

(b) Many candidates did not appreciate the difference between P and $n(P)$.

Answer: **(b)** 3

Question 4

(a) Most candidates attempted the simplest method of solution which was to convert both the terms into powers of 2. Those who attempted to convert into powers of 4, 8, or 16 were much less successful. A few candidates were able to obtain the solution by taking logs.

(b) This question was generally done well, and most candidates were able to use the laws of indices to gain full marks.

Answers: **(a)** $\frac{4}{3}$ **(b)** $p = 1,\ q = -\frac{4}{5}$

Question 5

(i) The majority of candidates realised that a sine curve between –2 and 4 was required. Many slips were made with regard to starting and finishing at y = 1 which resulted in there not being exactly one cycle of the graph.

(ii) The concept of modulus did not seem to be generally understood, and so this is an area where more practice would be beneficial.

(iii) Most candidates who had answered part **(ii)** correctly were able to relate the answer to this question to their graph in part **(ii)**. It was common to see attempts to list the solutions of the equation, rather than write down the number of solutions. Candidates should ensure that they read the question carefully to ascertain what is required.

Answer: **(iii)** 5

Question 6

(i) Most candidates solved the equations simultaneously leading to x = 2. A few correct solutions were also obtained by substituting x = 2 in both equations, leading to y = 4.

(ii) Most candidates differentiated successfully and substituted x = 2 to obtain the required result.

(iii) This part was generally well done by candidates.

Answer: **(iii)** $y = 4x - 4$

Question 7

Many candidates took the opportunity to present a well set out, step-by-step solution to this problem, and gained full marks. Any mistakes made were usually the result of not reading the question carefully enough, and a number of candidates found either the equation of the perpendicular through *B*, or through the mid-point of *AB*, or took *C* as a point on the *x*-axis.

Answer: 12.5

Question 8

(a) As well as the two correct matrix products, other incorrect ones were often included. A number of candidates wasted time by evaluating the matrix products despite the instruction in the question not to do so.

(b) Most candidates knew how to find $\mathbf{X}^{-1}$ and substitute it in the given equation, and this was the most popular method of solution. The alternative of pre-multiplying the equation by **X** was only occasionally seen.

Answer: $x = 4 \quad y = 12$

Question 9

(i) This part question was answered well, although $v = 0$ was a regularly occurring incorrect answer.

(ii) Most candidates realised the need to differentiate to find the acceleration and reached $\sin^{-1}(-0.5)$. A substantial number of candidates then lost marks by using degrees rather than radians.

(iii) The need for integration was generally appreciated, but again marks were lost due to the use of degrees rather than radians.

Answers: **(i)** 5 **(ii)** 0.916 **(iii)** 1.14

Question 10

(a) (i) Those candidates who realised the need for substitution found this an easy mark.

(ii) The integration was usually done well. A number of candidates omitted the $+ c$. A few chose, incorrectly, to propose a straight line equation for the curve, using the given derivative as the gradient.

(b) (i) This was generally well done. There were a number of incorrect attempts which involved integrating the term inside the bracket to $\frac{7}{2}x^2 + 8x$.

(ii) The majority of candidates knew how to use limits. A few missed the lower limit, assuming that it would come to zero. As this was a "Hence" question, correct answers from a calculator following incorrect working were given no credit.

Answers: **(a) (i)** 5 **(ii)** $y = -5e^{1-x} - x^3 + 10$ **(b) (i)** $\frac{3}{28}(7x+8)^{\frac{4}{3}}$ **(ii)** 25.7

Question 11

(a) (i) A good number of candidates were able to complete the square successfully. Others expanded f(x) and equated coefficients.

(ii) There were many possible correct domains which candidates could, and did, provide.

(b) (i) Many candidates knew how to find the range of g. Very few realised that the domain of h was the range of h^{-1}.

(ii) Most candidates could sketch the graph of $y = g(x)$ as a parabola starting at 4 on the y-axis. Far fewer were able to sketch the graph of $y = g^{-1}(x)$ using the symmetry about the line $y = x$.

(iii) The majority of candidates got the order of operations correct and most were able to obtain a quadratic equation which they solved. Very few realised that the solution $x = 4$ was inadmissible, as h(4) is negative, and therefore not in the domain of g.

