ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key Messages

Candidates should be encouraged to read the questions carefully and ensure that they are giving the actual answers required. They should also be encouraged to check that they are working to the required level of accuracy. Working in the solution of a question should be to more than 3 significant figures to ensure a final answer accurate to 3 significant figures is given, unless otherwise stated.

General Comments

Well prepared candidates were able to show their understanding of the syllabus objectives and apply the techniques they had learned both appropriately and correctly. Too many candidates showed a lack of revision or learning by producing papers with several questions not attempted.

Comments on Specific Questions

Question 1

Most candidates knew that differentiation was required and all but a few could correctly differentiate the first term. The second term, however, caused more problems because of the negative index which often resulted in an incorrect power or sign errors.

A number of weaker candidates thought that the product rule had to be used.

Most candidates set their derivative to zero and attempted to solve. The form of the equation, again, caused problems with many not knowing how to deal with the x^{-2} term correctly.

Many strong candidates mistakenly made the error that $x^3 = 8$ implied $x = \pm 2$. Extra solutions due to this error resulted in the loss of the final accuracy mark.

In spite of the above, many correct solutions were seen and the question provided most candidates with a confident start to the paper.

Answer: $(2, 12)$

Question 2

(a) Many graphs were drawn that were approximately correct. However, a sketch should convey important information: where a period starts and ends, at which points there is a zero gradient (at these points, the curve should be parallel to the *x*-axis) and the *y*-coordinate should also be clear at these points. Many candidates lost at least one mark due to a poorly shaped curve as a result of the omission of one of the above criteria.

(b) (i) Many incorrect responses of –4 were seen. Candidates should be aware that amplitude is not a negative quantity.

(ii) Candidates need to remember that the period of functions involving tangent differ from those involving sine and cosine. Most candidates stated an incorrect period of 120° or $\frac{2\pi}{3}$.

Answer: **(b)(i)** 4 **(ii)** 60°, $\frac{\pi}{3}$ or 1.05 rad.

Question 3

(i) Many candidates answered this question very well with the correct solution often seen. Errors made usually involved a numerical slip in evaluating the constant or the omission of the constant. The most frequently occurring error was when candidates substituted 6 into the function without integration, giving $\frac{dy}{dx} = \frac{2}{\sqrt{x+3}}$, so $\frac{dy}{dx} = \frac{2}{3}$. They then went on to substitute into the equation $y = mx + c$ using the point (6, 10) to obtain a linear equation. Other common errors seen were "integrals" of $2(x+3)^{\frac{1}{2}}$ and $-4(x+3)^{\frac{1}{2}}$, which were able to gain credit for the candidate recognising that $(x+3)^{\frac{1}{2}}$ was involved. Some candidates chose to differentiate the given equation making use of the quotient rule.

(ii) Candidates with the correct solution in part **(i)** usually gained the final mark. The main error occurred when candidates mistakenly assumed that if $(x+3)^{\frac{1}{2}} = k$ then $x + 3 = \sqrt{k}$.

Answer: **(i)** $y = 4(x+3)^{\frac{1}{2}} - 2$ **(ii)** 1

Question 4

(i) The given substitution was an obvious help to most candidates and there were many correct quadratic equations seen. The most significant errors were when the term involving 5^{2x+1} was incorrectly written as either y^3 or y^2.

(ii) The main misconception here was that candidates thought it was sufficient to solve for y and not x and so it was fairly common to see the solutions $y = 1$, $y = \frac{2}{5}$. The better prepared candidates usually went on to solve for x with the occasional error in the application of logarithms. Candidates need to be reminded that answers should be given to 3 significant figures unless otherwise stated. Many were unable to gain the final accuracy mark due to premature rounding and an answer of -0.57.

Answer: **(i)** $5y^2 - 7y + 2 = 0$ **(ii)** 0 and –0.569

Question 5

(i) The majority of candidates attempted some form of differentiation, evidenced by the number of terms of $3x^2$ seen. However, differentiation of the term in $\ln x$ caused problems for many. Several candidates attempted the product rule with the two terms, many of these candidates had made the same error in **Question 1**.

(ii) Many candidates found this part more challenging than had been expected. Many found the gradient or the equation of the line *AB*, which were not necessarily required to solve the problem. Others worked out either the mid-point of the line *AB*, or the coordinates of the point of intersection of the line with the tangent. Few candidates provided both of the required facts and hence made the correct conclusion from these facts.

Answer: **(i)** $y = 2x - 1$

Question 6

(i) Most candidates were able to make a reasonable attempt at the use of the Binomial Expansion. Many candidates correctly wrote down the term in x^2 as ${}^6C_2 \times 2^4 \times (px)^2$ or equivalent, but then went on to evaluate this as $240px^2$ rather than $240p^2x^2$. As a result of this, the solution $p = \frac{1}{4}$ was all too common.

(ii) This part was usually done well by most candidates. Recognition that two terms needed to be considered was common and those candidates who had made an algebraic slip in part **(i)** were usually able to obtain follow through and method marks for using their value of *p* correctly.

Answer: **(i)** $\frac{1}{2}$ **(ii)** 84

Question 7

(i) Most candidates were able to obtain full marks for writing down the inverse matrix.

(ii) Candidates should be reminded of the importance of the order in which the multiplication of two matrices is carried out. Most realised that they had to make use of the inverse matrix found in part **(i)**, but too many solutions involved pre-multiplication of matrix **B** by the inverse matrix from part **(i)**, rather than post-multiplication. Most candidates showed that they are able to perform actual matrix multiplication correctly and were usually able to gain credit for this. Candidates were expected to algebraically simplify their resulting matrix.

Answer: **(i)** $\frac{1}{5ab}\begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$ **(ii)** $\begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}$

Question 8

(i) There were few convincing arguments for the value of $x = 1$, but many candidates obtained the value of $y = 7$ having assumed the given *x* value. The question had been intended to test candidates' understanding of displacement vectors, but many chose to find the equation of the line *AB* and hence use the given value of $x = 1$ to obtain $y = 7$.

(ii) Most candidates could obtain the perpendicular gradient, and proceeded from there to find the equation of the straight line. However, too many candidates had not read the question correctly and used either the point (–2, 3) or the point (4, 11) rather than the point *P* obtained in part **(i)**, as required.

(iii) The coordinates of the point *Q* were usually found correctly. Most candidates used a matrix method for finding the area of the triangle, although other correct methods were also used. Candidates should remember to include the $\frac{1}{2}$ when making use of the matrix method.

Answer: **(i)** $y = 7$ **(ii)** $3x + 4y = 31$ **(iii)** 12.5

Question 9

(i) Many candidates would benefit from more practise at this type of equation. Far too many candidates appeared not to understand the concept of linear form. Time was wasted by candidates simply plotting a graph of the given points, not being deterred by the fact that a straight line was not usually obtained. Some candidates used imaginative scales which produced a straight line from the given points. Other candidates reduced the equation to the correct form using logarithms, but went on to plot a graph of $\log y$ against $\log x$. This is clearly a syllabus area with which many candidates are unfamiliar.

(ii) Many candidates did not attempt this part of the question, having realised that they did not have a straight line graph. Of those candidates that did have a graph to use, although most made use of the gradient of the graph to determine a value for b, many were unable to obtain a correct value for *a* using the intercept on the *y*-axis of their graph as they did not take into account that they had not started their *x* values at zero. Centres should be encouraged to concentrate more on this area of the syllabus.

Answer: **(ii)** $a = 3$, $b = 2.5$

Question 10

This question proved difficult for many candidates with blank responses or, what seemed to be, random answers appearing frequently.

(a) (i) This was answered fairly well with responses ${}^6P_4 = 360$ and $6 \times 5 \times 4 \times 3 = 360$ often seen. Most candidates realised that permutations needed to be used. The response $6! = 720$ was a common error.

(ii) This part was not as well answered as part **(i)** with 120 being a common incorrect response from $1 \times {}^6P_3$ rather than the required $1 \times {}^5P_3$. Candidates who gave more 'planned' responses usually did better e.g. fixing the first digit as <u>6</u> then 5 x 4 x 3 = 60.

(iii) Again, this part was not as well answered as part **(i)**. Good responses were again often well set out using a 'planned' approach, as in part **(ii)**. Other correct responses such as ${}^4P_2 \times 3$ were also seen.

(b) (i) This part was very well answered with many candidates who had not scored in earlier parts writing down ${}^8C_5$ and ${}^{12}C_5$. However, there seemed to be equal numbers of candidates who mistakenly added their two terms to obtain 848 as those who correctly multiplied to obtain the required answer. Most candidates realised that they needed to be considering combinations.

(ii) Very few fully correct solutions were seen. Most correct solutions were seen when candidates considered the case that gender was 'no longer important'. Several candidates spoiled their solutions by adding 4 to what was an otherwise correct answer to obtain give 8012. Some candidates attempted to use an alternative approach by listing the seven separate possible cases.

Answer: **(a)(i)** 360 **(ii)** 60 **(iii)** 36 **(b)(i)** 44 352 **(ii)** 8008

Question 11

(a) Most candidates were able to gain credit for stating that $\cot 3x = \dfrac{\cos 3x}{\sin 3x}$. However there were many candidates who used poor notation or had a poor understanding of trigonometry, mistakenly writing $\cot 3x$ as $\dfrac{\cos 3x}{\sin x}$ or $\dfrac{\cos}{\sin}3x$ or similar. Candidates should be encouraged to use correct notation throughout as this will reduce the likelihood of subsequent errors. Of those candidates that did express $\cot 3x$ correctly, most divided through by $\cos 3x$ thus 'losing' two solutions to the equation. Many answers of 10° and 50° were seen, gaining candidates 3 out of the 5 possible marks. Very few complete correct solutions were seen.

(b) More candidates were able to gain marks in this part than in part **(i)**, provided they were able to deal with $\sec\left(y+\frac{\pi}{2}\right)$ correctly. If this was not done, candidates were unable to proceed correctly nor to gain any further marks. Candidates should be encouraged to work in radians when they are used in a question rather than change to degrees as, too often, errors are made in conversion or with trying to add or subtract terms in both degrees and radians.

Answer: **(a)** $10^\circ, 30^\circ, 50^\circ, 90^\circ$ **(b)** $\frac{\pi}{6}, \frac{5\pi}{6}$

Question 12

Performance in this question was very variable. Some weaker candidates did very well on this question and, conversely, some very bright candidates made careless errors. The common source of error throughout was usually in sign, some candidates assuming that direction did not matter and that $\overrightarrow{BP} = \overrightarrow{PB}$, for example. Some candidates were only able to write their expressions in terms of directed line segments and seemed unable to translate this into expressions involving **a** and **b.** A number of candidates either did not attempt the question at all or gave up half way through, normally after attempting part **(iii)** badly.

(i) This was generally done quite well. However, the usual error was to get the vector(s) the wrong way round.

(ii) This was generally done quite well. However, the usual error was to get the vector(s) the wrong way round.

(iii) It was common to see the given ratio ignored or incorrect e.g. division by 4 rather than 3 for $\overrightarrow{AR}$.

(iv) It was common to see the given ratio ignored or incorrect e.g. multiplication by $\frac{8}{7}$ rather than $\frac{7}{8}$ for $\overrightarrow{BR}$.

There were many candidates who obtained one or both expressions for parts **(iii)** and **(iv)** correctly. The usual errors were due to sign errors, particularly from double negative signs when expanding brackets.

(v) This final part was attempted by only a few candidates. Less able candidates attempted to equate each of their vectors from parts **(iii)** and **(iv)** to zero. Many realised these two vectors had to be equated but did not know what to do next. Many candidates did not appear to understand the concept of equating 'like' vectors'. Completely correct solutions were seen rarely.