Answers: **(a)(i)** $2(x-2)^2 - 3$ **(ii)** $x \geq 2$ **(b)(i)** $g(x) \geq 4$ $h^{-1}(x) \geq 0$ **(iii)** 8.5

Question 12 EITHER

This alternative was attempted by the majority of candidates. It is possible that some were tempted into this by the relatively straightforward part **(i)** as many of them gained full marks in part **(i)** and were then unable to make a worthwhile attempt at parts **(ii)** and **(iii)**.

(i) Many candidates found this an easy 6 marks. $\frac{\mathrm{d}y}{\mathrm{d}x}$ was found either by the product rule or, more frequently, by expansion. Candidates should note that, when determining the nature of the stationary points, it is not sufficient to write down the answers without showing any working or explanation.

(ii) Most candidates realised that they should be using the chain rule and that $\frac{\mathrm{d}z}{\mathrm{d}t} = 10$, but few were able to obtain a correct value for $\frac{\mathrm{d}y}{\mathrm{d}t}$. In many cases the notation used was poor, and often confused with the small changes formula.

(iii) Although, in general, this part was also poorly done, more candidates realised that they needed to divide their answer to part **(ii)** by $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Answers: **(i)** $x = \frac{2}{3}$ maximum, $x = 4$ minimum **(ii)** $-\frac{5}{6}$ **(iii)** $\frac{5}{48}$

Question 12 OR

This alternative proved to be a much less popular choice but, on the whole, candidates performed better on it than on "Either".

(i) Most candidates showed ample working and recognised the need to produce clear evidence of simplification to produce the given result.

(ii) The differentiation was generally well done. A few candidates stopped at x = 3 and did not give the dimensions of the cuboid.

(iii) Most candidates were familiar with the procedures for evaluating small changes and interpreted their answer correctly.

Answers: **(ii)** 3 by 6 by 4 **(iii)** –38p decrease

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key message

In order to ensure that candidates receive credit where possible for incorrect answers, they should remember to show all their working. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown.

General comments

In general, most candidates appear to have been well prepared for this paper and attempted all the questions. There were many high scoring scripts showing that candidates had a thorough knowledge of the syllabus and were able to apply techniques learned both appropriately and correctly in the great majority of cases.

The new format of the paper was helpful to candidates as they were encouraged by the constraints of space to set out their work tidily and with greater thought. Errors in reading information from the questions were far fewer than in previous sessions.

Centres should note that additional sheets of paper, including graph paper, should only be issued if a candidate runs out of space for a particular question. Where candidates are required to draw an accurate graph, a grid is printed on the question paper.

Comments on specific questions

Question 1

Most candidates attempted to use a method using the discriminant of the quadratic equation formed when y was equated to zero. Errors occurred in the substitution of the terms involving k, but most candidates were able to achieve some measure of success.

Alternative methods involving the gradient function were usually less successful due to more involved algebraic manipulation. Candidates then tended to forget the actual aim of the question.

Answer: $k = -2$

Question 2

The required coefficients were usually identified correctly; however, errors were often made when dealing with the powers of a and $\frac{a}{3}$, and also with the relationship between the two coefficients. As a result, completely correct solutions were unusual although candidates were able to gain most of the marks available for using a correct method.

Answer: $a = \frac{1}{6}$

Question 3

(a) Most candidates were able to identify at least one of the unknowns from the given graph and give the correct numerical value required. Errors in identifying m and p were the most common, with candidates getting them the wrong way round.

(b)(i) Most candidates were able to obtain the correct amplitude.

(ii) Most candidates were able to obtain the correct period, although some chose to give the period in degrees rather than radians in terms of π, as asked for in the question.

Answers: **(a)** $k = 2$, $m = 3$, $p = 1$, **(b)(i)** 5, **(ii)** $\frac{2\pi}{3}$

Question 4

Candidates were instructed not to use a calculator for this question. The intention in such questions is for candidates to show all their workings thus demonstrating, in this case, their knowledge of how to deal with surds. Some evidence of expansion of brackets containing surds was required for each part of this question.

(i) A correct trigonometric equation was obtained initially in most cases, with candidates realising the need for rationalisation. The majority of candidates gave a completely correct solution.