Answer: **(i)** $\lambda\mathbf{b}-\mathbf{a}$ **(ii)** $\mu\mathbf{a}-\mathbf{b}$ **(iii)** $\frac{2}{3}\mathbf{a}+\frac{1}{3}\lambda\mathbf{b}$ **(iv)** $\frac{1}{8}\mathbf{b}+\frac{7}{8}\mu\mathbf{a}$ **(v)** $\mu=\frac{16}{21}$, $\lambda=\frac{3}{8}$

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

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Answer: **(i)** $y = 4(x+3)^{\frac{1}{2}} - 2$ **(ii)** 1

Question 4

(i) The given substitution was an obvious help to most candidates and there were many correct quadratic equations seen. The most significant errors were when the term involving 5^{2x+1} was incorrectly written as either y^3 or y^2.

(ii) The main misconception here was that candidates thought it was sufficient to solve for y and not x and so it was fairly common to see the solutions $y = 1$, $y = \frac{2}{5}$. The better prepared candidates usually went on to solve for x with the occasional error in the application of logarithms. Candidates need to be reminded that answers should be given to 3 significant figures unless otherwise stated. Many were unable to gain the final accuracy mark due to premature rounding and an answer of -0.57.

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Question 10

This question proved difficult for many candidates with blank responses or, what seemed to be, random answers appearing frequently.

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Answer: **(a)(i)** 360 **(ii)** 60 **(iii)** 36 **(b)(i)** 44 352 **(ii)** 8008

Question 11

(a) Most candidates were able to gain credit for stating that $\cot 3x = \dfrac{\cos 3x}{\sin 3x}$. However there were many candidates who used poor notation or had a poor understanding of trigonometry, mistakenly writing $\cot 3x$ as $\dfrac{\cos 3x}{\sin x}$ or $\dfrac{\cos}{\sin} 3x$ or similar. Candidates should be encouraged to use correct notation throughout as this will reduce the likelihood of subsequent errors. Of those candidates that did express $\cot 3x$ correctly, most divided through by $\cos 3x$ thus 'losing' two solutions to the equation. Many answers of 10° and 50° were seen, gaining candidates 3 out of the 5 possible marks. Very few complete correct solutions were seen.

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Answer: **(a)** $10°, 30°, 50°, 90°$ **(b)** $\frac{\pi}{6}, \frac{5\pi}{6}$

Question 12

Performance in this question was very variable. Some weaker candidates did very well on this question and, conversely, some very bright candidates made careless errors. The common source of error throughout was usually in sign, some candidates assuming that direction did not matter and that $\overrightarrow{BP}=\overrightarrow{PB}$, for example. Some candidates were only able to write their expressions in terms of directed line segments and seemed unable to translate this into expressions involving **a** and **b.** A number of candidates either did not attempt the question at all or gave up half way through, normally after attempting part **(iii)** badly.

(i) This was generally done quite well. However, the usual error was to get the vector(s) the wrong way round.

(ii) This was generally done quite well. However, the usual error was to get the vector(s) the wrong way round.

(iii) It was common to see the given ratio ignored or incorrect e.g. division by 4 rather than 3 for $\overrightarrow{AR}$.

(iv) It was common to see the given ratio ignored or incorrect e.g. multiplication by $\frac{8}{7}$ rather than $\frac{7}{8}$ for $\overrightarrow{BR}$.

There were many candidates who obtained one or both expressions for parts **(iii)** and **(iv)** correctly. The usual errors were due to sign errors, particularly from double negative signs when expanding brackets.

(v) This final part was attempted by only a few candidates. Less able candidates attempted to equate each of their vectors from parts **(iii)** and **(iv)** to zero. Many realised these two vectors had to be equated but did not know what to do next. Many candidates did not appear to understand the concept of equating 'like' vectors'. Completely correct solutions were seen rarely.

Answer: **(i)** $\lambda\mathbf{b}-\mathbf{a}$ **(ii)** $\mu\mathbf{a}-\mathbf{b}$ **(iii)** $\frac{2}{3}\mathbf{a}+\frac{1}{3}\lambda\mathbf{b}$ **(iv)** $\frac{1}{8}\mathbf{b}+\frac{7}{8}\mu\mathbf{a}$ **(v)** $\mu=\frac{16}{21}$, $\lambda=\frac{3}{8}$

ADDITIONAL MATHEMATICS

Paper 0606/13

Paper 13

Key Messages

Candidates should be aware that the rubric requires that non-exact answers should be given correct to three significant figures with the exception that angles in degrees should be given correct to one decimal place. Angles in radians should be given as decimals to 3 significant figures or as a multiple of π.

Candidates should be aware that, when an answer is given, particular care should be taken to show all working leading to that answer. Full working should also be shown if candidates have been asked not to use a calculator.

Candidates should be aware that identities such as $\tan A = \dfrac{\sin A}{\cos A}$ and $\operatorname{cosec} A = \dfrac{1}{\sin A}$ are not included in the list of formulae and should be known.

General Comments

Most candidates were able to attempt a high proportion of the questions and the standard of presentation was very good. Nearly all candidates planned their responses to fit into the working space provided. Although most candidates were careful to show working, further care would be appropriate for questions with answers given and where it is specified that a calculator should not be used.

Comments on Specific Questions

Question 1

Little working was seen for this question, but candidates obviously found this a difficult start to the paper. Candidates were required to be aware of how the constant *a* related to the amplitude, *b* to the period and *c* to the vertical translation in the trigonometrical graph. Candidates should be familiar with interpretation of trigonometrical graphs and would benefit from practising this technique.

Answer: $a = 3$, $b = 2$, $c = 4$

Question 2

Candidates showed a good understanding of what was required to solve the simultaneous equations. However, not all were aware that $x^2 = 16$ has two solutions and so were unable to find two points. Candidates should know the formula for finding the distance between two points. For an exact length the answer should be left in surd form.

Answer: (4, 3), (-4, 1), $\sqrt{68}$

Question 3

(i) Not all candidates were aware that n(*A*) meant the number of elements in the set *A* and the elements of the set were sometimes listed instead. Candidates should be aware that as the equation for the solution set *C* had no real solutions then n(*C*) = 0.

(ii) Candidates showed a good understanding of this question.

(iii) Candidates showed a good understanding of this question.

(iv) Many candidates were unaware of the implication that if C was an empty set then C' would be the universal set of all real numbers. It was not sufficient to state that C' contained elements not in C.

Answers: **(i)** n(A)=2, n(B)=3, n(C)=0 **(ii)** {–3, –2, –1, 3} **(iii)** {–2} **(iv)** 'universal set' or 'real numbers'.

Question 4

(a) Most candidates obtained $\tan x = -\frac{5}{3}$ and realised that there were two solutions. Candidates should remember that angles in degrees should be given correct to one decimal place.

(b) Candidates should know that $\operatorname{cosec} A = \frac{1}{\sin A}$ as this is not a given formula. Candidates should be aware of the order of operations to solve equations such as this. They should also be aware that in this case three further values were required for $\sin^{-1}\frac{1}{2}$ in addition to $\frac{\pi}{6}$ in order to find all the solutions in the given range. For a correct order of operations $\frac{\pi}{4}$ had to be subtracted from each of these values before division by 3. Most fully correct solutions came from the use of angles given as multiples of π by a fraction as candidates working with decimal equivalents tended to lose accuracy through premature rounding. Candidates should be advised not to try to work in degrees when the question asks for an answer in radians, as accuracy is inevitably lost through conversions.

Answers: **(a)** $121.0^\circ, 301.0^\circ$ **(b)** $\frac{7\pi}{36}, \frac{23\pi}{36}, \frac{31\pi}{36}$

Question 5

(a) (i) Candidates were not always aware of what they had been asked to do in this question. A 4 by 3 or 3 by 4 matrix representing days/drinks had to be multiplied by a compatible 3 by 1 or 1 by 3 matrix representing prices. The resulting price/day matrix had to be of an appropriate shape for the matrix multiplication.

(ii) Candidates should be aware that the only answer required here was for the total price.

(b) Most candidates knew that they needed to obtain an inverse matrix for **X** and showed a good knowledge of this technique. It was not always realised that multiplication by the identity matrix was unnecessary as it would leave the answer unchanged and, in fact, some answers were spoiled by attempts to multiply by an incorrect identity matrix. An incorrect identity matrix was also a source of error for candidates who formed and used simultaneous equations.

Answers: **(a)(i)** $\begin{pmatrix} 7.25 & 5.70 & 6.45 & 6.30 \end{pmatrix}$ or equivalent column matrix **(a)(ii)** \$25.70 **(b)** $\frac{1}{22}\begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}$

Question 6

(i) Most candidates identified a correct method to find OC but concluding that $OC = 9.65$ was not sufficient. Candidates should remember that as the given answer is stated as being to 3 significant figures they are required to show an answer to their calculation with at least 4 figures. This answer would then be rounded to the given answer. In order to obtain a fully correct unrounded answer, candidates should be aware that angles measured in radians should be used rather than risk introducing inaccuracy by converting to degrees.

(ii) A careful and logical look at the diagram was required and candidates did not always identify a correct strategy. Candidates were expected to use 12θ to obtain the length of the arc and it should be realised that $CB + AC = 12$ cm. Recalculating CB led to inaccuracy, as did conversion of 0.9 radians to degrees in calculating the length of the arc.

(iii) Candidates were expected to use the formula for the area of a sector as $\frac{1}{2}r^2\theta$ rather than converting to degrees to obtain a fraction of πr^2. There were several methods available to obtain the area of the triangle *OCB*. Calculating a height to find $\frac{1}{2}$ x base x height led to inaccuracy through premature rounding, particularly if an angle in degrees had been used. As in part **(ii)**, a correct strategy was required for this part and candidates should take care to subtract the correct triangle.

Answers: **(i)** 9.652 **(ii)** 22.8 **(iii)** 19.4

Question 7

There were many good solutions to this question with most candidates obtaining $1 = \log_5 5$ and $2\log_5 x = \log_5 x^2$. However, not all candidates knew that adding logs would give the log of the product. Candidates who obtained a correct quadratic equation using addition or subtraction of logs invariably went on to obtain both solutions correctly.

Answer: 3, 0.6

Question 8

(i) Candidates should be aware that full working should be shown to arrive at the given answer. Most candidates used the product rule. Those who realised that $x\ln x^3$ could be written as $3x\ln x$ found the product rule and subsequent manipulation to the final answer to be straightforward. Otherwise candidates had difficulty with the differentiation of $\ln x^3$ and could not proceed further.

(ii) Even if unsuccessful in part **(i)**, candidates were directed by the word 'hence' to use the given result in part **(ii)**. They should be aware that integrating the result from part **(i)** would give $x\ln x^3$ using the principle that integration is the reverse of differentiation. No further knowledge of integration was required.

(iii) Candidates had to realise that they should use their answer to part **(ii)** in $\int \ln x \, dx = \int (1+\ln x) \, dx - \int 1 dx$ and that no integration techniques were required in this part. Candidates who succeeded in substituting limits in a correct expression did not always express their answer in the requested form and would benefit from practising manipulations of this type.

Answers: **(ii)** $x\ln x$ **(iii)** $-1 + \ln 4$

Question 9

(a) There were many successful solutions to this question but care had to be taken with the signs within the binomial expansion, particularly when obtaining *q*. Most candidates realised that *r* would come from ${}^4C_2 5^2 (3x)^2$ but it was common for candidate not to square the 3 or to introduce a negative sign.