(ii) The area of the triangle was usually obtained correctly in an unsimplified form. Some errors in cancelling the 0.5 with single terms within one of the brackets were unfortunately too common.

(iii) Most candidates realised that they needed to work out AC^2. However, many did not realise that this gave the value of the area required and continued with further, unnecessary, calculations.

Answers: **(i)** $2\sqrt{2}$, **(ii)** $4 + 3\sqrt{2}$, **(iii)** $27 + 18\sqrt{2}$

Question 5

(i) This part was very well done by the great majority of candidates; the most usual method involved showing $f(0.5) = 0$. Methods involving long division were also used, usually correctly. Those candidates that used this approach had then done most of the work required for part **(ii)**.

(ii) Most candidates realised that a quadratic factor needed to be found. This was usually done by algebraic long division, but many adopted a method comparing coefficients and some were able to do the factorisation by observation. Most candidates were able to gain the first two marks available, but there was a great deal of misinterpretation of what was required after this. It was hoped that candidates would look at the discriminant of their quadratic factor (or equate the quadratic factor to zero and attempt to solve) and realise that there were no real roots, so the only real root to the solution of the initial cubic equation was 0.5. Many candidates attempted to factorise the quadratic factor arriving with real solutions. For those candidates that did use a correct approach, the statement of the value of the real root was often omitted.

Answer: 0.5

Question 6

(i) An equation of a straight line involving $\lg y$ and x was produced by most candidates, with the occasional error in the calculation of the constant involved.

(ii) Correct solutions for this part were rare. Candidates found it difficult to rearrange their answer to part **(i)** in terms of powers of ten. Some candidates were able to determine the correct value of *b*, but were unable to express *a* in a correct form.

Answers: **(i)** $\lg y = \frac{1}{5}x + 2$, **(ii)** $a = 100$, $b = \frac{1}{5}$

Question 7

(i) Most candidates obtained a correct answer.

(ii) Apart from those candidates who mistakenly used permutations rather than combinations, most realised that they needed the product of two combinations and were able to obtain a correct answer.

(iii) A common response was 336, with candidates having forgotten the case where there were no women on the team. Otherwise, as before, this part was usually done well.

Answers: **(i)** 3003, **(ii)** 1050, **(iii)** 364

Question 8

(i) Most candidates produced a correct sketch in the space given on the question paper. However, many did not state all of the coordinates of the points where the curve meets the coordinate axes as requested in the question. They were often not even marked on their sketch, which would have been acceptable. Some Centres issued graph paper for this question which was not necessary.

(ii) A surprisingly high number of candidates could not state the coordinates of the stationary point correctly.

(iii) Correct graphs showing the modulus function were common, although many candidates attempted to draw a 'modulus' graph using the line $y = 5$.

Answer: **(ii)** (1, –9)

Question 9

(i) This part of the question proved to be quite difficult for some candidates. They were required to use the properties of an isosceles triangle in order to gain the required result. Many attempts using the arc length were seen but these were ineffective.

(ii) The value of the radius found using the arc length was usually done correctly. Common errors included omitting π from the arc length 9π together with incorrect arithmetic.

(iii) Correct methods were employed by most candidates with the occasional error in calculation. Those that did not use the sine rule for the area of a triangle, but chose instead to find the length AB and use $\frac{1}{2} \times$ base $\times$ height, appeared to be more likely to make errors in calculation and use figures that had been rounded prematurely, thus resulting in an inaccurate value for the area.

Answers: **(ii)** 15, **(iii)** 105

Question 10

(i) Whilst most candidates were able to find $\overrightarrow{AB}$ correctly, there were many who did not understand the term unit vector. Many candidates did attempt to find the magnitude of their vector but were unsure as to what to do with this magnitude. Some candidates used incorrect notation. Centres should encourage candidates to be aware of the correct notation to use when dealing with vectors. Column vector notation and **i**~**j** notation is acceptable. Candidates should be discouraged from writing their vectors in a form that is similar to that used for coordinates.

(ii) More success was to be had by most in this part, with many applying a correct approach (of which there were many) to finding the position vector of C. The same comments regarding notation mentioned in part **(i)** also apply to this part of the question.