(b) Candidates had to realise that the fourth term of the binomial expansion was required where terms in x^9 would cancel. Care had to be taken in the expansion and evaluation of ${}^{12}C_3 (2x)^9 \left(\frac{1}{4}x^3\right)^3$ to obtain $\frac{220 \times 512}{64}$.

Answers: **(a)** $p = 4$, $q = 3$, $r = 1350$ **(b)** 1760

Question 10

(a) Most candidates knew that expressions in terms of powers of 5 and 3 were required. Not all candidates were able to replace 81 by 3^4 but most knew that addition and subtraction of indices were required. However, candidates would benefit from practising the skills required in simplifying expressions such as $x-2(3y-2)$ as care was required to obtain correct signs.

(b) Candidates should remember that the cosine rule is given in the list of formulae and are advised to start with substitution into the form given in that list and to rearrange as a second step. Not all candidates successfully related *a*, *b*, *c* and cos*A* to the triangle in the question. Candidates would benefit from practising substituting in the cosine rule and in applying the order of operations implicit in the formula. In particular, candidates should be aware that the entire coefficient of $\cos A$, $4\left(2+\sqrt{3}\right)$, had to appear as a denominator in an expression for $\cos A$. As candidates had been asked not to use a calculator all steps had to be shown including the rationalisation of the surd in the denominator.

Answers: **(a)** 6, $\frac{5}{3}$ **(b)** $-1+\frac{\sqrt{3}}{2}$

Question 11

(i) Some candidates appeared unclear about what is meant by a stationary point. Candidates who used the product rule for differentiation, rather than expanding and then differentiating, were more likely to avoid errors, particularly if (x – 1) was spotted as a factor in the product rule expression before expansion and simplification. There was no requirement to find the *y*-coordinates, but candidates who did rarely made use of them in part **(iv)**.

(ii) Candidates are not expected to have a method of integration for products available to them in this component. The only method expected of them was to expand the product and then integrate the resulting cubic expression term by term. Many candidates were at least partially successful in this. The omission of a constant of integration was condoned on this occasion.

(iii) Candidates should be aware that the area asked for in the question is obtained using the limits 1 and –5, the *x*-coordinates of the points where the curve cuts the *x*-axis. Many candidates used the *x*-coordinates of the stationary points. As this question is 'hence' from part **(ii)**, candidates should show the application of these limits to the expression obtained in part **(ii)**.

(iv) Few candidates realised that one real solution would be obtained for values of *y* above the maximum point. A line $y = k$ drawn above this point would cut the curve once only. Candidates who associated 'one solution only' with $b^2 = 4ac$ should be aware that this method is applicable to quadratic functions and cannot be used for cubic functions.

Answers: **(i)** 1, –3 **(ii)** $\frac{x^4}{4}+x^3-\frac{9x^2}{2}+5x\ (+c)$ **(iii)** 108 **(iv)** $k > 32$

ADDITIONAL MATHEMATICS

Paper 0606/21

Paper 21

Key Messages

Candidates are reminded that when dealing with Calculus questions angular measure must always be in radians.

General Comments

There are still a number of candidates who only achieve a few marks on this paper, suggesting that they perhaps should not be entered for such a demanding examination.

Candidates should remember that they need to work to an appropriate number of significant figures in order to avoid losing marks due to inaccuracies in both their working and their final answers.

The response to questions where the answer is given must always be complete and thorough with no missing steps as any short cuts or gaps in the argument cannot be given full credit.

Comments on Specific Questions

Question 1

(a) The second Venn diagram was often shaded correctly but the first one was usually incorrect. There were a number of common mistakes in the first diagram including shading $(A \cup B) \cap C'$ or $(A \cap B) \cap C'$ or $(A \cup B) \cup C'$ rather than the required region. Several candidates used the method of labelling each area and listing the possible sets, usually leading to all the correct sections being shaded.

(b) Most candidates sketched a Venn diagram to assist with their working. Those who labelled the sections representing F only and H only as $60 - x$ and $50 - x$ respectively, often went on to complete the calculation correctly, although some forgot to include the x term which resulted in an answer which was not an integer. The most common incorrect answer seen was 42 obtained from adding 60, 50, x and $30 - 2x$ and equating to 98.

Answer: **(b)** $x = 14$

Question 2

Most candidates expanded and collected the like terms with many achieving a correct two term expression. A number changed < to > when taking terms from the left hand side to the right hand side. Very few were able to successfully complete the final step and various incorrect final inequalities were seen with the most common being $x < \frac{1}{2}$ only or $x < \pm\frac{1}{2}$.

Answer: $-\frac{1}{2} < x < \frac{1}{2}$

Question 3

Candidates who were able to form a correct pair of linear equations from the given log equations usually solved them well. Errors in forming these equations were numerous. The first equation often became $x + 3 = y + 4$ or $x + 3 = y + 2$. The second equation was often transformed to either $x + y = 3$ or $x + y = 9$ (the 9 being 3^2 rather than 2^3). Candidates were able to score a method mark for attempting to solve their linear equations but no such credit was available for those who managed to manufacture a quadratic equation. Some assumed the base was 10 and equations with terms of 100 and 1000 appeared. Despite the large variety of mistakes there were, however, many completely correct answers.

Answers: $x = 5.8$, $y = 2.2$.

Question 4

(i) Most had little difficulty in finding f(37) and subsequently gf(37). Some attempted to find gf(x) before inserting 37 and this often resulted in an error due to omission of a bracket in the denominator. A common misunderstanding was to find f(37) and then multiply this by g(x). Very few candidates tried to find fg(x) rather than gf(x).

(ii) There were many successful attempts from candidates who rearranged first and then squared both sides.There were some poor attempts at squaring generally done prior to making a rearrangement. Some weaker candidates did not realise that there was a need to square and just transferred the square root sign to the other part of their function.

(iii) Most candidates rearranged and expanded $y(2x - 3)$ to get $2xy - 3y$ and subsequently collected like terms and proceeded to the correct answer. Others, having expanded correctly, were unsure how to proceed and either made just one x term the subject or returned to the original function.

Answers: **(i)** $\frac{1}{3}$ **(ii)** $f^{-1}(x) = (x + 3)^2 + 1$ **(iii)** $g^{-1}(x) = \frac{3x-2}{2x-1}$

Question 5

(i) A number of candidates did not realise that they had to insert $t = 0$ into the given function to find the initial number present. A common error was to substitute $t = 1$ and some even used $\frac{dB}{dt}$. Of those who used $t = 0$ a number arrived at the result of 500 or 501 rather than 900.

(ii) Many candidates omitted to give the answer as an integer.

(iii) A large number of candidates did not attempt this part. Of those who did attempt it, many did not find $\frac{dB}{dt}$. The majority of those who had found $\frac{dB}{dt}$ went on to find the correct answer but then subsequently multiplied it by 10. Weaker candidates often subtracted their answers to parts **(i)** and **(ii)** and divided this by 10.

(iv) More candidates were successful in this part than they were in the previous three parts. However, many candidates recognising that they had to take logs did so too early and wrote $\log 10000 = \log 500 + \log 400e^{0.2t}$. Even some of those who correctly got as far as $e^{0.2t} = 23.75$ subsequently went on to state that $t = \frac{\log 23.75}{\log 0.2}$.

Answers: **(i)** 900 **(ii)** 3455 **(iii)** 591 **(iv)** 15.8

Question 6

(i) Many candidates demonstrated their algebraic expertise and earned full credit with solutions which were clearly set out and easy to follow. Some candidates knew what to do but made algebraic slips whilst factorising resulting in incorrect values of both x and y.

(ii) Most candidates substituted $y = mx + 5$ into $x^2 + y^2 = 10$ but there were frequent mistakes in the algebra with many forgetting to square m. Most were aware of the need to equate the discriminant to zero but many did not appreciate the need to state both positive and negative **exact** values.

Answers: **(i)** (1, 3), (–3, –1) **(ii)** $m = \pm\sqrt{\frac{3}{2}}$

Question 7

(i) Many candidates did not appreciate what was required and used $v = \frac{s}{t}$ or even integrated. Of those who differentiated, many lost the 1 or got the sign of the trig function incorrect.

(ii) Most were aware that they needed to equate v to zero but the vast majority gave answers in degrees. Those who did answer in radians rarely gave an exact answer in terms of π.

(iii) Many were able to correctly find the second differential but those working in degrees were not able to score any further marks.

Answers: **(i)** $v = 1 + \cos t$ **(ii)** $t = \frac{2\pi}{3}$ **(iii)** $x = 3.83$ m, $a = -\sqrt{3}$ ms^{-2}

Question 8

(i) This was answered well by the majority. Most candidates knew how to use the quotient rule and did so effectively. Common errors were to omit brackets in the numerator, reverse the order of terms in the numerator or to make mistakes in simplification. A small number of candidates used the product rule and it proved more difficult to simplify the two terms to reach the final expression.

(ii) This proved challenging for many candidates including those who were awarded full marks in part **(i)** as they did not see the link with that part. Many did a lot of superfluous work which led nowhere as they had not appreciated the nature of the question.

Answers: **(i)** $k = 4$ **(ii)** $\frac{1}{4}\times\frac{x^2}{2+x^2}\ (+c)$

Question 9

Many candidates were able to correctly expand $(a+3\sqrt{5})^2$ to be awarded the first mark. However, the majority of candidates did not subsequently try to equate coefficients of the rational and irrational terms and many solutions came to an abrupt halt at this point. Others did continue to try to solve their quadratic equation by using the formula and got lost in a morass of algebra. Some equated the discriminant to zero. Occasionally candidates gained a further mark by spotting a correct pair of values.

Answers: $a = 2$ and $b = 12$, $a = -3$ and $b = -18$.

Question10

(i) A good proportion of candidates correctly changed the terms involving sec *x*, cosec *x* and cot *x* into terms of $\sin x$ and $\cos x$. Subsequent work was sometimes impressive but a number could not cope with adding the fractions and then using the required identity. A common error was to write $\frac{1}{\cos x}\times\frac{1}{\sin x}$ as $\frac{\sin x\cos x}{\cos x\sin x}$ before adding and it was not uncommon for candidates to try to use incorrect relationships such as $\sec x = 1 + \tan x$. The fact that the answer was given led to a number of incomplete solutions in which a complicated expression was suddenly equated to $\tan x$.

(ii) The intention of the question was to use the identity from part **(i)** to produce an equation in $\tan x$ and $\cot x$. However, many disregarded part **(i)**, in spite of the instruction in the question, and worked from the given equation changing all terms into $\sin x$ and $\cos x$ once more. This was sometimes a success but many obtained just two solutions as they neglected the negative root.

Answers: **(ii)** 54.7°, 125.3°, 234.7°, 305.3°

Question 11

(i) There were many good attempts at this part. Those who found *OR* and *SR* usually found the correct area of the triangle *ORS*. There was some confusion when candidates did not seem to be clear whether they were using $\frac{1}{2}$base × height or $\frac{1}{2}ab\sin C$ often resulting in a combination of both. Some preferred to work in degrees rather than radians but often lost required accuracy when changing to degrees from radians. A mixture sometimes occurred with 0.8 radians and 90° seen in the sine rule.

(ii) It was common to see correct calculations for the arc length *PQ* and many candidates used the answer given in part **(i)** to find *SQ* and *PR* and the correct final answer. A common mistake was to assume that *PR* equalled *SQ* and on some occasions *SR* was missing.

(ii) The vast majority used their triangle area from part **(i)** and calculated the sector area *OPQ* before subtracting. Very few simply stated that the desired area was four times the area of the triangle *ORS*.