Answers: **(i)** $\frac{1}{25}\begin{pmatrix}24\\-7\end{pmatrix}$, **(ii)** $\begin{pmatrix}41\\-16.5\end{pmatrix}$

Question 11

Candidates should be reminded that angles in degrees should always be given correct to one decimal place, whilst angles in radians should be given correct to 3 significant figures.

(i) Provided candidates used either the correct trigonometric identity and obtained a quadratic equation in terms of cosec x, or attempted to obtain a quadratic equation in terms of sin x, a successful solution was usually obtained. Most errors occurred with the substitution of an incorrect identity, together with poor manipulation of the trigonometric terms involved. Candidates should be reminded that these identities are given on the Mathematical Formulae page at the front of the paper.

(ii) Completely correct solutions were common, with most candidates recognising that the equation had to be written in terms of tan $2y$. Some candidates failed to deal with the double angle.

(iii) It was pleasing to see many correct solutions for this question. Questions of this type often prove difficult for candidates, as they have to deal with both the correct order of operations and the fact that radians are involved. There were of course exceptions, with some candidates using a mixture of both radians and degrees and some who dealt with $\cos\left(z+\frac{\pi}{6}\right)$ as a sum of two cosines.

Answers: **(i)** $19.5^\circ, 160.5^\circ$, **(ii)** $25.7^\circ, 115.7^\circ$, **(iii)** $\frac{\pi}{2}, \frac{7\pi}{6}$

Question 12 Either

Many candidates either misunderstood what was required of them, or misinterpreted the instructions or information given. They should be reminded to read the question carefully. A sketch of the situation drawn by the candidates themselves would have probably helped them in a question that was written to test the skills of taking the information given and determining a correct approach to adopt in order to achieve the required solution. Those candidates that did draw a sketch were usually able to gain full marks for this question.

(i) Many candidates did not attempt to find the tangent to the curve and invariably gave the coordinates of the point A as $(-1, 5)$. A sketch at this point would have helped most candidates.

(ii) Most candidates were able to state the coordinates of the point B correctly and go on to find the equation of the normal at B and make an attempt at finding the coordinates of the point D. Some correct methods to find the area of the triangle formed were seen. Candidates were usually more successful if they used the matrix approach rather than $\frac{1}{2} \times$ base $\times$ height. Again, a sketch would have helped many to sort out which point was which and what the area they were finding actually looked like.

Answers: **(i)** (0, 5) **(ii)** 50

Question 12 Or

This question was also written in order to test the candidates' ability to sort through information given and formulate a plan in order to obtain the required result. Provided a careful and logical approach was adopted, many candidates were able to gain full marks for this unstructured question.

Most candidates were able to find the coordinates of the points P, Q and R successfully using differentiation together with the fact that the given tangent is parallel to the x-axis. Integration was then required, a fact that most candidates recognised and, apart from the occasional error, this was usually done successfully with the correct area being identified and calculated.

Answer: 4

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key message

With the advent of more sophisticated calculators it has become even more important for candidates to show their working. They should be reminded that an incorrect answer with no working will not be awarded any marks, whereas candidates can be awarded marks for working even if they get an incorrect answer.

General comments

Some candidates produced high quality work displaying wide ranging mathematical skills, with well presented clearly organised answers. Presentation of answers was generally very clear to follow, the answer-book format helping the majority to organise their work. However, there were also some very weak performances. The majority of candidates made an attempt at most questions. Candidates need to read the questions carefully and ensure that they keep their working relevant. Where a question would benefit from a diagram – which is usually the case with vectors, relative velocity etc. – then candidates should be encouraged to produce as clear and as accurate a diagram as possible. Candidates should take care in the accuracy of their answers and Centres would be advised to draw attention to the rubric which clearly states the requirement for this paper. While interim answers within working do not need to be stated to the same accuracy, the value being worked with should be as accurate as possible to avoid losing marks at the end of the question. Candidates should also be aware of the need to use the appropriate form of angle measure within a question as this can lead to all further work being invalidated.

There was no single question which the majority found entirely straightforward, although there were several where candidates seemed able to score most of the marks with some regularity. **Questions 1, 4, 5,** and **11 Either** fell into this category. The use of calculators was evident on **Question 1** at times, despite the clear instruction not do so and Centres are asked to remind their candidates that using a calculator in such a question constitutes malpractice. Candidates would also be advised to use degrees/radians when instructed as in **Questions 10** and **11 Either** where the use of appropriate circular measure formulae was more successful than finding proportions of the full circle. The balance of choice on **Question 11** was uneven with the vast majority opting for **Either**.

Comments on specific questions

Question 1

There were many successful attempts at this question although inaccuracies in expansion, especially of the squared term led to a loss of marks. Most candidates seemed to know how to rationalise but often omitted the actual calculation. Simplifying the numerator before rationalising led to fewer errors. As there was an instruction to work without a calculator in this question, all steps of the working needed to be shown.

Answer: $14 + 3\sqrt{3}$

Question 2

(i) Although there were a large number of correct answers to this part there were also a range of common errors and unusual incorrect methods. Common errors were ignoring the minus sign and not cubing the denominator. Candidates still need to realise that ${}^{n}\mathrm{C}_{r}$ or $\binom{n}{r}$ notation is insufficient evidence and that coefficients must be evaluated. A large number of candidates found it necessary to give the full expansion in order to give a specific term but most clearly identified the value required. A small but noticeable minority tried to either differentiate or solve an equation to find the required value.

(ii) There were similar errors to those in part **(i)** in finding the coefficient of x^2 from the binomial. Many candidates were able to identify and combine the two relevant terms to be added although there were those who multiplied the wrong term by 4.

Answers: **(i)** –27.5 **(ii)** 38.5

Question 3

Some candidates omitted this entirely. This question was found to be difficult by those candidates who used only a single upper case letter as vector notation and there were those who seemed unfamiliar with the term 'position vector'. The problem for many others was that they did not appreciate the need to find vector $\overrightarrow{OC}$ having found vector $\overrightarrow{AC}$ or in some cases only $\overrightarrow{AB}$. It was very common for an answer to finish with the magnitude of vector $\overrightarrow{AC}$. There was a lack of a clear diagram in many cases; such a diagram would have helped candidates appreciate the basic geometry of the question.

Answer: $\overrightarrow{OC} = 5\,\mathbf{i} + 12\,\mathbf{j}$, 13

Question 4

For many this proved a standard type of question and was very well done, but with many stopping once the values of k had been obtained from the quadratic equation. The most popular method was to eliminate y and the small number who chose to eliminate x tended to get into difficulties with the algebra. Some candidates thought differentiation might lead to success, but this was not the case.

Few of those going on to present an inequality gave it in an exactly correct form. This was sometimes due to starting with an incorrect inequality for the discriminant, and sometimes due to expressing the final answer in an unacceptable form such as $-6 > k > 10$. A score of 5 marks on the question was very common.

Answer: $k < -6$ or $k > 10$

Question 5

(i) There were many fully correct answers to this question on the Remainder Theorem. However, some candidates appeared to misread the first sentence, taking $x - 1$ and $x + 2$ as factors and equating both $f(1)$ and $f(-2)$ to zero. This gave two values for p and made no use of simultaneous equations, although the question indicated that there was only a single value. The use of simultaneous equations was the most popular method of finding p, many finding R first although this was not requested.

(ii) Most candidates knew what to do here although it was surprising how many changed their methodology used in part **(i)** from long division to Remainder Theorem or vice versa. Those with two values for p were at a disadvantage in knowing which to use when logically they should have used both.

Answers: **(i)** $p = -17$ **(ii)** 71

Question 6

There were many accurate answers to parts **(a)(i)** and **(b)** but few correct solutions to part **(a)(ii)** were seen.

(a) (ii) The most common mistakes were to find $5! \times 3!$, $5! \times 3$ or $5! \times 4$. Even stronger candidates often made these errors.

(b) The values 35 and 10 were regularly found but these were then added too frequently. In this part also there was a greater occurrence of failing to evaluate ${}^nC_r$ and $\binom{n}{r}$ statements.

Answers: **(a)(i)** 40320 **(a)(ii)** 2880 **(b)** 350

Question 7

(i) This question was found to be difficult by many candidates. The gradient and intercept of the line were often calculated correctly, but then only a minority of candidates were able to proceed beyond $y = \frac{5}{2}x + 2$ to the correct $\log y = \frac{5}{2}\log x + 2$ and hence to $y = 100x^{\frac{5}{2}}$. In many cases the equation was incorrectly completed with log 2. Another common error was to assume that the 4, 12, 6 and 17 were *x* and *y* values rather than log *x* and log *y* values. The candidates who fared better often first rewrote the given form as log *y* = log *a* + *b* log *x* then identified *b* with the gradient and log *a* with the intercept.