Answers: **(ii)** 19.85 cm **(iii)** 25.0 cm^2

Question 12.

(i) Most candidates substituted $x = 2$ correctly in the function and obtained zero. On a few occasions $x = –2$ was used and long division was also seen on a few scripts.

(ii) Long division was more common here and was invariably successful. There were a number of algebraic and sign errors but most candidates managed to factorise the resulting quadratic successfully. A number who had obtained the three correct factors did not show all three multiplied together. Those who used the formula on the quadratic often gave the solutions of f(x) = 0 and did not return to show the function as a product of three factors.

(iii) Many candidates did not realise the link with part **(ii)** and proceeded to attempt to solve an equation once again in what was a very small answer space. A number who had not attempted part **(ii)** got the correct *x* values here.

(iv) The integration was correctly done by many although there were frequent errors in the last term with $\frac{1}{32x}$ and $+\frac{32}{x}$ being the most common. Another common error was to multiply through by x^2 before attempting the integration.

Answers: **(ii)** $(x – 2)(x – 4)(3x + 4)$ **(iii)** x = 2 and 4 **(iv)** 2

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key Messages

Candidates are reminded that when dealing with Calculus questions angular measure must always be in radians.

General Comments

There are still a number of candidates who only achieve a few marks on this paper, suggesting that they perhaps should not be entered for such a demanding examination.

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The response to questions where the answer is given must always be complete and thorough with no missing steps as any short cuts or gaps in the argument cannot be given full credit.

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Answer: **(b)** $x = 14$

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Answer: $-\frac{1}{2} < x < \frac{1}{2}$

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Answers: **(i)** $k = 4$ **(ii)** $\frac{1}{4} \times \frac{x^2}{2 + x^2}$ $(+c)$

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Many candidates were able to correctly expand $(a + 3\sqrt{5})^2$ to be awarded the first mark. However, the majority of candidates did not subsequently try to equate coefficients of the rational and irrational terms and many solutions came to an abrupt halt at this point. Others did continue to try to solve their quadratic equation by using the formula and got lost in a morass of algebra. Some equated the discriminant to zero. Occasionally candidates gained a further mark by spotting a correct pair of values.

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(ii) The intention of the question was to use the identity from part **(i)** to produce an equation in $\tan x$ and $\cot x$. However, many disregarded part **(i)**, in spite of the instruction in the question, and worked from the given equation changing all terms into $\sin x$ and $\cos x$ once more. This was sometimes a success but many obtained just two solutions as they neglected the negative root.

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Answers: **(ii)** 19.85 cm **(iii)** 25.0 cm^2

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(ii) Long division was more common here and was invariably successful. There were a number of algebraic and sign errors but most candidates managed to factorise the resulting quadratic successfully. A number who had obtained the three correct factors did not show all three multiplied together. Those who used the formula on the quadratic often gave the solutions of $f(x) = 0$ and did not return to show the function as a product of three factors.

(iii) Many candidates did not realise the link with part **(ii)** and proceeded to attempt to solve an equation once again in what was a very small answer space. A number who had not attempted part **(ii)** got the correct *x* values here.

(iv) The integration was correctly done by many although there were frequent errors in the last term with $\frac{1}{32x}$ and $+\frac{32}{x}$ being the most common. Another common error was to multiply through by x^2 before attempting the integration.

Answers: **(ii)** $(x – 2)(x – 4)(3x + 4)$ **(iii)** $x = 2$ and 4 **(iv)** 2

ADDITIONAL MATHEMATICS

Paper 0606/23

Paper 23

Key Messages

The work leading to an answer should always be shown so that marks for method can be awarded, even if the answer is incorrect. In particular, method marks cannot be given for solving an incorrect equation when the solutions to the equation are taken directly from a calculator, without showing any working.

Candidates need to be aware of the importance of the word "hence" in a question, which indicates that, in answering it, they must use what they have been instructed to find in the previous part of the question.

In "show that..." or "prove that..." questions, every step leading to the given answer must be clearly shown.

Candidates must avoid premature rounding of intermediate values in a structured calculation in order to obtain a final answer correct to three significant figures.

General comments

There was the usual wide variation in the quality of work seen. Some good marks were obtained by candidates who presented clear answers, displaying an impressive range of knowledge and skills.

It might seem superfluous to remark that the instructions in a question should be followed precisely if full credit is to be obtained. However, some candidates seem to proceed making their own assumptions of what they have to do, rather than reading instructions carefully (see **Question 1** below).

One of the general instructions on the front of the question paper says that non-exact numerical answers are to be given correct to 3 significant figures. If premature rounding is applied with any intermediate values (see **Question 4** below) this will not be achieved.

Fewer problems were encountered this year with first attempts at solutions overwritten with second attempts, something which can make the work presented unclear.

Comments on specific questions

Question 1

Good marks were obtained by candidates who followed the instructions of the question, using the given factor to find the quadratic factor in part **(i)**, then using the quadratic factor as directed by the instruction "hence..." in part **(ii)** to solve the given equation. A serious limitation in some attempts, at the outset, was the omission of setting f(2) = 0 in seeking to show that p = 10. Others set about 'solving' in part **(i)**, when at that stage of the question there was in fact nothing to solve. Some of these candidates had apparently either not read the instruction about a quadratic factor, or ignored it, for often in such attempts a quadratic factor was completely absent from their answer. Credit could not be given for the solutions themselves where they had been obtained directly from a calculator, and the specific instructions in the question ignored.

Answers: **(i)** $f(x) = (x - 2)(3x^2 + 14x - 5)$ **(ii)** $2, -5, \frac{1}{3}$

Question 2

There were many fully correct answers to parts **(i)** and **(ii)**, but few to part **(iii)**. Those achieving success in part **(iii)** partitioned the people into groups: Ken and the other 6 men, and Betty and the other 4 women. Their combinations formulas thus involved choices from 6 and 4, rather than, incorrectly, from 7 and 5.

Answers: **(i)** 495 **(ii)** 210 **(iii)** 96

Question 3

A good number of fully correct answers to this question were seen. Even when D was found incorrectly in part **(i)**, candidates showed good understanding of what to do in parts **(ii)** and **(iii)**. Otherwise excellent answers to part **(ii)** sometimes lost a mark due to the proof not being properly finished, the answer presented stopping at the point where the values of the gradients of the two lines had been found.

Answers: **(i)** $C(1, 6)$, $D(13, 15)$ **(iii)** 75 square units

Question 4

There were mixed answers to parts **(i)** and **(ii)**, with some candidates giving good clear answers in few steps, whilst others employed longer, invalid, manipulations of exponentials and logs. In part **(iv)**, some otherwise good work was spoiled by premature rounding in part **(iii)**. Candidates need to be reminded of the "3 significant figures" instruction on the front of the question paper, and that, for this accuracy to be achieved, any intermediate values used in the calculation of a final answer need to have at least three (and preferably more) significant figures. Some candidates even used a value for b from part **(iii)** correct to only one significant figure in part **(iv)**, leading to a needless mark loss.

Answers: **(ii)** $2a + b = \ln 3.297$ **(iii)** $a = 0.500$, $b = 0.193$ **(iv)** \$180 000

Question 5

Parts **(i)** and **(ii)** were reasonably well done, but part **(iii)** much less so. Good answers to part **(iii)** set the vector expressions found in parts **(i)** and **(ii)** equal to each other then, by equating coefficients of **a** and **b**, obtained two simultaneous equations in μ and λ. Weaker attempts showed little awareness of the method of equating coefficients, and often even indicated lack of awareness of the fact that, whilst **a** and **b** were vectors, μ and λ were scalars. Thus **a** and **b** were often treated as ordinary algebraic variables, sometimes with the consequence that expressions were presented showing the meaningless mathematical operation of the division of one vector by another. Candidates would be well advised in vector work to always use one of the conventions for writing vectors, to distinguish them clearly from scalar quantities.

Answers: **(i)** $\mu(\mathbf{a} + \mathbf{b})$ **(ii)** $3\mathbf{a} + \lambda(\mathbf{b} - 3\mathbf{a})$ **(iii)** 3:1

Question 6

Some good attempts at this question were seen, and a fair number of fully correct answers were obtained. Those showing good understanding of the given graph clearly appreciated that the given coordinates referred to values of 3^x and $\ln y$, not x and y. Those not recognising this obtained few marks. A common error seen in parts **(ii)** and **(iii)**, when the correct value of m had been found, was to replace $(4)(3^x)$ with 12^x.

Answers: **(i)** $\ln y = 4\left(3^x\right) + 3$ **(ii)** 20 500 **(iii)** 0.127

Question 7

This question was done exceptionally well, with many candidates obtaining nine of the ten marks available. In part **(i)** the procedure for finding the inverse of a function was generally well understood. In parts **(ii)** and **(iii)** there was hardly any confusion between gf and fg. In part **(iv)** the steps in the proof were usually clearly set out and straightforward for Examiners to follow. However, it was in this part that a mark was almost always lost as a result of two solutions being presented for the given equation.

Answers: **(i)** $\dfrac{2}{x-1}$ **(ii)** $\left(\dfrac{2}{x}+1\right)^2 + 2$ **(iii)** $\dfrac{2}{x^2+2}+1$ **(iv)** $x = 2$

Question 8

Overall, this was one of the least well answered questions. Whilst many candidates started satisfactorily enough in part **(i)**, others made sign or coefficient errors when carrying out the differentiations. Even more seriously, some made the false assumption that the angle in the expression for displacement was $(2t + 3)$, not the $2t$ given. Some, in attempting part **(ii)**, inexplicably set the velocity, rather than the acceleration, equal to zero. Answers to part **(iii)** were usually better than those to part **(ii)**, but a serious error very commonly made in both of these parts was to present values for t in degrees, rather than radians.

Answers: **(i)** $v = 5 + 6\sin 2t$, $a = 12\cos 2t$ **(ii)** 11 ms^{-1} at $t = \dfrac{\pi}{4}$ **(iii)** $t = \dfrac{7\pi}{12}$, $a = -6\sqrt{3}$ ms^{-2}

Question 9

In part **(i)**, candidates generally displayed good knowledge of how to find the turning points on a curve. Many very good solutions were seen, with the steps for locating the points clearly set out, then the second derivative test applied for determining their nature. Some, whilst still presenting correct mathematics, made things harder for themselves by first combining the two terms of the equation of the curve into one, then using the quotient rule for differentiation. This approach resulted in needlessly extensive, time consuming, algebraic work, especially when finding the second derivative later. Those who worked with the equation of the curve in its given form accomplished what had to be done far more economically.

Many clearly presented solutions to part **(ii)** were also seen, although generally this was less well answered than part **(i)**. The main difficulty was that some candidates did not know how to find the equation of the normal to the curve at the given point. They seemed to know that perpendicularity was involved somehow, but the line that the normal was perpendicular to was produced quite arbitrarily, often a chord of the curve, rather than the tangent at the specific point.

Answers: **(i)** (1.5, 4) maximum, (2.5, 12) minimum **(ii)** $\dfrac{29}{13}$

Question 10

The proof in part **(i)**, was very well done, almost always by combining the two terms on the left hand side, then using a Pythagorean identity. Good solutions to part **(ii)** used the result from part **(i)** to find a value for $\sin^2 x$, then for $\sin x$. It was at this point that many errors were seen, for in finding the square root it was often ignored that $\sin x$ could be equal to -0.5 as well as $+0.5$. Thus two solutions to the given equation, and corresponding marks, were lost.

Answer: **(ii)** 30°, 150°, 210°, 330°