(ii) Those candidates who had an answer to part **(i)** in the correct form usually made some further progress but too often a solution to a linear equation was given which was of a far more straightforward nature than that required.

Answers: **(i)** $y = 100x^{2.5}$ **(ii)** 1.55

Question 8

The first two parts of this question were reasonably well done by most candidates although there were omissions and others who seemed not to have an awareness of the exponential function and treated e as an algebraic variable.

(iii) Weaker candidates often demonstrated a lack of knowledge and the skills needed to solve an exponential equation. Many tried to take the log of each of the three terms first and others used lg rather than ln. Dividing by 55e was another common tactic. More disappointing was the number of candidates unable to give their answer to the correct degree of accuracy.

(iv) Frequently it was not appreciated that differentiation was required to find a rate of change. Others thought that it was about small changes. Where differentiation did take place the inclusion of *t* being multiplied in addition to the (–)0.1 and/or a constant term (15 or 16) was a little too common. A significant number substituted the 16 prior to an attempt at differentiation.

Answers: **(i)** 70 **(ii)** 39.7 **(iii)** 17 **(iv)** –1.11

Question 9

This question was found to be extremely challenging and only a handful of accurate solutions were seen. This is a question where an attempt to construct a diagram of the situation would have been extremely beneficial. Some attempts at the question started with an incorrect diagram and there was also a lot of confusion between distances and speeds. Equations were frequently seen with terms of different dimensions. A common answer was to determine that 30 km travelled in 40 minutes indicated the lifeboat's speed to be 45 km/hr with no further working. Others took the counter argument that the ship travelled 10 km and tried to calculate the speed based on a right-angled triangle. This topic is one where more experience would be beneficial to candidates.

Answers: **(i)** *V* = 39.7 or 39.8 **(ii)** 251

Question 10

Overall there was a wide range of ability shown on this question. A significant minority of candidates made no attempt at all to any part while for a good number of candidates it seemed to be reasonably straightforward. Candidates are recommended to look carefully at the angle measure required which is always identified by the range of solutions and are advised to work in this unit throughout. Where candidates quite sensibly work from an angle in the first quadrant they should take care that they clearly indicate which angles form their final answer.

(i) For many, rearranging to $\tan x$ was a straightforward step and but for careless use of signs or giving the incorrect number of angles this led to good solutions. There were those who tried squaring methods – usually with little success – or used the forms $R\sin(x \pm \alpha)$ or $R\cos(x \pm \alpha)$.

(ii) Most attempts involved replacing $\sec y$ with $\dfrac{1}{\cos y}$ although there were incorrect identities applied also. Candidates by and large realised they needed to form a quadratic equation, but careless multiplication at times led to these only having two terms causing the subsequent solution to be relatively simple. Those who formed a three term quadratic equation solved it well whether by factorisation or by use of the formula.

(iii) This was the least successful part of the question. There were three main causes for the loss of some or all of the marks. Working in degrees as a first step and then applying algebra was a common error. Others wrote $\sin(2z - 3) = \sin 2z - \sin 3$ negating any further work. The other common loss of marks was due to working to an insufficient degree of accuracy with final answers being incorrect to the three decimal places specified in the general rubric.

Answers: **(i)** 126.9, 306.9 **(ii)** 48.2, 311.8 **(iii)** 1.89 and 2.68

Question 11 EITHER

Solutions to this question often gained full marks even from candidates who had scored poorly across the rest of the paper. It was pleasing to see that there was hardly any confusion between the perimeter and area formulae. This was also clearly the most popular option.

(i) This proof was usually successfully accomplished, although as might have been anticipated, there was sometimes an assumption of the result in order to provide a “proof”. Others candidates used a lengthy solution when splitting the given triangle gave a neat proof using basic trigonometry.

(ii) There were many precise and accurate solutions to this part. It was encouraging to see that only a few candidates made the standard errors of only finding the minor arc or including all three sides of the triangle.

(iii) This was completed with even greater success than part **(ii)** although some solutions were lengthier than others due to subtracting the minor arc from the triangle first.

Conversion to degrees tended to lead to a loss in accuracy and candidates would be advised to work in radians and apply the appropriate formulae. It is also advisable to give values for areas/perimeters found as part of the solution independently as when the final answer is incorrect it is often too difficult to identify how many of the errors may have taken place in calculation. Giving earlier values often allows method marks to be awarded in the situation where the final answer is inaccurate or incorrect.

Answers: **(ii)** 98.6 cm **(iii)** 504 cm^2

Question 11 OR

Attempts at this question were relatively rare in comparison. Although there were some good solutions many were poorly done and often difficult to follow.

(i) Those candidates who realised the proof referred to the normal and not just the line *PQ* fared the best. There was a need to find the equation of the normal at *P* and to show this passed through *Q*. Differentiation was required and when attempted, usually by the product rule, it was carried out correctly. Too many solutions found the equation of *PQ* using *P* and *Q* and concluded by showing this passed through *Q*.

(ii) It was pleasing to see a good number of clearly explained answers, with the steps properly set out without dropped or redundant integral signs. A few used integration by parts but not always with success. Some candidates arrived at the correct answer, but notation lacked rigor and the logic was difficult to follow. Other solutions which missed the point of the given equation were equally lacking in logic and generally failed to gain credit.

(iii) Most candidates attempted some substitution but not always with the integral found previously. The choice of limits was often incorrect, however, with 0 to π often used – or worse 0 to 180. The most secure method was to find the area under the curve and of the triangle separately then subtract. Those who favoured doing the whole thing with a one step $y_1 - y_2$ method struggled if they had the wrong integral.

Answers: **(ii)** $\sin x - x\cos x$ **(iii)** 0.908 or $\pi - 1 - \frac{\pi^2}{8}$

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key message

With the advent of more sophisticated calculators it has become even more important for candidates to show their working. They should be reminded that an incorrect answer with no working will not be awarded any marks, whereas candidates can be awarded marks for working even if they get an incorrect answer.

General comments

There was a lot of very impressive work from candidates. There were also significantly fewer very poor scripts, with even the weakest candidates demonstrating a knowledge of some of the syllabus content. **Questions 6** (simultaneous equations), **5** (differentiation and integration) and **8** (logarithms and indices) were particularly well done. The only question that was generally answered poorly was **Question 2** on set notation and Venn diagrams.

Almost all candidates found that the question/answer booklet contained sufficient space for their solutions.

Comments on specific questions

Question 1

This proved to be an easy starter question and many candidates scored full marks. A number of candidates differentiated correctly but then used degrees rather than radians when calculating the approximate increase.

Answers: **(i)** $3\cos 3x$ **(ii)** $1.5p$

Question 2

This was the least well done question on the paper.

(a) (i) Some candidates did not use brackets as shown in the notation section of the syllabus.

(ii) The sign for "is an element of the set" appeared frequently.

(b) Some of the candidates who were successful in part **(i)** produced very complicated, correct, alternatives in part **(ii)**.

Answers: **(a)(i)** $n(\mathscr{E})=72$ **(ii)** $R \subset W$ or $R \cap W = R$ or $R \cup W = W$ or $R \cap W' = \varnothing$ or others
(b)(ii) $(X' \cap Y)'$ or $(X \cap Y) \cup Y'$ or $(X \cup Y)' \cup X$ or $(X' \cap Y') \cup X$ or others

Question 3

Strong candidates were able to complete part **(i)** and link this successfully to part **(ii),** but it was surprising how many wrote down the identity matrix incorrectly. Some candidates struggled with the matrix algebra in part **(ii)** writing $2\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$. There were some who failed to answer part **(i)** but then proceeded to set up and solve simultaneous equations in part **(ii)**.

Answers: **(i)** $\frac{1}{20}\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Question 4

(a) This part was generally well done with the majority of candidates scoring full marks.

(b) While the majority of candidates knew the relationship between sine and cosecant only the strongest found an expression for cosine in terms of *p*.

Answer: **(b)** $\frac{1}{2p\sqrt{1-p^2}}$

Question 5

The recent improvement seen in the way candidates answer this type of question was maintained.

(i) Most candidates answered this part correctly.

(ii) Many candidates recognised the relationship between the two parts; a few multiplied by 3 instead of dividing. There were a few who ignored the instruction to find the indefinite integral and found the answer to the definite integral by calculator, for which they received no credit.

Answers: **(i)** $\frac{3(x+5)}{\sqrt{2x+15}} (\Rightarrow k = 3)$ **(ii)** $\frac{34}{3}$

Question 6

This question was done exceptionally well with the majority of candidates scoring full marks. Some candidates did not show any working when solving the quadratic equation. This was fine when the equation was correct but where candidates had made an error they could only be rewarded for working that they showed.

Answer: 22.1 or $\sqrt{490}$ or $7\sqrt{10}$

Question 7

This question discriminated well between candidates of different abilities. Strong candidates answered parts **(i)** and **(ii)** well. Those who chose to use the quotient rule in part **(ii)** often had problems.

Part **(iii)** presented the most difficulties, with even the very strong candidates ignoring the constant of integration. There were some who did introduce the constant *c* but assumed that as *t* and *s* were both zero then so was *c*.

Answers: **(i)** 3.75 **(ii)** -0.36 **(iii)** $5 - \frac{20}{3t+4}$

Question 8

(i) This part was generally well done even by the weakest candidates.

(ii) Again this was well done by the majority of candidates; the main difficulty involved converting the 2 to 25 when the logarithms were removed from the left hand side.

(iii) Candidates handled the power of z much better than they did the powers of 2 and 3.

Answers: **(i)** 4.82 **(ii)** –0.5 **(iii)** $a = 4, b = -2, c = 5$

Question 9

This was one of the least well answered questions on the paper. Those candidates who used coordinate geometry to find the coordinates of C and then M and D followed by the “array” method for area were the most successful. Others found the coordinates and used the sum of the areas of two triangles correctly but there were those who assumed that one of the angles of triangle ACD was a right-angle. It was quite common for candidates to make false assumptions about lines being equal, perpendicular or parallel. Using these false assumptions it was quite common for D to be found as (14, 22). Some proved that the area of triangle ABC was equal to the area of triangle ACD, but those who merely assumed this fact did not receive full credit.

Answer: 150

Question 10

(a) This part was done less well than part (**b**). The idea was not understood by a number of candidates and of the three parts **(ii)** was answered correctly most frequently. This was usually done by calculus rather than by using part **(i)**. Again, many candidates did not realise the significance of the stationary point when they sketched the curve; frequently a parabola was not drawn.

(b) Both parts were very well answered although a few did find the derivative of g rather than the inverse.

Answers: **(a)(i)** $50 - 2(x-4)^2$ **(ii)** (4, 50) **(b)(i)** $\sqrt{x+7} - 3$ **(ii)** 18

Question 11

“**OR**” proved to be far more popular than “**EITHER**” although there was little difference in the performance of candidates, which was generally good.

EITHER

(a) Some candidates did not realise what was needed in this part and wrote down a value for $\sin 60$ to three decimal places.

(b) (i)(ii) Only the better candidates were able to reach the required expression for A without error, although most did earn partial credit.

(iii) The value of x was correctly found by the majority of candidates, even those who had not reached the expression for the area. Many correctly showed that the stationary point was a maximum.

Answers: **(a)** $\sin x = \frac{\sqrt{3}}{2}$, $\cos x = 0.5$ **(b)(i)** $y = \frac{60 - 3x}{2}$ **(iii)** $x = 15$, maximum

OR

(i)(ii) Again, only the better candidates were able to reach the required expression for A without error, although most did earn partial credit. Common errors were to fail to halve the formulae for a sphere to make the formulae for a hemisphere, or forgetting to include the circular end of the cylinder.

(iii) This part was generally well answered although the resulting equation proved more difficult to solve than the equivalent equation in **EITHER**.

(iv) This part was well done.

(v) Some candidates did not fully understand the process; they found the second derivative, equated it to zero and solved for r. They then used the sign of this value of r to distinguish between maximum and minimum.

Answers: **(i)** $h = \dfrac{8640 - 2r^3}{3r^2}$ **(iii)** $r = 12$ **(iv)** 720π **(v)** minimum